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Middle School Students' Conceptions of the Usefulness of Mathematics: A Sociocultural Approach to the Study of Utility Value

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Abstract

The purpose of this research is to apply a sociocultural lens to examine students' perceptions of usefulness in mathematics. It is important to explore perceptions of usefulness, as teachers are increasingly being encouraged to help their students view mathematics as useful. However, we know little about how students – and in particular, students in underserved communities – think about what it means for mathematics to be useful. This study addressed four main goals: (a) illuminate students' conceptions of usefulness in mathematics, (b) identify key features of students' engagement with mathematics in meaningful everyday activities, (c) highlight the ways in which students' conceptions of usefulness and engagement with mathematics in everyday activities can inform the design of problem-solving tasks that students will view as useful, and (d) explore the relationship between students' perceptions of usefulness and their engagement in mathematical problem solving.

Participants in this study were 108 sixth-and seventh-grade students, 75% of whom identify as Latin@, from a working class suburb of a large Midwestern city. Questionnaires were administered and interviews were conducted to explore students' goals, values, and conceptions of usefulness. Additionally, ethnographic observations and semi-structured interviews were used to examine similarities and differences between students' engagement with mathematics in the classroom and in everyday activities involving mathematics. Finally, the data collected were used to design two problem-solving tasks. Students' engagement on these tasks was then compared to their engagement with "typical" mathematics curriculum tasks. Mixed methods were employed for analyses, and triangulation across methods and data sources was used whenever possible.

Findings highlight that the students thought about usefulness not only in terms of how mathematics content can be applied to their future jobs/careers, everyday life, and specific money-related activities, but also in terms of how particular methods or strategies for learning mathematics can be useful (e.g. working collaboratively). Additionally, findings illustrate that the students had strong interdependent values and emphasized the importance of working with others and helping their families when participating in everyday activities involving mathematics. Design principles for developing problem-solving tasks that students view as useful are proposed, and affordances and constraints of those principles are considered. This study contributes to our understanding of the ways students conceive of usefulness, as well as the wide range of influences on middle school students' conceptions and perceptions of the usefulness of mathematics.

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1. Introduction

The purpose of this research is to apply a sociocultural lens to the study of perceived usefulness, or utility[1], in mathematics. Applying a sociocultural lens entails beginning with an exploration of students' own values, experiences, and ways of thinking about usefulness before engaging in the design of learning environments. Additionally, this research takes an ecological perspective, considering students' engagement with mathematics both within and across settings to provide an even stronger foundation for designing learning environments that are built around the experiences and values of the learners.

Usefulness is an important domain to study in mathematics, as teachers are increasingly being encouraged to help their students view mathematics as useful (National Council of Teachers of Mathematics, 2000; National Governors Association Center for Best Practices, 2010). However, we know little about what it would look like through students' eyes for mathematics to seem useful. Is it useful for students to hear about how mathematics can help them to split bills with friends at restaurants? Or do students find it useful to learn about the mathematics engineers use in their jobs? This research examines not only the ways in which students think about usefulness but also how students' conceptions of usefulness align with their own goals, values, and experiences.

Existing research examining usefulness in education has primarily focused on students' *perceptions* of usefulness, or whether students think subjects or tasks are useful, but not the criteria they use to make judgments about usefulness. Such studies typically utilize rating scales

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[1] Throughout this paper, the words usefulness and utility will be used interchangeably.

to measure students' perceptions of usefulness and identify correlates and outcomes of high perceived usefulness. Some researchers also create interventions intended to enhance perceptions of usefulness by either showing/telling students how a topic or technique can be useful, or by allowing students to reflect on its usefulness. While all of this work is certainly important, it typically does not consider what "useful" means to students, or what types of usefulness students find most relevant. Additionally, little attention is paid to the personal values and prior experiences of students. In this research, I apply a sociocultural lens to the study of utility to account for students' own perspectives and illuminate the various ways in which middle school students think about usefulness in mathematics. Furthermore, I introduce new qualitative methodologies to this body of work to examine multiple ways of eliciting students' conceptions of usefulness.

In addition to exploring middle school students' conceptions of usefulness, I examine the ways in which students engage with mathematics in their mathematics classrooms versus in everyday activities. Through semi-structured interviews, I explore defining features of students' engagement with useful mathematics. I then compare those features of engagement with the ways students engage with mathematics in their classrooms to examine similarities and potential disconnects. Finally, I use findings regarding students' conceptions of usefulness and features of their engagement with useful mathematics to consider principles for designing problem-solving tasks that students perceive as useful. I conclude by describing the design of two problem-solving tasks I developed, as well as findings regarding students' engagement with and perceptions of usefulness of the tasks.

In this research, I examine the following questions:

RQ1. a) What are students' perceptions of usefulness, and what is the relationship between those perceptions and students' interest and performance? *(Chapter 3)*

b) How do students conceptualize usefulness in mathematics, and in what ways are those conceptions similar to or different from existing conceptions of utility? *(Chapter 3)*

RQ2. a) How do students engage with mathematics in everyday activities, and what are key features of their engagement with mathematics in those settings? *(Chapter 4)*

b) How do the features of mathematics learning in the students' classrooms compare with the features of their engagement with mathematics in everyday activities? *(Chapter 4)*

RQ3. a) How can an understanding of students' conceptions of usefulness (RQ1) and engagement with mathematics both in and out of the classroom (RQ2) be used to design problem-solving tasks that students perceive as useful? *(Chapter 5)*

b) What is the relationship between students' perceptions of utility of a mathematics task and their interest in, engagement with, and performance on the task? *(Chapter 5)*

To explore these questions, I draw on a corpus of data from research with four classes of seventh-grade students and one class of sixth-grade students who live in a working-class suburb of a large Midwestern city and many of whom identify as Mexican, Mexican-American, or Chican@. The students' teacher, Ms. Sanchez, expressed a desire to help her students connect with mathematics and find meaning in the mathematics they were learning. She also reported her students frequently questioning her about the usefulness of mathematics. Therefore, this is an apt setting for examining students' conceptions of usefulness and considering ways of drawing on students' ideas about usefulness to design meaningful problem-solving tasks. In the sections that follow, I first reflect on why it is worthwhile to study usefulness. Then I describe the overall study design and ways in which it aligns with these reasons for studying usefulness. I conclude the chapter by providing a description of the local context where data collection takes place, as well as the participants involved in this research.

Why Study Usefulness?

Before describing the design for this study, it is important to first establish why usefulness is worth studying at all. I will briefly discuss three main reasons – benefits of having high perceptions of usefulness, emphases on usefulness in mathematics education practice, and potential areas of exploration that have not yet been examined. Subsequently, I will add a brief caveat to address potential concerns.

First, prior work examining usefulness in education has established that having high perceptions of usefulness in mathematics can contribute to numerous positive outcomes. In particular, research has highlighted the relationship between high perceptions of utility and improved academic performance (Bong, 2001b; Durik, Vida, & Eccles, 2005; Hulleman, Durik, Schweigert, & Harackiewicz, 2008; Mac Iver, 1991; Simons, Dewitte, & Lens, 2004), greater enrollment in mathematics courses (Durik et al., 2005; Updegraff, Eccles, Barber, & O'brien, 1996), and enhanced interest (Harackiewicz, Durik, Barron, Linnenbrink-Garcia, & Tauer, 2008; Hulleman et al., 2008). Several studies have also illustrated the ability to manipulate students' perceived utility value and for enhanced perceptions of utility to increase interest and course performance (Canning & Harackiewicz, 2015; Harackiewicz, Rozek, Hulleman, & Hyde, 2012; Hulleman, Godes, Hendricks, & Harackiewicz, 2010; Hulleman & Harackiewicz, 2009). Further, some of these studies have found that enhanced perceptions of utility were particularly advantageous for low performers in mathematics (Hulleman et al., 2010; Hulleman & Harackiewicz, 2009). Given these potential benefits for academic performance, persistence, and interest, a greater exploration of how to improve perceptions of utility in the mathematics classroom is worthwhile.

Second, it is important to study perceptions of usefulness in mathematics specifically because of the current emphasis on usefulness in mathematics education practice. In 2000, the National Council of Teachers of Mathematics stated in the Principles and Standards for School Mathematics that a mathematics curriculum "should offer experiences that allow students to see that mathematics has powerful uses in modeling and predicting real-world phenomena" (p. 15-16). Since then, the Common Core State Standards in Mathematics (CCSSM; National Governors Association Center for Best Practices, 2010) has continued this push for students to perceive mathematics as useful. CCSSM draws on the *Adding It Up* report (Kilpatrick, Swafford & Findell, 2001) to state that teachers should help students develop a *productive disposition*, which includes a "habitual inclination to see mathematics as sensible, useful, and worthwhile" (p.5). Furthermore, CCSSM's fourth mathematical practice states, "Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace." If we are to help students view mathematics as useful and learn to apply mathematics to everyday life, it would be beneficial to further explore what uses of mathematics are especially meaningful to students and how best to enhance students' perceptions of the usefulness of mathematics.

Third, this work aims to contribute to several underexplored spaces in research on usefulness, or utility value, that might provide helpful information for optimizing mathematics instruction. While such research frequently focuses on *perceptions* of utility, or *whether* students think mathematics is useful, this work seeks to better understand students' *conceptions* of usefulness, or *how* individuals think about what it means for something to be useful. Additionally, existing work primarily involves white, middle class high school or college students. Little research has explored utility value in K-8 school settings or with students of different ethnicities. Expanding to a younger population is important because of the current focus on usefulness in K-8 mathematics education, and findings will enable us to consider differences between adults' and adolescents' conceptions of utility. Furthermore, moving beyond white, middle class populations is also crucial in order to capture potential variation in ways of thinking about usefulness. This expansion is particularly important because educational reforms disproportionately target groups of students who are already experiencing success and fail to consider the needs of students of color and low-income students who may not be experiencing success (Berry, Ellis, & Hughes, 2013). Thus, this work aims to serve those populations of students and allow new voices and perspectives to enter discussions of the usefulness of mathematics.

A final underexplored area in utility research is methodological. While existing studies primarily rely on quantitative analyses of rating scales, this work adds a qualitative analysis of students' talk about utility. Including qualitative analyses of students' discussions of usefulness will offer a richer picture of students' conceptions of utility and a more nuanced understanding of what it means for a student to view mathematics as useful or not useful. Furthermore, techniques that have been used in high school and university settings might be less successful with middle school students. Thus, this work includes new forms of data collection to explore what techniques are most effective and equitable for utility research with middle school students.

Finally, despite presenting these reasons for studying usefulness in mathematics, I do not intend to claim that all mathematics can, or should, be viewed as useful by students. In some cases, an emphasis on the usefulness of mathematics can come at the expense of students' socialization into and deep engagement with mathematical practices (Dowling, 2002). Thus, rather than trying to highlight "real world" uses of all types of mathematics, I suggest that we begin by examining the types of utility on which students place the greatest importance. From students' conceptions of usefulness, we can then identify mathematics content that is most conducive to promoting perceptions of utility. Certainly, there are mathematics topics that students are required to learn that will not be useful in students' lives outside the mathematics classroom. For such topics, I do not propose falsely claiming their usefulness. In those cases, other points of leverage should be used to help engage students with the material. However, given middle school students' emphasis on usefulness, as well as the potential relevance of middle school mathematics content to students' lives, I argue that connections between classroom mathematics and their real world uses should be highlighted when relevant, appropriate, and aligned with the types of uses that students value.

Finally, throughout this research, I wish expand the range of ways we think about what it means for mathematics to be useful. Mathematics content need not be based in out-of-school practices to be considered useful; rather, a student might find the elegance of a particular problem-solving method to be useful. Furthermore, even within a given classroom, students might conceive of usefulness in different ways. Thus, this research seeks to both broaden the literature's conception of usefulness and enhance our understanding of what it might mean *to students* to see mathematics as useful.

Study Design

The approach I take in this work is in line with Bronfenbrenner's (1977) call for ecologically valid experiments, which highlights the importance of considering sociocultural context and human development when engaging in study design. This research is grounded in literature that takes a sociocultural and developmental perspective, and participants were selected with that framing in mind. For example, sixth- and seventh-grades were chosen as the focal grades due to students' desire to see mathematics as useful yet tendency to experience declining perceptions of usefulness during the middle school years. Additionally, student demographics were considered in the selection of the study site, as one goal of this work was to explore the perspectives of students whose views and values are often not represented in utility value research.

Working from a sociocultural and developmental perspective, I started this research with an exploration of students' goals, values, participation in cultural practices, and current engagement with mathematics before attempting to design tasks that promote perceptions of usefulness. As with Saxe's (1994) work with Brazilian candy sellers, only once we understand children's and adolescents' goals, interactions, and prior experiences can we engage in thoughtful and authentic design. Thus, this research began with interviews, surveys, and observations of students in their mathematics classrooms to explore, among other things, students' conceptions of usefulness and experiences with mathematics in the classroom. In order to capture a range of student perspectives and allow new conceptions of usefulness to emerge, it was important to introduce new methodologies to this body of research. Open-ended response questions and interview tasks, including a card-sorting task and video response task, were incorporated into this research to place students' perspectives at the center and enable students to express their views about utility in different ways. For example, in a video response task, students observed and commented on moments of useful and not useful instruction in videos of classroom mathematics, which provided a new pathway for students to express their conceptions of usefulness. A discussion of students' survey and interview responses, with particular emphasis on conceptions of usefulness, is the focus of Chapter 3.

Subsequently, survey results were used to identify activities students participated in that they perceived as involving mathematics. These activities then became the focus of semi-structured interviews that explored students' engagement with mathematics in everyday activities. Protocols for the interviews were designed to explore the mathematical practices and features of engagement, including goals and supports, that emerged as key components of students' participation in these activities. In Chapter 4, I discuss the themes that emerged regarding students' engagement with mathematics outside the classroom and compare those themes with findings regarding students' engagement with classroom mathematics.

Finally, having gained an understanding of students' values, ideas about usefulness, and engagement with mathematics in and out of the classroom, I was then able to engage with task design in a meaningful, authentic way. Findings from the first two phases of this work were used to design two problem-solving tasks that connected with students' values and experiences and that were intended to promote perceptions of usefulness among students. Those two tasks were administered to students along with four typical curriculum tasks, and both students' perceptions of the usefulness the tasks and students' engagement with the tasks were measured. In Chapter 5, I discuss the design process, as well as the findings that emerged regarding students' perceptions of and interactions with the tasks.

Local Context and Participants

Before turning to discuss the conceptual framework that guides this research, it is important to have a sense of the context in which this work took place. Below I describe the school, students, and teacher who participated in this research.

School

The focal school, Legacy Middle School[2], is located in the suburbs of a large Midwestern city in the United States. The school contains approximately 600 students spread across grades six through eight. According to the state report card, nearly 75% of students at the school are considered low-income, as defined by eligibility to receive free or reduced-price lunch, living in substitute care, or belonging to families that receive public aid. Just below 10% of the students have been identified as English Language Learners, while nearly 13% of the students have Individualized Education Programs. Additional information about the subset of students who are involved in this research is provided below.

Students

Since data was collected over the course of two school years, and the participating teacher had new students each year, there were two groups of participants in this study. The first group included seventh-grade students who participated during the 2014-2015 school year, and

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[2] The name of the school, as well as the names of any students or teachers referenced by name, are pseudonyms.

the second group included sixth-grade students who participated during the 2015-2016 school year. Sixth and seventh grades were chosen as the focal grades due to the potential real world applicability of the mathematics content at these grade levels and the difficulty many students face in mathematics after transitioning to middle school, including decreased perceptions of utility (e.g. Midgley, Feldlaufer, & Eccles, 1989).

The first group of participants was drawn from four classes of seventh-grade students (*N*=84, mean age = 12.7 years). Participating students were asked to describe their race or ethnicity using as many identifications as they felt appropriate. Of the 84 students, 76% identified as Mexican, Mexican-American, or Chican@; 18% identified as Puerto Rican; 18% identified as White/European-American; 10% identified as African American; 5% identified as Other Hispanic; and 8% provided other identifications. Of the 83 students who completed a question about gender identification, 47 identified as male, and 36 identified as female. Students were also asked to report the languages they felt most comfortable speaking. Eighty of 83 students selected English, and 32 selected Spanish; only two students listed both English and a language other than Spanish.

In addition to drawing on schoolwide data regarding socioeconomic status (SES), students in the study were asked about their housing accommodations as a proxy for SES. Students reported whether they lived in an apartment or house and whether their parents owned the apartment or house. Nineteen students reported living in an apartment, while 64 reported living in a house. Fifty-two students reported their parents owning the apartment/house, while 24 said their parents did not own the apartment/house, and six didn’t know. To provide some context for these reports, in the school district of which LMS is a part (which contains LMS and one other middle school), 66% of the housing units are one-unit structures, 13% are 2-unit structures, 6% are 3-4 unit structures, and the remainder include 5 or more units (United States Census Bureau / American FactFinder, 2014). Additionally, approximately 69% of residences are owned, while 31% are rented.

The second group of participants consisted of one class of sixth-grade students (*N*=28, mean age = approximately 11.25 years[3]). Participating students were again asked to describe their race or ethnicity using as many identifications as they felt appropriate. Of the 24 students who reported demographic information, 88% identified as Mexican, Mexican-American, or Chican@, 29% identified as White/European-American, 8% identified as Puerto Rican, and 8% identified as Other Hispanic/Latino. Seventeen of the students identified as female, while seven identified as male. Of the 24 students, 100% reported English as one of the languages they felt most comfortable speaking, while 46% also felt comfortable speaking Spanish. Eighteen students reported living in a house and six in an apartment, while 17 students reported that their parents/guardians owned the residence and seven stated that their parents/guardians did not own their place of residence.

Teacher

All five participating classes were taught by the same teacher, Justine Sanchez, who was a first year teacher during the 2014-2015 school year. During that year, she taught four seventh-grade mathematics classes and one English class. At the start of the 2015-2016 school year, Justine was moved to sixth-grade and asked to teach only mathematics. At the same time, the

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[3] Only 24 of the 28 students reported demographic information, such as age and ethnicity, as four of the students joined the class later in the year.

school decided to implement “block scheduling,” which for them meant that mathematics classes would be approximately twice as long as they were during the prior year, and science and social studies classes would be shorter in length. As a result, Justine was assigned to teach only three mathematics classes during the 2015-2016 school year.

Justine was born in Mexico and identifies as Latina. She lives in an urban neighborhood approximately 30 minutes from Legacy, though the demographics in both neighborhoods are similar. Ms. Sanchez is somewhat unique compared to typical middle school mathematics teachers, as she studied social justice mathematics education in her teacher preparation program and discussed wanting to incorporate themes of social justice into her lessons at Legacy. Justine responded to an email inquiring about her interest in participating in this research because she was eager to find ways to help her students connect with the mathematics they were learning and to see mathematics as useful for their lives.

Dissertation Overview

The remaining chapters are laid out as follows: In Chapter 2, I present a review of the literature that guides this work, focusing in particular on research that examines utility value, sociocultural context, and student engagement. In Chapter 3, I address RQ1 and examine students’ perceptions and conceptions of usefulness in mathematics. Chapter 4 focuses on RQ2 and highlights features of students’ engagement with classroom mathematics versus mathematics in everyday activities. Drawing on findings from these two chapters, I then present a rationale for designing two high utility problem-solving tasks in Chapter 5. I discuss the study that was designed to examine these tasks in use and highlight findings regarding students’ engagement with and perceptions of usefulness of the tasks. Within each of these three findings chapters, I describe the methods used to gather and analyze the data from which findings are drawn and offer discussions of key themes that emerged from the findings. Finally, in Chapter 6, I consider themes that cut across the chapters and implications of my findings, as well as limitations and future directions for this work.

2. Literature Review

In this research, I apply a sociocultural lens to the study of utility value to explore students' own conceptions of usefulness and uses of mathematics outside the classroom. I also examine students' ways of engaging with mathematics in and out of the classroom to learn about what it looks like for students to engage with useful mathematics. Findings are then used to inform the design of problem-solving tasks that students are likely to view as useful and to examine students' ways of engaging with these tasks. Given these areas of focus, this study builds on three key bodies of research. First, I examine the construct of utility value and its origins in the expectancy-value model. I highlight key themes in the utility value literature, present a framework that identifies existing definitions of utility, and describe my own conceptualization of utility for the purposes of this study. Next, I introduce the sociocultural lens I apply to utility value research and explore the ways in which that lens has been applied to study mathematics education. I present the constructs of culturally relevant mathematics instruction and funds of knowledge, both of which are important traditions on which this work builds. Finally, I unpack the construct of engagement and describe the importance of engagement as an outcome to explore in this work. I conclude by drawing attention to features of the classroom context that influence student engagement and desired forms of engagement in mathematics classrooms.

Utility Value

In this section, I present the construct of utility value. I first describe the expectancy-value model, where the construct of utility value is situated. Then I present a framework I created to understand the different ways in which utility is being conceptualized in the literature. Finally, I highlight two types of utility value interventions – directly communicated and self-generated – and point to the importance of adding a sociocultural lens to this work.

Expectancy-Value Model

As a way of understanding how values and beliefs influence one's achievement-related choices and performance, Atkinson (1964) created the expectancy-value model. Since then, Eccles and colleagues have amended and expanded this model, focusing especially on its application to the field of education (Eccles & Wigfield, 2002; Eccles et al., 1983; Wigfield & Eccles, 1992). In Eccles and Wigfield's expectancy-value model, there are a multitude of direct and indirect influences on one's achievement-related choices and academic performance. Some indirect influences are personal beliefs, such as an individual's perceptions of others' expectations for herself or her views of her own self-concept, while others are features of one's environment, such as family demographics and existing cultural stereotypes. All of these beliefs and features of the environment influence the two factors that directly affect achievement-related choices and performance: expectations of success and beliefs about the value of a task. While both of these factors are certainly important, in this paper I focus on utility value, one of the four beliefs one holds about the value of a task.

In the expectancy-value model, Eccles and colleagues consider four components of task value: interest-enjoyment value (the degree to which one finds a task interesting and fun to engage in), attainment value (the importance to the individual of doing well on the task, including importance for confirming or disconfirming aspects of one's self-concept or identity), relative cost (negative effects of engaging in the task, such as the onset of anxiety or need for intense prolonged effort), and utility value (the relevance of a task for one's current/future goals or other aspects of one's life) (Eccles & Wigfield, 2002; Eccles et al., 1983; Wigfield & Eccles, 1992). A significant body of work has collapsed these four variables into one factor and illustrated the relationship between task value overall and both interest and achievement-related choices, such as course enrollment and career choices (e.g. Anderman et al., 2001; Bong, 2001a; Jacobs, Lanza, Osgood, Eccles, & Wigfield, 2002; Wigfield et al., 1997). Fewer studies, however, have examined the role played by particular task values, such as utility value.

In research that has isolated perceptions of usefulness from the other task values, utility value has been shown to be positively associated with achievement-related choices such as course enrollment decisions (Durik et al., 2005; Updegraff et al., 1996), career aspirations (Durik et al., 2005), and interest in the subject (Hulleman et al., 2008). Additionally, research has demonstrated a positive association between utility value and performance (Bong, 2001b; Cole, Bergin, & Whittaker, 2008; Durik et al., 2005; Hulleman et al., 2008; Simons, Dewitte, & Lens, 2004). While these relationships suggest important benefits of promoting perceived utility value, one question that has not been investigated is how researchers have conceptualized utility and whether conceptions of utility are consistent across studies.

In existing research, there is considerable variation in the specificity of definitions of utility value. For example, while Harackiwicz et al. (2012) broadly define utility value as the degree to which a task is "useful and relevant for other aspects of [one's] life" (p. 1), George (2006) narrows the scope to consider usefulness "in everyday life and for future careers" (p. 572). In this research, I align with Harackewicz and colleagues' broader definition so as to allow for a range of conceptions of utility to emerge from students. Regardless of the level of specificity used by researchers, however, I argue that it is important to be explicit about the ways in which usefulness is conceptualized to better understand what is being measured and manipulated in utility value interventions. In the sections below, I present a framework I developed to categorize existing conceptions of utility in the utility value literature.

Conceptions of Utility Framework

Since most research on utility value does not attend in any significant way to the conceptions of utility that inform the work, it is crucial to shed light on this underexplored area. Particular conceptions of utility – even if never explicitly acknowledged – guide the design of all items and scales that measure or attempt to influence one's perceived utility. Thus, to better understand what exactly is being measured and manipulated in these studies, we must unpack the conceptions of utility that currently guide utility value research.

To explore these conceptions, I developed a framework by gathering items used to measure participants' perceptions of utility in 15 current and/or highly cited articles examining utility value in education, as well as the two most commonly cited utility value scales. I continued gathering items until I had reached a point of saturation and the same items began surfacing repeatedly. Then I grouped the items according to their underlying conceptions of utility and found four main categories: general utility, utility for one's job or career, utility for everyday life, and utility for school/college. Below I elaborate on the categories and provide typical examples of each. I conclude by presenting the few additional conceptions of utility that surfaced and did not fit into any of the four main categories.

General utility. The first conception of utility that arises in research on utility value is the broadest – a sense of general utility, without further specification. This conception of usefulness underlies a variety of scale items designed to measure perceived utility value (e.g.

Durik et al., 2005; Fennema & Sherman, 1976; Hulleman et al., 2008). Some studies exclusively use items that measure a sense of general utility. For example, both of the items used by Anderman et al. (2001) measure perceptions of usefulness that fall in the general utility category: "In general, how useful is what you learn in math?" and "Compared to most of your other activities, how useful is what you learn in math?" Similarly, Bong (2001b) and Durik et al. (2005) each used one item to measure perceived utility, and both asked about a general sense of usefulness. Finally, Xiang, McBride, Guan, & Solmon (2003) and Xiang, Chen, & Bruene (2005) each used two items to measure perceptions of usefulness, both of which asked students about general utility (e.g. "In general, how useful is what you learn in running/Roadrunners?").

In other studies, scales included a variety of items, at least one of which asked about a general sense of usefulness. For example, one of the four items in Parsons' (1980) Importance and Usefulness of Math scale states, "In general, how useful is what you learn in math?" Similarly, the frequently used Fennema-Sherman Usefulness of Mathematics scale (Fennema & Sherman, 1976) includes twelve items that tap into several different conceptions of utility. The most common is a sense of general utility, which is implicit in items such as "I study mathematics because I know how useful it is." Comparable items can also be found in a variety of other scales (Canning & Harackiewicz, 2015; Harackiewicz et al., 2012; Hulleman et al., 2008, 2010).

One variation that arises within this category is whether the item specifies a time scale. While all of the above items ask about general utility in the broadest sense, some items inquire about general utility for the future. For example, George (2006) asked respondents to agree or disagree with the statement, "I will use science in many ways as an adult," and Hulleman et al.

(2010) asked for agreement with the statement, "I don't think this technique would be useful to me in the future." Similarly, the Fennema-Sherman Usefulness of Mathematics scale includes several items focused on general utility for the future, such as "I will use mathematics in many ways as an adult" (Fennema & Sherman, 1976). Other work that utilizes some version of the Fennema-Sherman scale has also included a focus on general utility for the future (Chouinard, Karsenti, & Roy, 2007; Chouinard & Roy, 2008).

What is important to note here is not simply the difference in whether studies focus on general utility for the future or general utility more broadly, but the fact that researchers typically do not highlight or justify their choices to include one type of item versus the other. Rather, items are generally presented without an explanation of what kind of utility they intend to measure or why the particular items were selected for inclusion. Furthermore, potential implications of those choices are not acknowledged.

Utility for one's job or career. A second, more specific conception of utility that arises in utility value research is utility for one's future job or career (Hulleman et al., 2008; Simons et al., 2004; Updegraff et al., 1996). For example, in the same Parsons (1980) scale cited above, one item asked, "How useful do you think the math you are learning will be for what you want to do after you graduate and go to work?" while another item adapted that same question to ask specifically about the usefulness of high school mathematics for future work. Additionally, a conception of usefulness for one's career is the second most common conception underlying Fennema and Sherman's (1976) Usefulness of Mathematics scale, as four of the twelve items reference the usefulness of mathematics for one's future work. For example, two such items are

“Knowing mathematics will help me earn a living” and “I'll need mathematics for my future work.”

While items often inquire about usefulness for one’s job or career rather broadly, some items focus in on either the utility of mathematics for *getting* a job or the utility of mathematics for *performing* particular jobs. For example, both George (2006) and Battle and Wigfield (2003) included items about utility for getting jobs: “It is important to know science to get a good job” (George, 2006) and “A graduate education is important to me because it will provide better job opportunities” (Battle and Wigfield, 2003). Meanwhile, one item used by Canning and Harackiewicz (2015) asked about utility for performing one’s job: “This technique could be useful to me in my future career.” It is important to note such distinctions, as perceptions of usefulness for different purposes are being measured, yet those differences are once again not acknowledged.

Utility for everyday life. Another prevalent conception of usefulness that is implicit in measures of utility value is usefulness for everyday life. For example, George (2006) asked respondents whether they agree with the statement, “Science is useful in everyday living,” and Hulleman et al. (2010) asked for agreement with the statement, “This technique could be useful in everyday life.” Similarly, Canning and Harackiewicz (2015) included the item, “This technique could be useful to me in daily life.” Some scale items in studies of utility value also use the language of relevance to life to question participants about everyday utility: “What I am learning in this class is relevant to my life” (Hulleman et al., 2008) and “Math is of no relevance to my life” (Fennema & Sherman, 1976). As with the general utility category, study participants are sometimes – though not as frequently – asked about utility for everyday life *in the future* specifically. For example, the Fennema-Sherman scale includes the item, “I see mathematics as a subject I will rarely use in my daily life as an adult.”

Utility for school or college. A fourth prevalent way of thinking about what it might mean for a subject, topic, or skill to be useful focuses on usefulness for one’s education. Though this conception emerges less frequently than the other three, it does arise in several different measures. In one study, participants were asked to rate whether a particular course was useful for their schooling, without specification to now or in the future (Simons et al., 2004). In contrast, other studies focused specifically on usefulness for other coursework now or for one’s future education. For example, one of Hulleman et al.'s (2008) scales included the item, “I can apply what we are learning in Introductory Statistics to some of my other courses.” Meanwhile, Canning and Harackiewicz (2015) focused on whether a particular technique might be useful in one’s future schooling: “This technique could be useful to me in my future classes.”

Additional conceptions of utility. Though additional conceptions of utility rarely surfaced in the measures of perceived utility value that were examined in this research, a couple items did emerge that suggest different ways of thinking about usefulness. First, in addition to the items cited above, George (2006) included one item that does not fit into any of the above categories: “Science helps in logical thinking.” This item suggests a conception of utility for developing particular skills or competencies, which is not typically considered in utility research. Additionally, Battle and Wigfield (2003) included items that focus on usefulness for accomplishing specific goals related to money, status, career, and support for future family. Of particular interest is the support for future family category, as conceptions of usefulness related to helping others are not typically considered. Gaspard et al. (2014) did include a “social utility” category that considered one's relation to others, but items focused on the usefulness of mathematics for impressing others, rather than helping or providing for others. Although these studies highlight a few places where new conceptions of utility have surfaced, the four main conceptions of utility seem to guide the development of most utility value scales and interventions.

Summary. The key takeaway from this framework is that while researchers appear to be conceptualizing utility in four primary ways, these conceptions are not explicitly stated, nor are potential implications of using particular items over others. It seems important to consider these implications, though, as respondents' ratings of perceived utility value will be tied to the conceptions that underlie scale items. Thus, if a respondent has a strong sense of the utility of a task for her career, but no career-related items are included, her ratings of utility value might be much lower than they would otherwise be. This issue is similarly present in interventions intended to enhance perceptions of utility. Interventions often focus on certain types of usefulness without attention to why those types of usefulness were chosen or consideration of whether those conceptions of usefulness align with participants' conceptions. In the section below, I describe some of these studies, as well as new directions for utility value interventions that begin to consider participants' own conceptions of usefulness.

Utility Value Interventions: Directly Communicated vs. Self-Generated

While most studies involving utility value have focused on measuring, rather than attempting to alter, perceptions of utility value, interventions to enhance perceived utility value have been increasing in recent years. Thus far, two main types of interventions have been conducted: directly communicated utility value interventions and self-generated utility value interventions. Below I describe each type of intervention in turn, highlight key examples of each, and draw attention to some strengths and limitations of each type of intervention.

In interventions involving *directly communicated utility value*, participants are told about the ways in which the techniques they are learning or courses they are taking can be useful – for either everyday life, current or future schooling, or one's career. In many studies, participants are told about the utility of particular topics or courses in a laboratory setting. For example, in Durik & Harackiewicz's (2007) research, college students were taught how to use a particular technique for solving two-digit multiplication problems. In one condition, participants also received information about the utility of the technique for "everyday situations that would be relevant to college students" (p. 603). They were provided examples such as "doing personal banking, tallying grocery bills, and taking notes during math lectures" (p. 603), as well as calculating restaurant tips and store discounts. In research using the same math technique, Durik, Shechter, Noh, Rozek, and Harackiewicz (2014) focused on usefulness for both career and schooling, informing college participants of how the technique could enhance their academic performance or be used in one of six particular careers. While the choice to include applications that are relevant now versus in the future have not typically been acknowledged, in one of Durik et al.'s (2014) studies, the researchers did separate conceptions of utility into two groups – immediate utility and future utility. For the immediate condition, participants were informed of the technique's usefulness for college classes and for everyday activities; in the future condition, participants were told about the usefulness of the technique for careers and future graduate school admissions exams. While no differences between the two conditions were found, this study is one of only a few that attend to different types of utility.

While the number of laboratory studies on utility value is increasing, utility value interventions that occur outside the laboratory or with populations other than college students remain scarce. Harackiewicz et al.'s (2012) research serves as one promising example in this domain. The researchers examined whether high school students would take more mathematics and science courses in high school if they saw value in the subjects. Parents were enlisted to communicate to their children the utility of science and mathematics outside of school hours. As resources for their discussions, parents received two brochures and a link to a website. The resources described ways in which mathematics and science apply to everyday life (including activities such as playing video games, driving, and using cell phones), preparation for future schooling, and careers such as engineering or computer science. They also gave parents tips on approaching these conversations with their children. At the end of the 15-month intervention, mothers' perceived utility value had increased overall, and students reported an increase in the number of conversations they had with their parents about the value of STEM courses. Students' perceived utility value increased only when their mothers had higher perceived utility value and when they had more conversations with parents. Additionally, compared to the control group, students who received the intervention took significantly more mathematics and science classes during their final two years of high school.

While these results are certainly promising, there are some potential issues with the directly communicated utility value model. First, as one might imagine, information that is communicated might not resonate with the individuals who are receiving the information. For example, if you are told that mathematics is useful for engineers, but you have no desire to become an engineer, then that information might not influence your views about the usefulness of mathematics. Shechter, Durik, Miyamoto, and Harackiewicz (2011) attend to this issue by highlighting the role of culture and individual differences in the types of utility that motivate students, though this line of work has not been taken up in any significant way by other researchers. Second, studies have shown that direct communication of utility value can actually have a negative effect on interest for individuals who have low confidence, perceived competence, or expectancies of success (Durik et al., 2014; Godes, Hulleman, & Harackiewicz, 2007). Durik et al. (2014) suggest that this effect might result from direct communication of utility being perceived as controlling by these individuals or being suggestive of a certain expected level of competence or performance. This negative effect on individuals with low confidence, combined with the potential misalignment between the value that is communicated and the values of the individual, highlights the need for a new type of utility value intervention.

One type of intervention that begins to combat some of these issues involves the use of self-generated utility value statements, rather than direct communication of utility value. In such interventions, participants are asked to make their own connections between the material they are learning and its usefulness in other contexts. For example, Hulleman et al. (2010) introduced college participants to the aforementioned two-digit multiplication technique; however, rather than being told how the technique could be useful, participants were asked to write a short essay "describing the potential relevance of this technique to your own life, or to the lives of college students in general" (p. 22). In contrast to directly communicated utility value interventions, having students generate their own ideas about the utility of the technique was especially productive for enhancing interest and perceived utility value among participants with low perceptions of competence. In another self-generated utility value intervention with high school students, Hulleman and Harackiewicz (2009) similarly found that the intervention improved both course grades and interest in science for high school students who had low expectations of success.

Recently, Canning and Harackiewicz (2015) compared the effectiveness of interventions involving the use of directly communicated utility value to self-generated utility value. Their findings provide further evidence that direct communication of utility value undermines the interest and performance of students with low perceptions of competence, while self-generated utility value interventions are beneficial to that same subset of students. However, they found that effects were most powerful for college students with low perceptions of competence when both direct communication and self-generation techniques were used. In other words, students who were told about the value of a technique and also asked to write about how it might apply to their lives reaped the greatest benefit from the intervention.

Additionally, Canning and Harackiewicz included a manipulation to examine the impact of focusing on utility for everyday activities versus utility for school or career, a second example of attention to different types of utility. Through this manipulation, the authors found that directly communicated utility value was effective for participants with low confidence only when examples of utility for everyday activities were provided. They hypothesized that everyday examples are more accessible and least threatening to individuals with low perceived competence in school and that future interventions involving direct communication should tailor information “to the characteristics and needs of the individual” (p. 65). The current study aims to contribute to such efforts by digging more deeply into the characteristics and needs of middle school learners. One way to begin this process is to apply a sociocultural lens to the study of utility value, as I describe in the section below.

Sociocultural Perspectives in Education

Sociocultural perspectives have their origins in the work of Vygotsky (1978) and have been especially helpful in examining learning and development across the life span (Cole, 1998; Lave & Wenger, 1991; Lee, 2001; Rogoff, 2003; Wenger, 1998; Wertsch, 1998). A sociocultural approach is grounded in the proposition that social and cultural processes mediate thought and activity. Thus, rather than solely examining the thinking and learning of an individual, we must examine the individual in interaction with his/her environment, including any tools or artifacts that mediate interaction. Context plays an especially significant role, as cognition is situated in activity (Goodwin, 1997; Lave & Wenger, 1991). In other words, an individual's thinking and learning cannot be separated from the context in which that thinking and learning occurs and the tools that are used in those contexts. As such, before considering the resources that students bring with them to the classroom, it is crucial to acknowledge the social, cultural, and political nature of learning and knowing mathematics, as well as the issues of power and privilege that arise in the mathematics classroom.

Mathematics Knowing and Learning as Social, Cultural, and Political

Knowing and learning mathematics are social, cultural, and political processes (Martin, Gholson, & Leonard, 2010; Nasir, Hand, & Taylor, 2008). As Moses discusses in his work on the Algebra Project (Moses & Cobb, 2001; Moses, Kamii, Swap, & Howard, 1989), mathematics – and algebra, in particular – often serves as a gatekeeper for students, providing some students with access to and closing others out of a range of opportunities. Moses compares this issue of access to voting issues during the Civil Rights Movement, arguing that full citizenship and the ability to make change in the world are dependent upon students' ability to develop mathematics literacy and gain economic access. The Algebra Project is one way to help students – and Black students, in particular – gain that access to change the existing narrative. Similarly, Gutstein (2006) discusses how teaching mathematics for social justice (discussed in greater detail in a subsequent section) can give students the mathematical tools needed to achieve "liberation from oppression" (p. 22). Teaching for social justice involves helping learners use mathematics to interrogate the social, cultural, and political contexts of their lives and to make change in society (Gutstein, 2003).

Zooming in to particular contexts further illustrates the social, cultural, and political nature of mathematics knowing and learning. Since different contexts produce different knowledge (Martin et al., 2010), we must recognize the many contexts in which students spend their days and get to know our students as "sociocultural beings" (Gutierrez, 1999, p. 271). A large body of work in education has illustrated the link between cognition and context by studying everyday mathematics (e.g. Brenner, 1998; Nasir, 2000; Saxe, 1991). For example, Taylor (2004) studied the connection between children's place value understanding and shopping at the corner store. He found that when children were able to use currency, rather than base-10 blocks, in the school setting, they demonstrated deeper place value understanding. However, these kinds of rich mathematical understandings that are developed in out-of-school contexts often go unnoticed in the classroom. To bridge this gap, Taylor (2009) proposed that purchasing strategies used by children in the store setting could serve as a foundation for future learning and be drawn upon as resources to address misconceptions in the classroom.

At the classroom level, acknowledging mathematics learning as a social and cultural activity cues researchers to consider the many factors that impact learning and to view mathematics learning as situated in a particular space and community of practice (Cobb & Hodge, 2002; Lave & Wenger, 1991; Nasir, 2002). Using this lens, teachers and researchers have noted the many resources students bring to the classroom with them (often described as "funds of knowledge"; discussed in more detail below) and acknowledged the impact of classroom norms and interactions on learning (Cobb, Wood, & Yackel, 1993; González et al., 1995; Moll, Amanti, Neff, & Gonzalez, 1992). For example, Esmonde and Langer-Osuna (2013) examined in fine-grained detail the norms and practices of a particular mathematics classroom and found that it was a very "racialized" and "gendered" space (p. 24). The authors identified multiple issues of power and privilege in student groups that emerged when students from many different backgrounds were mixed together. Although those interactions resulted in a great deal of conflict among students, the researchers found that the interactions also enabled students to use personal resources as a point of access to engage in valued mathematical practices.

Given the many resources and perspectives students bring with them to the mathematics classroom, researchers have adopted a variety approaches for documenting and utilizing those perspectives and experiences. For example, Martin (2009) urges researchers to provide rich descriptions of African American and Latin@ students' experiences, including their strengths and the circumstances under which they succeed in mathematics. Identifying such positive moments will enable teachers and researchers to build on students' strengths and engage in more equitable instruction moving forward. Similarly, Hand (2012) highlights three features of mathematics instruction that creates equitable opportunities for mathematics learners:

"promoting dialogic space in classroom interaction, blurring distinctions between mathematical and cultural activity, and reframing the system of mathematics education" (p. 236). These features involve challenging existing power hierarchies in the school setting, validating the many forms of participation that occur in the mathematics classroom, and noticing the connections learners make to their out-of-school experiences. As this latter component is especially relevant for the present research, below I focus in to highlight several bodies of work that have explored the connection between student learning in the classroom and relevant everyday experiences in out-of-school settings.

Drawing on Students' Everyday Experiences

A variety of approaches have been used to explore ways of drawing on students' everyday experiences, cultural backgrounds, and understandings developed outside the classroom to improve school learning and motivation. For example, Lee (1995) drew on African American students' experiences with the practice of signifying to help develop their literacy analysis skills, a process she refers to as *cultural modeling* (e.g. Lee, 2003; Lee, 2001). In mathematics education, researchers have engaged in similar processes that are grounded in the understanding that mathematics knowledge and learning are tied to cultural practices (Nasir, Hand, & Taylor, 2008). Several of these approaches include drawing on funds of knowledge (e.g. Civil, 2007; González et al., 1995; Moll et al., 1992), engaging in culturally relevant mathematics instruction (e.g. Enyedy & Mukhopadhyay, 2007; Ladson-Billings, 1995; Ladson-Billings, 2014), studying mathematics in everyday practices (Brenner, 1998; Nasir, 2000; Saxe, 1988; Taylor, 2009), and teaching mathematics for social justice (Gutstein, 2003, 2006). Each approach will be described in greater detail below, as I discuss the benefits of building bridges between out-of-school and school settings.

Funds of knowledge. One body of research that examines the knowledge and experiences students bring to the mathematics classroom with them is work centered on students' *funds of knowledge.* Research on *funds of knowledge* has examined how knowledge developed in families and communities over time can serve as a rich resource in the classroom (e.g. González et al., 1995; Moll, Amanti, Neff, & Gonzalez, 1992). Drawing on funds of knowledge in the school setting has the potential to improve students' participation and interest in mathematics (Civil, 2002; Civil, 2007). For example, Civil (2007) analyzed second grade students' experiences participating in a series of lessons in which they created garden enclosures and explored the sizes of their gardens. She found that as students participated in the lessons, they drew on everyday experiences, engaged with challenging mathematics, and exhibited personal interest while working on tasks.

Similarly, researchers have highlighted the importance of drawing on everyday experiences and language when teaching mathematics to English Language Learners (ELLs; Moschkovich, 2002, 2012). In particular, Moschkovich (1999) highlights several effective tactics for approaching mathematics instruction with ELLs including knowing your students, honoring their diversity, and focusing on their strengths and resources rather than their deficits. Gutierrez (2002) echoes the importance of these practices with ELLs and highlights their effectiveness with English-dominant Latin@ students, as well.

Culturally relevant mathematics pedagogy. In a similar vein, research on *culturally relevant mathematics pedagogy* highlights the importance of "using cultural referents to impart knowledge, skills, and attitudes" that "empower… students intellectually, socially, emotionally, and politically" (Ladson-Billings, 2009, p. 18). Ladson-Billings describes culturally relevant pedagogy as being comprised of three propositions: "(a) Students must experience academic success; (b) students must develop and/or maintain cultural competence; and (c) students must develop a critical consciousness through which they challenge the status quo of the current social order" (p. 160). To illustrate culturally relevant mathematics pedagogy in practice, she identified eight teachers who enacted these principles and described the features of their instruction, as well as its impact on students (Ladson-Billings, 2009). Ladson-Billings found that the teachers saw themselves as part of the students' communities, worked to give back to the community, viewed teaching as an art, believed that all students can succeed, drew on students' prior knowledge and experiences, and helped students to make associations between their various identities. These teachers had high academic expectations for their students, and students in turn achieved at higher levels than their district counterparts despite the low ranking of the district (Ladson-Billings, 1995).

Other research has similarly highlighted the value of culturally relevant and critical mathematics instruction (Frankenstein, 1983, 2001; Gutstein, Lipman, Hernandez, & de los Reyes, 1997; Matthews, 2003; Tate, 1995). For example, Tate (1995) describes the ways in which one mathematics teacher built her pedagogy around "an awareness of the problems African American children face in education and society" (p. 171). She connected with students' worlds outside of school by first asking them to highlight problems in their own communities. These problems were then used as a basis for explorations in which students applied mathematics to research the problems and then proposed strategies for achieving community change.

Engaging with culturally relevant mathematics pedagogy is not without struggle, however. Enyedy and Mukhopadhyay (2007) applied the principals of this approach when teaching a summer seminar for 25 African American and Latin@ high school students. They found that while students were able to engage with meaningful, personally relevant topics as they conducted their own social sciences research, tensions also emerged. In particular, local knowledge was privileged at the expense of thorough consideration of counterevidence and, as a result, more complex, rigorous mathematics was sometimes not taken up. In another study utilizing culturally relevant mathematics pedagogy with middle school students engaging in a community mapping project, Enyedy, Danish, and Fields (2011) identified a similar struggle. In particular, although students exhibited more agency and experienced a greater sense of relevance when this pedagogy was applied, statistical concepts were not addressed as deeply as the researchers had hoped. One conclusion stated by the researchers is that negotiation between teachers and students must be a key component of culturally relevant pedagogy, as the goals of both the social justice component of the project and the mathematics might not be enacted as expected.

Mathematics in everyday practices. Often guided by the ideas of culturally relevant mathematics pedagogy, many researchers have explored the ways in which students use mathematics in everyday practices. For example, Saxe (1988) illustrated the power of drawing on everyday understandings in his study of Brazilian child candy sellers. He found that the children used complex mathematics to solve problems related to candy selling and explored the degree to which that knowledge might influence their performance on school problem-solving tasks. While sellers and non-sellers performed comparably on certain traditional tasks, Saxe found that sellers were able to apply their candy selling strategies to solve school mathematics problems that were closely linked to the selling practice. In particular, candy sellers outperformed non-sellers on tasks that required them to do arithmetic calculations with bills and to compare two pricing ratios. This finding, as well as the results of other studies such as Taylor and Dobie's (forthcoming) examination of mathematical understandings learned through the practice of tithing, highlight achievement-related benefits of creating tasks and assessments that draw on and are aligned with students' own experiences and cultural practices.

Many other researchers have documented the use of mathematics in a variety of everyday practices (Brenner, 1998; Carraher, Carraher, & Schliemann, 1985; Guberman, 1996; Lave, Murtaugh, & de la Rocha, 1984; Nasir, 2000; Nunes, Schliemann, & Carraher, 1993; Rogoff, 2008; Scribner, 1984; Taylor, 2009). For example, Brenner (1998) examined money exchange among Native Hawaiian children, while Lave, Murtaugh, and de la Rocha (1984) highlighted the ways in which adults use mathematics while grocery shopping. Meanwhile, Nasir (2000, 2002) examined adolescents' use of mathematics while playing basketball. In all of these studies, a key component was highlighting the social and cultural nature of mathematics and the ways in which different groups of people use mathematics in different settings in their everyday lives.

Teaching mathematics for social justice. Finally, research on teaching for social justice is similarly guided by the notion of culturally relevant mathematics pedagogy. In particular, students engage with problems that are relevant to their lives in order to gain the mathematical tools necessary to both interpret issues and affect change in society (Gutstein, 2006). Gutstein has reported on his work teaching mathematics for social justice with middle school students who attend a predominantly Latin@ urban public school. Through real world projects that drew on a standards-based curriculum, students interrogated “the conditions of their lives and the sociopolitical dynamics of their world” (Gutstein, 2003, p. 39). Specifically, he developed 17 projects that built on students’ experiences and out-of-school lives, focusing on topics such as racism in housing data, distribution of world wealth, and effects of gentrification on communities. These topics allowed students to use mathematics to engage with complex ideas and gain sociopolitical awareness.

In this work on teaching mathematics for social justice, Gustein (2003, 2006) has identified numerous benefits but also some challenges. When working with the units described above, students were able to connect mathematical analyses with sophisticated critiques of complex situations and prior assumptions. Engaging with the projects also helped students to develop mathematical power, improve their attitudes towards mathematics, and enhance their sociopolitical awareness by using mathematics. In other words, students felt that they were better able to apply mathematics to understand the world (Gutstein, 2006). However, while Gutstein found that students engaged deeply with both the contexts and the mathematics of the social justice units, some students still felt that they could not connect with the situations presented to them (Gutstein, 2003, 2006). These students reported that the contexts did not feel relevant, which highlights the fact that presenting a real-life context is not enough to ensure that it is meaningful to students.

Across all these bodies of work, researchers have highlighted the importance of grounding instruction in students’ own daily practices and lived experiences to enact pedagogy that is socially and culturally relevant to students. This practice of building bridges between school mathematics and everyday cultural practices can be especially powerful in working towards achieving equity in mathematics education (Tate, 1994, 1995). It is worth mentioning, however, that in addition to considering students' everyday experiences with mathematics, it is important to consider the narratives about mathematics to which students are exposed through significant others, such as their parents (Martin, 2007; Martin, 2006). Martin found that while many African American parents reported experiencing a struggle for mathematical literacy, their narratives about and characterizations of their experiences differed. Such narratives might play an important role in children's interpretations of their own daily experiences and might also be connected to the values that children develop over time. Related, in the next section I discuss the importance of considering students' goals and values when applying a sociocultural lens to examine students' perceptions of the usefulness of mathematics.

Drawing on Students' Goals and Values

In order to engage in equitable research that is sensitive to sociocultural context, another important area of consideration is students' goals and values. Exploring students' goals and values can provide insight into valued domains that influence the ways students think about usefulness. This information can then allow us to target inventions to particular students, as discussed by Shechter et al. (2011). For example, the theory of identity-based motivation operates from a perspective of cognition as situated – or dependent upon context – and highlights the fact that students' in-the-moment decisions and interpretations of their experiences are guided by the identities they dynamically construct in the moment based on the sociocultural context (Oyserman & Destin, 2010; Oyserman, 2009). By better understanding students' goals and values, we can work to help students construct identities in the mathematics classroom that feel connected to their out-of-school lives and make them motivated to pursue challenges.

In addition to exploring students' goals and values at large, in this research I examine values related to interaction with others, in particular. Such values arise in developmental research on adolescents, as well as work on individuals' motives and views of the self in different communities. First, in developmental research examining the goals of adolescents, Wentzel (1989, 1993) found that middle school students emphasized not only academic goals in the classroom but also social goals. These goals included exhibiting skills such as cooperation, dependability, and responsibility. Furthermore, working to achieve such social goals was a predictor of one's academic achievement, as measured by classroom grades and standardized test scores (Wentzel, 1989, 1993, 2004). Similarly, research examining different communities' values and views of the self also highlights the role of connection to others. In such work, researchers have examined contexts that promote independence versus interdependence, where independence is characterized by a focus on personal motives and distinctiveness from others and interdependence is conceived of as an emphasis on connectedness and the needs of others (Grossmann & Varnum, 2011; Stephens, Markus, & Townsend, 2007). These two orientations describe communities' views of the relationship between one's self and society at large. Research has illustrated that while American middle-class contexts typically promote independence, working-class contexts typically promote interdependence (Grossmann & Varnum, 2011; Stephens, Markus, & Townsend, 2007). Thus, students from working-class backgrounds might be less likely to be motivated by independent goals. As such, it seems especially important to examine these values given the community in which this work takes place – a working class, predominantly Latin@ community.

Interdependent goals have also been shown to influence one's academic experience. Stephens, Fryberg, Markus, Johnson, & Covarrubias (2012) illustrated the relationship between educational context and one's view of the self when they studied students who attend American colleges. The researchers discovered a mismatch between American colleges, which typically promote norms of independence, and students from working-class backgrounds, who most often identified with interdependent norms. The latter group experienced greater difficulty on academic tasks and lower grades in college when the culture of the university was described in terms of independent norms, such as exploring one's own personal interests, expressing one's opinions, participating in independent research, and developing one's own intellectual journey. However, when the university culture was presented as promoting interdependence – characterized by being part of a community, connecting with other students and faculty members, working together, and taking part in collaborative research – students from working-class backgrounds performed equivalently to students from middle-class backgrounds. In other words, students' academic success increased and experience of difficulty decreased when college was framed in a way that aligned with their interdependent motives.

Related, researchers have highlighted the emphasis on interdependent motives in particular ethnic communities. For example, Gay (2002) notes that "many students of color grow up in cultural environments where the welfare of the group takes precedence over the individual" (p. 110). In particular, *familism* – or having a strong identification with and loyalty to one's family – is a central value among Latin@s (Esparza & Sanchez, 2008; Sabogal, Marín, Otero-Sabogal, Marín, & Perez-Stable, 1987), and Mexican and Mexican-American communities often exhibit strong family values (Valdés, 1996). Thus, the presence of interdependent versus independent motives seems especially important to examine in this study.

In existing research on utility value, the primary focus is on independent goals and usefulness *for the individual* (e.g. for getting accepted into college or performing a job). However, for students who live in communities that promote interdependence over independence, seeing the usefulness of mathematics for achieving interdependent means might be especially motivating. For example, in Gutstein's (2006) work on social justice mathematics, using mathematics for interdependent purposes – such as to liberate one's community – had the potential to improve students' attitudes towards mathematics and ability to use mathematics to better understand the world. Similarly, Moses' work on the Algebra Project (Moses & Cobb, 2001; Moses et al., 1989) connects mathematics literacy with economic access in Black urban and rural communities, highlighting implications of learning algebra for equality and citizenship. By emphasizing the social and political implications of being mathematically literate, Moses encouraged students to pursue mathematics as a tool to liberate their communities. The current research follows in the footsteps of such work by first examining students' interdependent versus independent values and then considering how we can align utility value measures and interventions with those values.

Methodological Implications

Taking a sociocultural approach to the study of utility value in mathematics education requires important methodological considerations. First, students' interactions and social relationships with others must be considered as part of this work. Thus, qualitative methods are used to examine student interactions in the mathematics classroom and to explore the role of relationships with others in out-of-school activities involving mathematics. Second, it is crucial to consider the culturally based knowledge and practices that students possess to better understand the resources that students bring to discussions of utility. To address this need, I examine not only students' interactions and perceptions of usefulness in the mathematics classroom but also students' engagement with mathematics in everyday life. And third, multiple methods of eliciting students' perspectives on usefulness should be used to provide students with different stimuli to provoke discussions of usefulness. In this work, open-ended questions and prompts are included so as to not constrain the types of answers that students might provide. This open-endedness is especially important for conducting research that is equitable and grounded in students' own social and cultural experiences.

This section highlighted the deep connection between cognition, learning, and social context, and underscored the rich mathematical understandings that students often develop outside the classroom. However, as Nasir and Hand (2008) highlight, we must work to better understand the ways in which out-of-school settings support student learning and engagement with mathematics. Identifying features of students' engagement with learning and mathematics outside the classroom can provide a lens for examining modes of learning and engagement in the mathematics classroom. Furthermore, while existing research on utility value has examined performance, interest, and course enrollment as outcomes, the relationship between utility value and engagement has rarely been examined. Thus, this work tackles engagement in two contexts – first, features of adolescents' engagement with mathematics in their everyday lives, and second, the relationship between perceptions of utility and student engagement in the mathematics classroom. As a foundation for examining these topics, I now move on to explore current research on student engagement and, in particular, features of classroom contexts that influence engagement.

Student Engagement

While many studies examine student performance or course-taking as outcomes of high perceived utility value, I argue that student engagement is an important outcome to consider, as well. Engagement is a predictor of academic achievement (Dotterer & Lowe, 2011; Marks, 2000; Wang & Holcombe, 2010), and engagement in school has been linked to students' expectancies of success, sense of relatedness to others, and perceptions of control (Eccles, Wigfield, & Schiefele, 1998; Furrer & Skinner, 2003; Patrick, Skinner, & Connell, 1993). Furthermore, positive relationships have been documented between particular components of engagement and students' hope for the future, commitment to learning, and intrinsic motivation to learn (Connell & Wellborn, 1991; Shernoff & Hoogstra, 2001; Skinner & Belmont, 1993; Van Ryzin, Gravely, & Roseth, 2009). Thus, examining ways to improve student engagement through perceptions of utility has the potential to also impact students' learning and motivation to learn.

In the sections below, I first define the construct of engagement and highlight two ways in which it has been conceptualized in the educational literature. Then I focus in on features of classroom contexts that are likely to influence student engagement, followed by some defining features of productive engagement with mathematics. Finally, I conclude by circling back to connect with the utility value literature and lay the foundation for the findings chapters to come.

What Is Engagement?

Engagement is a multifaceted construct that describes one's participation in and commitment to a given activity. When discussed in the context of schooling, engagement includes three dimensions: behavioral, emotional/affective, and cognitive (Fredricks, Blumenfeld, & Paris, 2004; Jimerson, Campos, & Greif, 2003). The behavioral dimension includes aspects of engagement related to participation, while the emotional dimension encompasses positive and negative reactions to people and things in a given setting, Finally, the cognitive dimension includes perceptions and beliefs about one's own ability and the expectations of others, among other aspects. Researchers have been increasingly interested in the construct of school engagement because engagement is viewed as malleable, rather than fixed, so there are opportunities to influence the environment in ways that will in turn affect student engagement (Fredricks et al., 2004). Below I highlight some of those environmental features that have been shown to influence students' engagement in learning.

Features of Classroom Context that Influence Student Engagement

In Fredricks et al.'s (2004) review of literature on student engagement, five features of classroom contexts that influence student engagement are cited: teacher support, peers, classroom structure, autonomy support, and task characteristics. Researchers have highlighted a variety of ways in which these features can enhance – or detract from – students' engagement in the classroom. In the paragraphs below, I highlight some of the key themes and articles discussed by Fredricks et al., occasionally incorporating additional research that is especially pertinent to mathematics.

First, teacher support has played a role in influencing behavioral, emotional, and cognitive engagement (Battistich, Solomon, Watson, & Schaps, 1997; Blumenfeld, Puro, & Mergendoller, 1992; Marks, 2000; Ryan & Patrick, 2001; Skinner & Belmont, 1993; Weber, Radu, Mueller, Powell, & Maher, 2010; Wentzel, 1997). For example, while examining schools that were experiencing reforms, Marks (2000) found that elementary, middle, and high school students' engagement was related to both teacher and peer support. In another study, when teachers created an atmosphere of respect and social support, students were more behaviorally engaged (Stipek, 2002). Conversely, students who experienced negative relationships and conflict with teachers exhibited lower engagement and poorer student performance than students who had positive relationships with their teachers (Baker, 2006; Ladd & Burgess, 2001).

Peers also play a role in student engagement, though that role has been less frequently examined in research. Peer acceptance has been linked with aspects of both emotional and behavioral engagement (Berndt & Keefe, 1995; Wentzel, 1994), while rejection from peers has been shown to increase dropout rates and result in decreased classroom participation and interest in school (Buhs & Ladd, 2001; French & Conrad, 2001). In engagement with mathematics outside the classroom, one related factor that might be important to consider is the distributed nature of problem solving (Hutchins, 1995). As studies of everyday mathematics have illustrated, individuals typically work together to solve problems they encounter (Nasir & Hand, 2008; Saxe, 1991; Taylor, 2009). Thus, the role of peers, as well as other significant others, in adolescents' engagement in these settings is important to consider.

Next, classroom structure can significantly impact students' engagement in learning. Fredricks et al. (2004) discuss components of structure including the communication of teachers' expectations for students and students' perceptions of work norms in the classroom. When expectations and work norms are clearly stated and consistently enforced by teachers, student engagement is enhanced (Connell & Wellborn, 1991; Fredricks, Blumenfeld, Friedel, & Paris, 2002). Additionally, Cobb and Yackel (1996) propose that students' mathematical beliefs are strongly tied to the social norms that exist in classrooms, and Yackel (2001) found that classroom norms played a role in encouraging students to engage in mathematical explanation and justification. Building on that notion, Weber et al. (2010) found that student engagement was related to the presence of particular classroom norms and students' perceptions of expectations regarding their roles in the classroom. Specifically, student engagement was enhanced by teachers asking students to justify their answers, students presenting their own work on overhead projectors, students convincing their classmates of answers, and students having a say in guiding classroom investigations. On that last note, and connected with autonomy support – the fourth feature of classroom contexts that influence engagement – Shernoff et al. (2003) found that students were most engaged when they were involved in active, student-directed learning (such as group work) rather than passive learning (such as listening to a lecture or watching a video).

Autonomy support refers to the availability of choice for students, as well as shared control and decision-making in the classroom. As mentioned above, different participation structures have the opportunity to inhibit or promote autonomy, as they provide students with different levels of control over their learning (Nasir & Hand, 2008; Shernoff et al., 2003). Similarly, Perry (1998) found that elementary school students were most engaged when they were provided choice by their teachers, and Langer-Osuna (2015) highlighted the important role of autonomy in a student's ability to successfully engage with complex mathematics and be positioned as competent by his peers. It is worth noting that this sense of autonomy should not be equated with a focus on independence, as discussed above. Autonomy support can be provided in contexts where either interdependent or independent values are promoted.

Finally, task characteristics influence student engagement in learning. This component is especially important for the current research, as the final phase of this study involves the design of tasks to promote student engagement and perceptions of usefulness. Several main features of tasks have been discussed as having the ability to promote student engagement: authenticity, opportunities for creativity and student ownership, opportunities for collaboration, and level of challenge. First, Newmann, Wehlage, and Lamborn (1992) proposed that students are most engaged by authentic tasks, which has been illustrated in research by Marks (2000). Additionally, providing students with opportunities to take ownership over their learning and encouraging creativity have also been shown to support student engagement (Nasir & Hand, 2008; Weber et al., 2010). Related, Weber et al. found that students were most engaged when completing open-ended tasks that allowed for multiple solutions and were novel for students. Third, participation structures also influence the ways in which students can interact with tasks. Researchers have found that tasks that provide opportunities for collaboration also tend to promote student engagement (Helme & Clarke, 2001; Newmann et al., 1992). And finally, students' perceptions of the level of task challenge also have an influence on their cognitive, affective, and behavioral engagement with tasks (Fredricks et al., 2002; Shernoff et al., 2003)

Hybridity and Engagement

Given the sociocultural and ecological focus of this work, I propose the possibility that students' engagement with mathematics outside the classroom might influence their engagement with mathematics in the classroom. In prior research, differences have been found in adolescents' engagement with various subjects and activities depending on features of each setting. For example, Nasir and Hand (2008) examined high school students' engagement in the mathematics classroom and on the school basketball team. In particular, they considered adolescents' access to the domain, opportunities for taking up integral roles, and opportunities for self-expression and unique contributions in each context. The researchers found that students engaged in different ways in the two contexts and that those differences in engagement were likely linked to the participation structures in each setting and the types of connections and identities they afforded.

Other research has illustrated that hybridity, or encouraging students to draw on out-of-school experiences in the classroom and make connections between school and their own lives, promotes student engagement and positive identities in the classroom (Ballenger, 1997; Civil, 2007; González, Andrade, Civil, & Moll, 2001; Michael, Andrade, & Bartlett, 2007). Researchers have studied hybrid spaces to explore the ways in which students' experiences with different socially and culturally constituted worlds can influence their engagement in a given setting (Dagenais, Day, & Toohey, 2006; Gutiérrez, Baquedano-López, & Tejeda, 1999; Holland, Lachicotte, Jr., Skinner, & Cain, 2001; Wortham, 2006). For example, Langer-Osuna (2015) examined how one student's ability to position himself in relation to different worlds and draw on multiple identities in the mathematics classroom afforded him stronger engagement with the mathematics and allowed him to be positioned as competent among his peers.

To consider the different ways these everyday experiences and identities might be drawn on in the mathematics classroom, we can turn to Taylor's (2012) Multi-Approach Engagement Framework. This framework highlights four approaches teachers use to draw on students' out-of-school practices in the mathematics classroom. Each approach is characterized by a) whether it involves the use of activities that are personally authentic or inauthentic to students and b) whether it involves mathematics that is connected to or disconnected from the activity itself. Taylor proposes that teachers will most effectively engage students when tasks are personally authentic and when mathematics content is connected to the activity. This framework provides a lens for considering how a teacher's approach can influence the types of connections students are able to make and highlights the role that interactions with mathematics in everyday settings can play in students' classroom engagement.

Mathematical Engagement

In addition to considering the features of classroom contexts that promote engagement and the ways teachers might draw on students' out-of-school practices to enhance engagement, it is important to consider what productive engagement looks like in a mathematics classroom. Sengupta-Irving and Enyedy (2015), as well as many others (e.g. Barron, 2003; Zhang, Scardamalia, Reeve, & Messina, 2009), have framed their work using Engle and Conant's (2002) four principles that foster productive disciplinary engagement – problematizing, authority, accountability, and resources. Problematizing suggests that students "take on intellectual problems," while authority refers to students being "given authority in addressing such problems" (Engle & Conant, 2002, p. 400). Meanwhile, accountability focuses on the fact that "intellectual work is made accountable to others and to disciplinary norms," and resources highlights the need for students to be "provided with sufficient resources to do all of the above" (p. 401). Sengupta-Irving and Enyedy (2015) connect those principles to the Common Core State Standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), proposing that CCSSM's mathematical practices mediate the relationship between the design of a learning environment and students' interest and engagement.

Furthermore, they found that drawing on the four principles offered an opportunity to improve students' affective engagement in addition to fostering productive disciplinary engagement.

Related, CCSSM's mathematical practices provide a helpful lens for considering some of the goals for student engagement in the classroom (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). There are eight practices that CCSSM encourages teachers to develop in students: persevering in problem-solving, reasoning abstractly, argumentation and critiquing others' reasoning, modeling with mathematics, using appropriate tools strategically, attending to precision, identifying and utilizing structure, and finding regularity in repeated reasoning. Many aspects of these practices have also been explored in existing research on student engagement with mathematics. For example, in their discussion of features of math-talk learning communities, Hufferd-Ackles, Fuson, and Sherin (2004) identified questioning, explaining one's thinking, and clarifying and critiquing others' ideas as key student behaviors in strong, engaged learning communities. Additionally, many researchers have explored ways to promote students' justification of their own reasoning and ability to make abstract generalizations (e.g. Bieda, 2010; Ellis, 2007; Lannin, 2005).

Finally, Helme and Clarke (2001) identified moments of deep cognitive engagement in middle school classrooms and saw that many of the aforementioned mathematical practices were indicative of high cognitive engagement. For example, when students worked in collaborative small groups, actions such as questioning, exchanging ideas, justifying arguments, and giving directions were associated with high engagement. Connecting back with the earlier discussion of participation structure, Helme and Clarke also found that student-to-student interactions offered a greater opportunity for high levels of cognitive engagement – in the form of these desired mathematical practices – than student-teacher interactions.

Engagement and Utility Value

Despite the important role of student engagement in learning, little existing research on utility value has considered student engagement as an outcome. When researchers have examined the relationship between engagement and utility value, engagement has been measured through student reports of maintained situational interest (Hulleman et al., 2010). While such measures are certainly helpful in learning about the potential influence of utility value on one component of student engagement, namely maintained interest, additional research is needed to examine utility value's potential effect on other components of engagement, such as the ways in which students engage with material that they perceive as useful versus not useful. In particular, considering the relationship between utility value and domain-specific forms of engagement, such as the mathematical practices described above, will be helpful for better understanding the forms of engagement that might be promoted by increased perceptions of utility.

In sum, this research draws on three main bodies of research to consider middle school students' perceptions and conceptions of the utility of mathematics. First, I ground this work in the construct of utility value, which is situated in Eccles and colleagues' expectancy-value theory. I aim to expand the literature's conceptualization of utility value by uncovering students' own perspectives on the usefulness of mathematics and, in particular, the perspectives of students in underserved and often silenced communities. Second, I apply a lens of sociocultural context, building upon the work of many others who have examined the power of drawing on students' everyday knowledge in the classroom. Examining utility value through a sociocultural lens requires me to listen to the voices of students, allow multiple perspectives to emerge, examine utility value through both students reports and observations of interactions, and consider the influence of students' values and everyday experiences on their classroom engagement. Finally, I draw on research on engagement to highlight the importance of attending to student engagement in this work and to consider features of classroom contexts that might promote engagement. Additionally, I examine students' modes of engagement with mathematics not only in the classroom but also in everyday activities to gain insight into practices that promote engagement and perceived utility in different settings. In turn, I apply those findings to the ecologically valid design of problem-solving tasks that are intended to enhance students' perceptions of utility. In the next three chapters, I detail the findings that emerged when applying these lenses to examine middle school students' perceptions and conceptions of the usefulness of mathematics.

3. Middle School Students' Perspectives on the Usefulness of Mathematics

"If we don't learn math that is useful, why learn it at all?"-Omar, 7th grade

In order to design ecologically valid interventions that both enhance students' perceptions of the usefulness of mathematics and are tailored to students' own values and experiences, it is important to begin by exploring the ways in which students think about usefulness. Existing research has primarily highlighted correlates and outcomes of perceived usefulness but not students' conceptions of what it means to engage with mathematics that is useful (e.g. Canning & Harackiewicz, 2015; Durik, Shechter, Noh, Rozek, & Harackiewicz, 2014; Harackiewicz, Rozek, Hulleman, & Hyde, 2012; Hulleman, Godes, Hendricks, & Harackiewicz, 2010; Hulleman & Harackiewicz, 2009). In other words, we know a considerable amount about students' *perceptions* of the usefulness of mathematics, or whether students think mathematics is useful, but very little about students' *conceptions* of usefulness in mathematics, or the ways in which students define usefulness.

In current and past research, usefulness (or utility) is often defined somewhat broadly, linking useful tasks to "other aspects of [one's] life" (Harackiewicz et al., 2012, p. 1). However, examining the ways in which the construct is taken up in survey items and interventions reveals four primary underlying conceptions of usefulness: general utility, utility for one's job or career, utility for everyday life, and utility for school/college. Although these conceptions seem to guide the design of survey items and interventions, it is unclear whether these ways of thinking about usefulness align with the kinds of usefulness that students care about or would want to see. Thus, this study begins by broadening the ways we tend to think about usefulness to allow new conceptions of usefulness to emerge.

In order to accomplish this goal, it is necessary to introduce new methodologies to research on usefulness. Open-ended response questions and interview tasks, including a card-sorting task and video response task, emphasize students' perspectives and allow students' own definitions of utility to emerge. These tasks were designed with issues of equity in mind, as the open-ended nature of the tasks allows students to draw on their own perspectives and ways of defining usefulness (in contrast to rating scales, which constrain possible definitions of usefulness). The use of varied methodologies also enables students to express their views about utility in different ways, which is especially important for the current study, as techniques that have been used in high school and university settings might be less successful with middle school students. Thus, a secondary goal for this work is to identify measures and methods of data collection that are effective and equitable for utility research with middle school students.

In this chapter, I explore the following questions:

1. *What are Ms. Sanchez's students' perceptions of usefulness, and what is the relationship between those perceptions and students' interest and performance?*

2. *How do Ms. Sanchez's students conceptualize usefulness in mathematics, and in what ways are those conceptions similar to or different from existing conceptions of utility?*

By drawing on student survey and interview data, I highlight the ways in which students in Ms. Sanchez's four seventh-grade mathematics classes think about what it means for mathematics to be useful. I also compare students' conceptions with the underlying conceptions of usefulness in existing literature. Below I will elaborate on the methods and data analyses that were used in this study before diving into the main findings of this work.

Methods

This study draws on analyses of several different data sources – a student survey, student interviews, and a video task. Table 3.1 provides an overview of the participants involved in each, as well as the constructs that each data source was designed to measure. Below I elaborate on the procedures involved in collecting each form of data.

Table 3.1

Overview of Data Sources

Data Source	Sample	Designed to Measure…
Survey	84 students (from 4 classes)	Perceived usefulness, situational and personal interest, perceived competence/expectancy of success, conceptions of usefulness, definition of mathematics, friends' and parents' attitudes towards mathematics, self-construal, personal values
Interviews	12 students (from the 3 observed classes)	Conceptions of usefulness, attitudes towards mathematics, definition of mathematics, current/future uses of mathematics by self or others
Video Task	12 students (same students who participated in interviews)	Conceptions of usefulness

Survey

Eighty-four students across the four participating seventh-grade classes completed a 15-minute survey. The only students who did not complete the survey were students who were absent, one student who had special needs and did not have the support required to participate, and one student whose parent opted out of the study on her daughter's behalf. Students completed the survey on the computer, as the school provided all students with a laptop to use during the school year. The survey consisted of a combination of open-ended items and rating scales. The six open-ended items inquired about potential benefits of learning mathematics, whether/why students think they will use mathematics in the future, what it looks like to do mathematics, students' perceptions of the most and least useful subjects, and students' perceptions of themselves as mathematics students. These questions were used to tap into students' conceptions of usefulness in mathematics, explore factors that might influence conceptions of usefulness, and provide added depth to students' responses to rating scales.

Various rating scales were adopted and modified to measure the following: perceptions of usefulness – independent, interdependent, and now/future (Doepken, Lawsky, & Padwa, 2004; Fennema & Sherman, 1976); perceived competence (Eccles, Wigfield, Harold, & Blumenfeld, 1993); personal and situational interest (Mitchell, 1993); relative interdependence (Hardin, Leong, & Bhagwat, 2004; Singelis, 1994); friends' and parents'/caregivers' view of mathematics (OECD, 2013); important life values (Schneider, 2013); and reasons for wanting to do well in school (Stephens, Fryberg, Markus, Johnson, & Covarrubias, 2012). Since the demographics of this population of students are different from the demographics of participants in most prior studies on utility value, these scales were used to examine similarities in and differences from expected relationships between variables. For example, once scales were determined to be reliable, results were used to examine whether students' interest in mathematics was correlated with their perceptions of the usefulness of mathematics, as prior research has found (though predominantly with high school and college students). Results from these scales will also be used in the future to examine the relationship between different conceptions of usefulness and other student beliefs and values, such as perceptions of their own competence. Finally, in addition to including the aforementioned scales (or modified versions of those scales), items were also developed to measure the following: importance placed on various features of mathematics, students' future school and career plans, prior math grades, housing accommodations (house vs. apartment; owned vs. rented), parents'/caregivers' jobs or careers, favorite and least favorite school subjects, languages spoken, age, gender, and race/ethnicity.

The initial survey went through several rounds of pre-testing, and two main forms of pre-testing were used. First, expert reviews were used to identify potential response problems and gather recommendations for improvement prior to field testing (Presser & Blair, 1994). Second, two rounds of field testing at three different sites with participant debriefing were used to determine practical problems with administering the survey, the amount of time to complete the survey, response problems related to question comprehension, and the reliability of scales. Prior to field testing, an a priori power analysis was conducted to determine the number of pilot participants needed, and a sufficient number of participants was surveyed (N=161). Overall, student demographics were fairly similar to the demographics of the focal students. The mean age of pilot participants was 11.61 years, and the mean grade level was 6.29. Of the 161 respondents, 125 reported being in either sixth- or seventh-grade, while the remainder were primarily in 5^{th} or 8^{th} grade. The majority of pilot participants identified with non-dominant ethnicities, with the greatest number of students identifying as Hispanic/Latino (n=72). Primary differences from the focal sample are the wider range in grade levels of participants and the lower percentage of students who identified as Hispanic/Latino (though a comparable percentage who identify with non-dominant ethnicities).

After pilot testing was complete, three types of changes were made following the analysis of pilot data: First, words or items that students had difficulty comprehending were changed to be more developmentally appropriate. Second, all scale points on rating scales were labeled, so as to require less interpretation of numbers from participants, and most scales were changed to have five scale points, as that proved to be the number most appropriate and reliable for this age range. Third, reliability and factor analyses were used to drop items that did not fit well with other scale items and to make the survey a more manageable length for students. More detailed information on the items that were used in this study, as well as the modifications that were made, can be found in Appendix A.

Upon completion of field testing, the survey was administered to focal participants. After collecting responses, items were collapsed to form the following scales: Perceptions of Usefulness (α=.911), Perceptions of Usefulness-Interdependent (α=.831), Interest (α=.911), Perceived Competence (α=.868), Significant Others' Mathematics Attitudes (α=.796), Independent Self-Construal (α=.628), and Interdependent Self-Construal (α=.581). Due to the low alphas on the latter two scales, those scales are not considered further in analyses. Instead, items focused on Reasons for Wanting to Do Well in School are used to gain insight into students' independent and interdependent values. For the purposes of this chapter, analyses from the following survey items are considered: perceptions of usefulness scales, interest scale, students' reported grades, ratings of the importance placed on various features of mathematics, and short-answer questions about whether/why students think they will use mathematics in the future and students' perceptions of the most useful subject.

Interviews

In consultation with Ms. Sanchez, four to six students from each of the observed classes were selected to participate in a semi-structured interview. Students were selected to maximize diversity in achievement level, gender, and attitudes towards mathematics. Of the invited students, twelve returned consent forms and agreed to participate in an interview. Each interview lasted between 16 and 34 minutes, with the mean interview length being approximately 26 minutes. Interviews were structured around four main themes: definitions of and attitudes towards mathematics, ideas about the usefulness of mathematics, experiences in mathematics class this year, and mathematics in one's daily life. Question types included imagined scenarios, card-sorting tasks, open-ended questions, application questions, and follow-up questions on survey responses (to clarify ambiguous responses or gather additional detail from students). An example of each can be found in Table 3.2.

Table 3.2

Examples of interview tasks

Task Type	Example
Follow-up on survey response	In your survey when you listed the benefits of math, one of the things you said was that it can _______. Can you tell me a little bit about why you think math will help with that?
Imagined scenario	Imagine that a new student is coming to your school and that student has never taken a math class before or even heard the word "math." Can you pretend I'm that new student and tell me what is math?
Card-sorting task	I'm going to show you five pictures, and I'd like you to put them in order from the one that involves the least math to the one that involves the most math.
Open-ended question	Thinking about your own day-to-day life, in what ways, if any, do you use math each day?
Application question	I'm going to show you a list of math topics. I'd like you to go down the list and tell me where you think you might use each of these concepts. And if you don't think you would ever use it, just say that.

Three interview items are used in this chapter's analyses. First, one item asked students to elaborate on survey responses in which they rated the importance of mathematics being useful. Specifically, students were asked to explain why they said it was either "important" or "very important" that the mathematics they learn is useful. Second, a card-sorting task required students to view six cards that showed pictures of students engaging in what could be considered as mathematical activity (sample pictures can be found in Figure 1). Pictures were selected for variation in the activity students were engaged in, the materials involved, and the people involved. The goal was to select pictures that represented a variety of forms of mathematics and ways of engaging with mathematics to allow students to consider multiple criteria that might influence their perceptions of usefulness. Students were first asked to select the cards containing pictures in which students were doing mathematics, as I did not want to assume that students viewed all pictured activities as mathematical. Then the interviewees were asked to identify in which of those pictures students were engaged in "useful mathematics" and in which pictures students were engaged in mathematics that was not useful. Finally, students were asked to explain the reasons for their selection, and responses were used to gain insight into students' conceptions of usefulness.

Figure 1. Sample Pictures for Card-Sorting Task

The third interview task I draw on for analyses in this chapter is a question in which students were asked to identify where, if anywhere, they imagine using six different mathematics topics. The topics were drawn from those that appeared in students' curriculum throughout the 2014-15 school year. The six topics included adding and multiplying fractions, writing equations, finding equivalent ratios, commission and mark-up, making graphs, and finding perimeter and area. Students were shown a list of the six topics and then asked to look through the list and tell the interviewer "where you think you might use each of these concepts." They were also told that if they didn't think they would ever use the topic, they should feel free to say so.

Video Response Task

All students who participated in interviews also completed a ten-minute video task on the computer. The task was designed based on prior work that has been done with teachers viewing videos, as research has not typically involved students responding to videos. The goal of the task was to allow students to express their ideas about the usefulness of mathematics using a different stimulus material. The following steps were taken to identify four video clips of mathematics classrooms that varied in problem context and participation structure. First, I identified videos and video clips that could be freely accessed – all came from either the 1999 TIMSS study or the Teaching Channel's website. I watched the videos and recorded several pieces of information for each class segment: the problem context(s) (contextualized or decontextualized), participation structure (direct instruction, whole class discussion, or small group work), and mathematics content. I then looked across all segments to identify any clips that aligned in mathematics content and participation structure and thus could be used to isolate differences in problem context. Only one pair of clips achieved such alignment and was thus selected for inclusion. The clips matched on mathematics content (writing and evaluating variable expressions) and participation structure (teacher-led/direct instruction), and involved variation in problem context only. I also attempted to find clips that matched on mathematics content and type of problem context but varied in participation structure; however, no such freely available clips could be identified. Instead, two additional clips were selected that included mathematics content familiar to the participants of this study and involved different participation structures from the first two clips. Both of these clips involved small group work, though one showed a student helping another student while working in pairs, and the second showed a teacher talking to a small group.

Again, the problem context of the clips differed. Details of each of the four clips used in this study can be found in Table 3.3.

Table 3.3

Descriptions of Video Clips

	Mathematics Content	Participation Structure	Problem Type
Clip #1	Writing and evaluating variable expressions	Teacher-led/direct instruction	Decontextualized
Clip #2	Writing and evaluating variable expressions	Teacher-led/direct instruction	Contextualized (purchasing hot dogs at a baseball game)
Clip #3	Proportional relationships	Partner work	Contextualized (Cost of Internet service)
Clip #4	Graphing linear equations	Small group work (with teacher)	Decontextualized

Wearing headphones, students individually watched each of the four short video clips. Students were randomly assigned to one of two versions that differed only in the order of the clips. Six students watched the clips in the order listed, while the other six watched the clips in the following order: #4, #3, #1, #2. After watching each clip, students were asked to type responses to the following questions:

1. In this lesson, do you see any moments where students are learning something useful or doing something useful?
2. If yes, what parts of this math lesson do you think seem useful, and why?
3. If no, why do you think this lesson is NOT useful?

Student responses ranged in length from a few words to a couple sentences.

Data Analysis

The collection of multiple data sources and analysis using multiple methods allows for triangulation by both method and data source to increase the robustness of findings. For example, students' conceptions of usefulness (the focal construct in this study) are examined using survey items, interview questions, and the video response task. Students' beliefs about their own competence are explored through rating scales, interview questions, and classroom observations. Additionally, as described in Chapter 4, features of learning environments that influence engagement are examined both in the classroom and in students' everyday activities.

In this work, quantitative analyses were used to evaluate survey ratings, while qualitative analyses were used to provide added depth to quantitative findings and analyze responses to short answer questions in the survey, interviews, and video task. For rating scales, I first conducted factor analyses to identify components of each scale. Subsequently, I calculated bivariate correlations to examine the relationships between variables. For survey, interview, and video task short answer responses, items were coded to examine students' conceptions of usefulness. These analyses began with a round of open coding to identify a variety of codes within each category. Codes were then collapsed, and a final coding scheme was created (see Appendix B). Two coders independently coded 20% of the responses and then compared results; a satisfactory measure of inter-rater reliability was achieved (Cohen's Kappa = .764). For all responses that had been coded differently, the two coders discussed differences and agreed upon one code for each segment. One coder then independently coded the remainder of the data. Finally, pseudonyms were provided to the twelve interview participants only. The other 72 survey respondents did not receive pseudonyms and so will be referred to in more general terms in the analysis.

Findings

Drawing on students' survey, interview, and video task responses, three main themes are explored. First, I highlight students' perceptions of the usefulness of mathematics and compare perceived utility value to other variables with which we expect utility value to be correlated based on prior research. Second, I examine students' value of usefulness in mathematics and their reasons for seeing usefulness as an important feature of mathematics. Finally, I explore students' conceptions of the usefulness of mathematics, or ways that students think about what it means for mathematics to be useful. I highlight two main categories – applicability of the content and features of the learning experience – and discuss the types and frequency of responses within each.

Student Perceptions in Mathematics

Before examining students' conceptions of the usefulness of mathematics (or what it means to students for mathematics to be useful), it is important to highlight students' reported perceptions of usefulness (or *whether* students think mathematics is useful). Students' perceptions of usefulness were measured using a modified version of the Fennema-Sherman Usefulness of Mathematics scale (Doepken et al., 2004; Fennema & Sherman, 1976). Students rated 12 different items on a scale of 1 to 5, where 1= strongly disagree and 5= strongly agree. The mean of all scores was then calculated to obtain an overall perceived utility value score for each student, in line with prior research and the intended use of the scale. As can be seen in Table 3.4, the mean perceived utility value was 4.08 (N=84; SD=.666), indicating that students on average agreed that mathematics is useful.

Table 3.4

Descriptive Statistics and Correlations for Perceived Usefulness, Interest, and Performance

Measure	*M*	*SD*	α	Range	1	2	3	4
1. Perceptions of usefulness	4.08	.666	.911	2.5-5	–			
2. Perceptions of usefulness – interdependent	3.75	.743	.831	2-5	.633**	–		
3. Interest	3.89	1.117	.911	1-6	.560**	.556**	–	
4. Reported grade	9.27	2.585	–	1-13	.290*	.191	.204	–

* Correlation is significant at the 0.05 level (2-tailed).
** Correlation is significant at the 0.01 level (2-tailed).

In addition to administering the Fennema-Sherman scale to students, I also altered the scale to create an interdependent version of the scale that focused on measuring students' perceptions of the usefulness of mathematics for helping others. As previously described (and detailed in Appendix A), the scale was validated during pilot testing and consisted of five items that used the same rating scale as the Fennema-Sherman Usefulness of Mathematics scale. When items were collapsed to form one mean score for each student, the mean rating of perceived usefulness for interdependent purposes across all students was 3.75 (N=83, SD=.743), suggesting moderate agreement that mathematics is useful for achieving interdependent goals. It is also worth noting that results from a paired samples t-test indicate that there is a significant difference between students' ratings of perceived usefulness in general and perceived usefulness for interdependent outcomes; $t(82)=4.863$, $p=.000$. This finding suggests that students overall see mathematics as more useful for their jobs and daily activities than for helping others.

Next, since studies of perceived utility value typically occur at the high school or college level, it is important to examine whether expected correlations between utility value, interest, and performance (as discussed in the literature review) surfaced at the middle school level. Interest was measured using a seven-item scale, three items of which measured situational interest and four items of which measured personal interest. Students rated their agreement with the items on a six-point scale, where 1=strongly disagree and 6=strongly agree. Factor analyses revealed only one factor for these items, so the mean of all seven scores was calculated to obtain one interest score for each student. Across all students, the mean interest score was 3.89/6.00 (N=83, SD=1.117), which corresponded to a level of interest that was slightly above the midpoint of the scale.

Performance was measured via students' reported mathematics grades from winter 2015. Students reported letter grades, which were recoded as numbers on a scale from 1-13, with 1 representing an F and 13 representing an A+. The mean grade among participants was a 9.27 (N=71, SD=2.585), which corresponded to between a B and B+. To check for accuracy of student reports, the correlation between actual and reported grades was calculated for ten students who gave permission for their grades to be released by the school. Reported and actual grades were significantly correlated, r=.5844, $p<.10$.

Table 3.4 highlights the correlations between utility value (independent and interdependent), interest, and performance. Consistent with prior literature, interest was very strongly correlated with perceived usefulness (both general and interdependent; p=0.01). The correlation between performance, as measured by reported grades, and general perceptions of usefulness was also statistically significant, though at the 0.05 level. However, reported performance was not correlated with interdependent perceptions of usefulness. Overall, using these measures with middle school students revealed that expected correlations with utility value did exist at the middle school level, suggesting alignment with prior research at the high school and college level.

Value of Usefulness

Before exploring ideas about usefulness or considering ways we might leverage students' conceptions to improve instruction, it is important to first establish that usefulness matters to students. In this section, I will draw on findings from a) a survey item that asked students to rate the importance of different values related to mathematics and b) an interview item asking students to explain their ratings. On the survey task, students used a scale of 1-5 (1=Strongly Disagree, 5=Strongly Agree) to rate their agreement with the statement, "It is important to me that the math I'm learning is…" As shown in Table 3.5, six different sentence completers were provided for rating – including "fun," "useful," and "easy." The statement with the highest mean rating of agreement and smallest standard deviation was "It is important to me that the math I'm learning is useful" (*M*=4.35, *SD*=.692), illustrating that students have a strong desire to learn mathematics they perceive as useful. In fact, only two of the eighty-two students disagreed with the statement, and only four were neutral; all other students either agreed or strongly agreed that it is important for the mathematics they learn to be useful.

Table 3.5

Mean Student Ratings of Important Features of Mathematics (1=Strongly Disagree, 5=Strongly Agree)

It is important to me that the math I'm learning…	Mean Rating (SD)	n
…is useful	4.35 (.692)	82
…challenges me	4.10 (.951)	82
…relates to my everyday life	3.88 (.992)	81
…is fun	3.78 (.930)	82
…is easy	3.01 (1.024)	82
…reflects my culture	2.94 (.934)	82

While we might entertain the fact that students' calls for usefulness are actually expressions of boredom or frustration with the difficulty of mathematics, these results suggest otherwise. Paired samples t-tests were conducted to compare students' ratings of the importance of mathematics being useful to their ratings of the importance of mathematics being fun or easy. There was a significant difference in students' ratings of usefulness and fun, $t(81)=5.58$, $p=.000$, as well as usefulness and easiness, $t(81)=11.019$, $p=.000$. Furthermore, the item with the second highest mean was "It is important to me that the math I'm learning challenges me" ($M=4.10$, $SD=.951$). Ratings of the importance of challenge were also statistically significantly higher than ratings of the importance of mathematics being easy, $t(81)=6.816$, $p=.000$, or fun, $t(81)=2.323$, $p=.023$. Thus, it is unlikely that students primarily emphasize usefulness because they find mathematics too challenging or boring.

To explore students' emphasis on usefulness in greater depth, interview participants were asked to explain why they either agreed or strongly agreed that it is important for the mathematics they are learning to be useful. Of the twelve students, six expressed that they don't see a purpose in learning mathematics that is not useful when they could instead be learning mathematics they will use in the future. For example, Rachelle described how she will be happier in the future if she learns useful mathematics now, rather than being taught concepts that she has no reason to learn: "If I learn math that's not useful, then it will be kind of like pointless to learn it, but if I learn math that is useful, then I can use it and it can be like very helpful for whatever I do and I'll be really glad that I knew it." Omar similarly echoed that he doesn't see a purpose in learning mathematics that is not useful since there are many types of mathematics he believes he needs to use in his life: “Math is in effect our everyday lives… So I feel like… if we don't learn math that is useful… why learn it at all?”

Meanwhile, five students focused on the fact that it is important to them that mathematics is useful specifically because of its applicability to future jobs or careers. While two students mentioned the importance of learning mathematics that will be needed to “get a better job” or for a career broadly speaking, three students mentioned specific careers. For example, Victoria stated that she wants to be an engineer, so learning useful mathematics “will help me do better in my job and like help me get hired.”

One student blended together these two types of explanations, stating that it is important to learn mathematics that will be needed in the future and mentioning that careers are one place where mathematics might be needed. This student, Carrie, also connected her perceptions of usefulness to her motivation in mathematics:

> If I’m learning things that I’m actually not going to use later on in my life, you know, I’m not going to know the things that I do need to know, for like careers and stuff, but if I learn things that are useful to me, I feel like—I mean, I pay attention in class but it would make me a little bit more you know, I’m going—you need this, so I better pay attention.

In this quote, Carrie not only emphasized the importance of learning things she will need to know in the future but also explained that learning useful material will increase her focus.

This sample of responses provides a small window into the meaning students place on usefulness. However, these comments also raise an important issue: Students talked about usefulness in rather different ways. Some emphasized wanting to learn mathematics that they will use in their careers, while others more generally expressed wanting to use math that will be needed in the future. Still others discussed using mathematics to develop “a better understanding of life” or wanting to see mathematics they are taught “outside of class.” If we aim to enhance students’ perceptions of usefulness, it is crucial that we first understand the ways they think about what it means for mathematics to be useful. The next section will explore the various ways students think about usefulness, as well as the frequency with which each conception surfaced.

Conceptions of Usefulness

In order to gain insight into the ways students thought about what it means for mathematics to be useful, I examined student responses on four different items drawn from the survey, interviews, and video task. In particular, the data drawn on for these analyses includes students’ survey responses to the questions a) *What is the most useful subject? Why?* and b) *Do you think you will use math in the future? Why or why not?* as well as interview participants’ responses to c) the card sorting task about useful versus not useful mathematics and d) the video task about the usefulness of four mathematics lessons. Coding these responses elucidated students’ conceptions of usefulness by providing a window into the criteria students used to make judgments about the usefulness of mathematics given different prompts.

Analysis of students’ responses to these tasks revealed that at the broadest level students’ conceptions of usefulness fell into one of two categories: *applicability of the content* and *features of the learning experience*. Responses that focused on *applicability of the content* highlighted one of four situations in which mathematics content might be applied: in a future job/career, in other classes or future schooling, in everyday life, or for specific everyday activities. These categories closely align with the conceptions of usefulness referenced in existing literature. Meanwhile, responses that fell into the second category, *features of the learning experience*, were substantially different from categories that currently exist in the literature. Students who discussed usefulness in this manner reflected on aspects of the learning experience, rather than the content itself, that were useful. Students primarily considered one of the following two features of the learning experience when making judgments about usefulness: the interaction involved in learning (*Are we working with others? Is the teacher there for support?*) and the usefulness of the task or structure of the activity (*Am I actively involved? Are students explaining their thinking?)*. Several students also considered the usefulness of particular mathematical representations, such as graphs and charts.

Below I will elaborate on each category, describing the prevalence of each conception of usefulness and providing rich examples of each. I will also briefly describe several less common conceptions at the end of the section. It is worth noting that while some students were consistent in using a particular criterion to make judgments about usefulness across all tasks, others considered the applicability of content on some tasks and features of the learning experience on others. I will address possible reasons for these changing conceptions in the discussion.

Applicability of content. This section explores the first way in which students discussed the usefulness of mathematics – in terms of applicability of the content. Responses that fell into this category described the usefulness of mathematics in terms of whether and where mathematics skills and concepts can be applied. Such responses emerged mainly from analyses of students' survey responses about a) the most useful subject (e.g. science, mathematics, foreign language) and b) whether students think they will use mathematics in the future and why. In the sections below, I elaborate on the five sub-categories within applicability of content, describe the frequency with which each emerged, and then examine the two most commonly occurring categories in greater depth.

Categories of responses. When thinking about usefulness in mathematics in terms of the applicability of its content, students' responses primarily fell into five main categories: everyday life (general), daily activities, everyday life + daily activities, job/career, and current or future schooling. The *everyday life (general)* category includes broad claims about how math is everywhere or can be used for everything. Examples include, "Because you use a lot of math in your daily life" and "Because where ever you go there is gonna be math[4]." Responses that were classified as *daily activities* described specific activities that mathematics is useful for (but did not include larger claims about overall usefulness in everyday life). For example, one student wrote, "Because without math it would be almost impossible to make, build, or buy anything," while another replied, "Because in the future we will need to know how much something cost and to do taxes." Responses that were coded as *everyday life + daily activities* included a combination of the first two types of responses. In other words, students made statements about the broad applicability of mathematics in everyday life but also provided examples of specific daily activities for which mathematics is useful. One such response was "it's a skill that your going to most likely use everyday in your lifestyle. It's a must have. Whether you go to the store and you need to pay there is math involved or when you grow older and need to start paying bills it all involves math." Finally, some responses referred to applicability of content for a future *job/career* (e.g. "Because in the future we all have to use math for any job" or "it will be helpful

[4] Since surveys were completed by students on the computer, all survey responses are copied exactly as students typed them.

if I am a casher or a banker.”), while others discussed usefulness in terms of applicability for *current or future schooling* (e.g. “Helps me with other classes” or “It helps teach you the hard math questions and when you're in high school you will understand it more”).

Frequency of responses. When students were asked to discuss the usefulness of mathematics on survey questions[5], responses across the five aforementioned categories were provided. I will consider responses to the survey items asking about the most useful subject and about whether students expect to use mathematics in the future. Responses considered include the 45 students out of 84 total (53.6%) who listed mathematics as the most useful subject, as well as the 82 students who provided responses about how, if at all, they expect to use mathematics in their future. The frequency of responses in each category can be viewed in Figure 2, and students' responses to each question will be considered separately below.

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[5] I only consider responses on survey questions here, as some of the videos in the video response task and pictures in the card-sorting task included problem contexts that influenced students' statements about usefulness. Since I intend to explore here how students think about usefulness without any cueing of particular contexts, I will limit this discussion to the two relevant survey items.

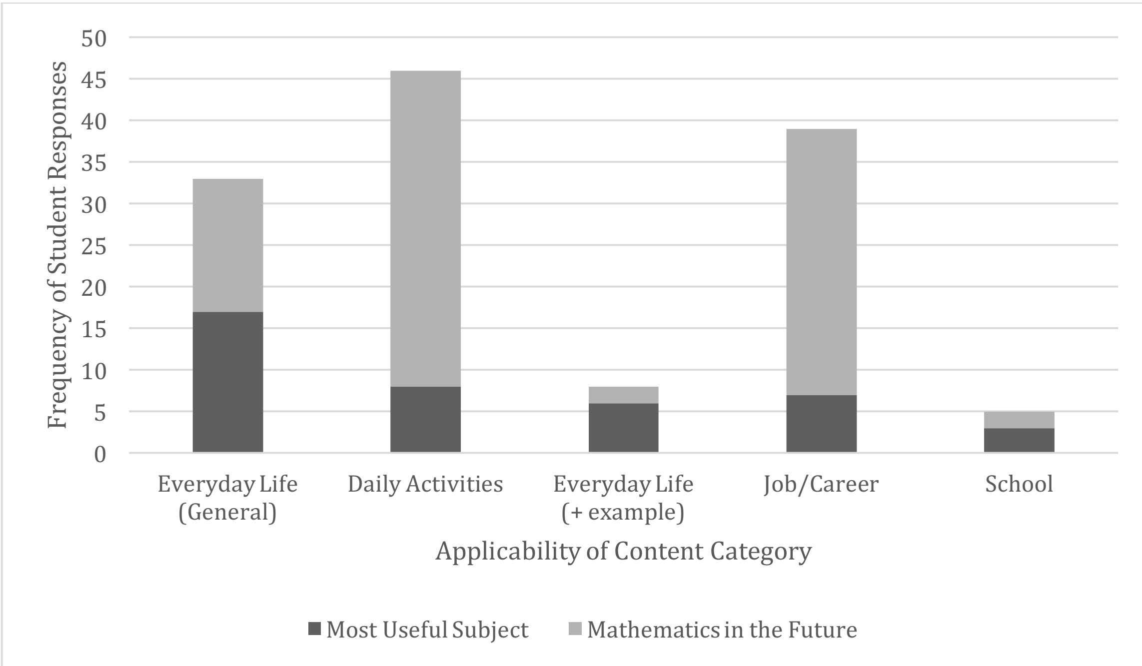

Figure 2. Students' Conceptions of Usefulness Related to the Applicability of Content

Of the five categories, usefulness for *everyday life (general)* was the most commonly cited reason for why mathematics is the most useful subject (*n*=17; 37.8%), followed by usefulness for accomplishing specific *daily activities* (*n*=8; 17.8%). Seven of the forty-five respondents (15.6%) cited *job/career* reasons, such as that mathematics will be helpful for performing particular jobs in the future, while six (13.3%) provided *everyday life + daily activity* explanations that represented a sort of hybrid between the first two categories. Finally, three of the forty-five respondents (6.7%) mentioned usefulness for current or future schooling.

When students wrote about how, if at all, they expect to use mathematics in their future specifically, students emphasized the same categories, albeit with slightly different frequency. Students most often wrote about usefulness for their future *job/career* (*n*=38; 46.3%), though they still frequently mentioned both usefulness for *daily activities* (*n*=32; 39.0%) and usefulness for *everyday life (general)* (*n*=16; 19.5%). While it is helpful to know the general categories into which students' explanations fell, it is also important to consider the variation within categories. Since there is little variability in the *everyday life (general)* category, and there are few examples in the *current or future schooling* category, responses related to *job/career* and *daily activities* – including the specific activities mentioned in *everyday life (+ example)* responses – will be examined in greater depth below.

A closer look: Daily activities and job/career. In this section, I explore the variation of student responses within the *daily activities* and *job/career* categories. Due to the different types of daily activities students discussed and the different ways in which students described usefulness for one's job/career, all responses in those two categories were coded a second time at a more granular level. Tables 1.5 and 1.6 provides breakdowns of the different types of responses provided in each of the categories.

One consistent pattern in the *daily activities* category across both of the aforementioned questions was that students who mentioned specific daily activities nearly always mentioned something money-related (e.g. doing taxes, buying/selling things, and paying bills) as one of their activities (see Table 3.6). In fact, while ten students mentioned other types of daily activities, four of those students also referenced the utility of mathematics for working with money. Thus, a total of 42 of the 48 responses focusing on specific activities referenced money-related actions. Zooming in a bit further, students were nearly as likely to mention shopping-related activities (*n*=20; 41.7%), such as purchasing items and calculating discounts or tax, as they were to reference home management-related money activities (*n*=22; 45.8%) including doing one's taxes and paying bills. Six students mentioned usefulness for money more broadly, and two students wrote about using mathematics to make sure they weren't cheated out of their money.

Table 3.6

Applicability of Mathematics for Specific Daily Activities

Sub-Category	Percentage of responses (*N*=48)	Examples
Money – management (bills, budgeting, taxes, mortgage)	45.8% (*n*=22)	"We have to pay bills and do taxes so we need math" "Because in life you need to know about how much money you make and how much you spend so you wont waste more than you earn"
Money – buying/ selling (incl. discounts)	41.7% (*n*=20)	"When I'm shopping I will know how much I will spend if there is 50% off" "When going to buy stuff you have to see what is a better buy or if there is a discount and want to know how much it will be"
Money – general	12.5% (*n*=6)	"In the future you have to deal with money" "When you you get older, you need to use math like with money for example"
Money – being cheated	4.2% (*n*=2)	"When we grow up, we want to make sure that we aren't robbed from our money that we earn or that we should get back"
Other (music, measurement, time, building, cooking, sports)	20.8% (*n*=10)	"Based on what we make we always use math to see how tall it is using numbers by inches, feet, and how far by kilometers" "You will need to know how to cook for your kids"

In the career category, students offered three types of career-related explanations of the usefulness of mathematics*:* usefulness in a future job (general), usefulness in a future job (specific), and usefulness for getting a job (see Table 3.7). Usefulness in a future job (general) included broad statements about the possibility of having to use mathematics in a future job (e.g. "Well maybe because you might need to do math in work in the future"), the expectation that all jobs will use some mathematics (e.g. "I think I will need math in the future because every job involves somewhat math”), or the idea that some people will need mathematics in their jobs while others might not (e.g. “Depending on what you what to be when you grow up.”). Approximately 63% of career-related responses included such explanations. Meanwhile, nearly one-third of the career-related responses were coded as usefulness in a future job (specific). Students providing such responses referenced specific jobs for which mathematics would be useful including cashier, teacher, doctor, graphic designer/game editor, business owner, architect, and banker. The most commonly referenced job was cashier, with six of the 15 students providing comments such as, “If you get a job as a cashier or something you have to count money and use math.” Finally, only four responses (8.7%) referenced the usefulness of mathematics for getting a job (e.g. “I think it is the most useful because you are more likely to get a job if you are good at math”).

Table 3.7

Applicability of Mathematics for One’s Job/Career

Sub-category	Percentage of responses (*N*=46)	Examples
In future job (general)	63.0% (*n*=29)	“Well maybe because you might need to do math in work in the future” “because you need it for your job”
In future job (specific)	32.6% (*n*=15)	“When I grow up I want to be a graphic designer or game editor, that in itself is basically all math” “If you're a cashier, you have to know how much change to give them”
For getting a job	8.7% (*n*=4)	“Knowing and being good at math will help you get a job easier” “You are more likely to get a job if you are good at math”

Within the category of usefulness in a future job (specific), we might question whether the jobs students mentioned are the jobs they intend to pursue in the future. Only three of the 15 students clearly indicated that the job they mentioned is one they would like to have when they get older. For example, one student wrote, “When I grow up I want to be a graphic designer or game editor, that in itself is basically all math.” In contrast, the other 12 students typically provided hypothetical scenarios in which one would need mathematics *if* one worked in a particular job. For example, one student commented that mathematics would be useful to her in the future “because what if you want to be a math teacher and teach these subjects,” while another wrote, “You never know what you will work in when you grow up. What if you end up being a casher your going to need to know math.” Additionally, when asked later on in the survey about where they see themselves in ten years, none of these twelve students mentioned the jobs they referenced when describing why they will need mathematics in the future. Rather, the six of those 12 who mentioned using mathematics if they become a cashier reported that in ten years they expect to be “making millions,” have a “good job” with “a good amount of money, be an “explorer,” and/or be attending college. While it might be advantageous that students can view mathematics as useful for careers they do not necessarily intend to pursue, such perceptions might also be less motivating since they do not align with students’ own goals. A case study is provided below to illustrate how one student’s conception of usefulness related to career influenced the way she thought about the usefulness of particular mathematics topics.

A case study: Katie. Katie is a 13-year-old female in Ms. Sanchez’s honors mathematics class. On her survey, she reported viewing mathematics as the most useful subject because “it mostly has everything in it and you learn a lot in math too.” Additionally, she thinks she will use mathematics in the future “because if I become a teacher or something else that relates to math than I would need that knowledge of learning math in middle school.” Given these responses, one might draw the conclusion that Katie's perceived utility value of mathematics is high, which is likely to positively impact her achievement-related outcomes in mathematics. However, examining Katie's responses about the use of particular mathematics topics paints a slightly more complex picture. During her interview, Katie was asked whether she imagined using six different mathematics topics that her class worked with during the year. If she thought she might use a topic outside of class, she was asked to state where she imagined using each. Katie's responses can be viewed in Table 3.8.

Table 3.8

Katie's Views on the Usefulness of Specific Mathematics Topics

Mathematics Topic	Use of Topic in Life (Katie's perspective)
Commission and Mark-up	"Commission and markup can be but I don't think it is—<u>for like cashier work</u>. 'Cause they're, like-they're, like, using a much, like, money they have to give or something. I don't know, yeah."
Adding and multiplying fractions	"Um, <u>I don't think you use this for any jobs</u>, I'm not sure."
Writing equations	"Um, no? I'm not sure."
Finding equivalent ratios	"Well, maybe those are like, <u>for math teachers</u>. But yeah, that's all."
Making graphs	"Uh... um, like, maybe making a graph for uh... for let me see, like-like for science? Like something-something that works for science. Like <u>a job that is for science</u>."
Calculating perimeter and area	"For <u>construction</u>, 'cause you need to see how much perimeter has and what area you're gonna use."

In these responses, Katie illustrated a strong and rather stable conception of usefulness in terms of the applicability of mathematics for one's career. Katie reported that four of the six topics might be used in future careers, and she mentioned a different career for each. Additionally, Katie stated that one of the remaining two topics would not be used "for any jobs." It is important to note that no prompting to think about jobs or careers was provided, so Katie could have referenced ways in which she might use these topics for a variety of different purposes. However, for five of the six topics provided she spoke solely about uses – or lack thereof – for jobs. Another important note is that none of the jobs Katie mentioned are jobs that she actually imagines herself doing. After Katie reported that she could imagine making graphs in “a job that is for science,” she was asked whether she was imagining that for herself or for others, and she clarified that she meant “just other people.” She was asked the same question after she reported that calculating perimeter and area would be useful “for construction,” and she again said that she imagined that for “other people.” Furthermore, at the end of Katie’s interview, she was asked whether she knew what she wanted to do for work when she got older, and she replied that she has “two choices” – either a lawyer or a veterinarian. Katie reported that she will “probably not” need mathematics for either.

This case study illustrates a tension between students’ career-related conceptions of usefulness and their own goals and values. Katie’s strong emphasis on the usefulness of mathematics for careers that she does not plan to pursue raises the question of just how motivating Katie’s perceptions of usefulness will be. In the discussion, I will consider potential implications of this lack of alignment in terms of Eccles and colleagues’ expectancy-value model.

Finally, it is worth noting that some students reported that they do not imagine using mathematics in the future particularly because they do not expect their future jobs to involve mathematics. In contrast to Katie, these students’ perceptions of the usefulness of mathematics were affected by the lack of alignment between mathematics and their imagined future careers. In particular, it is worth noting that of the 82 students who responded to the question about whether they anticipate using mathematics in the future, only four reported that they did not. All four of those students cited career-related reasons for why they did not expect to use mathematics in the future. For example, students provided responses such as “I don't think that the occupation I choose will involve too much math, maybe typical everyday math but not algebra” and “Depending on what you what to be when you grow up.” These responses suggest that while some students – like Katie – are able to maintain a sense of mathematics as useful for careers despite its lack of connection to her own desired career, other students view that disconnect as a signal that mathematics will be less useful for them. Thus, viewing the usefulness of mathematics in terms of one’s job/career can have several different outcomes regarding one’s overall sense of the usefulness of mathematics, the implications of which will be explored in the discussion.

Features of the learning experience. While considering the applicability of mathematics content was the most common way of thinking about what it means for mathematics to be useful, a second way of thinking about usefulness emerged: considering whether the features of the learning experience provided some value. This conception of usefulness has not been considered in prior literature on perceptions of usefulness and thus is a new addition. Conceptions of usefulness focused on features of the learning experience emerged primarily in interviewees’ responses to the card sorting task and video task. Thus, those two tasks will be considered in the section below, as I describe the ways in which students discussed usefulness in terms of features of the learning experience.

Categories of responses. Students who described usefulness in terms of features of the learning experience provided explanations focused on the *method of interaction, structure of the activity, or representation being utilized.* Responses focused on the *method of interaction* included descriptions of how interacting with others while learning can be useful. For example, one card in the card-sorting task showed a picture of a student writing on a whiteboard while his teacher looked on from behind. One interview participant said that the boy was doing useful mathematics because “the teacher’s showing the kid how to do it, but also the kid is showing what the answer could be, and the teacher’s right there to like say that’s not the right answer but you can do-you can use this to figure it out.” The second type of response in this category focuses on the usefulness of the *structure of the activity.* Such responses included references to the forms of activity students were engaged in. While different forms or structures also involve people interacting in different ways, the *structure of the activity* responses emphasized aspects of the structure other than interaction with others. For example, one student viewed a picture of students writing on the board together and thought that seemed useful because the kids were “drawing on the board” and “showing like, what they think can be the answer to the question.” Here, the boy emphasized the acts of writing on the board and showing one’s thinking, rather than the interaction with the other students at the board. Finally, responses focused on the usefulness of the *representation being utilized* located the usefulness of the task in the mathematical representation being used by the student. For example, an interview participant who viewed a picture of students working with a chart noted that useful mathematics was happening because “charts just help represent data and math” and that “it helps you understand how to… put it so that other people can visualize.”

Frequency of responses in each category. Responses focused on features of the learning experience occurred with differing frequency across the tasks. For example, on the most useful subject question, none of the forty-five students who listed mathematics as the most useful subject provided reasons related to features of the learning experience. However, on the card sorting task and video response task, which were designed to allow different conceptions of usefulness to emerge, seven of the 12 students (58%) considered features of the learning experience at least once – and often numerous times – to describe their reasoning. Overall, 20 of the 96 coded card-sorting and video task responses considered features of the learning experience. Ten of those responses cited the usefulness of the structure of the activity, eight mentioned ways in which the form of interaction or participation involved in the learning was useful, and three focused on the usefulness of representations being used. Responses within the two primary categories – *structure of the activity* and *method of interaction or participation* – will be explored further below.

A closer look: Structure of the activity. Half of the students who were interviewed (*n*=6) provided responses focused on the usefulness of the structure of the activity. Four of the ten responses occurred during the card sorting task, and the remaining six responses were provided in reaction to video clips. On the card-sorting task, Jose responded to three different pictures by describing the usefulness – or lack thereof – of the structure of the activity with which students were engaged. For example, when viewing a picture of students sitting at their desks and watching their teacher at the front of the room, Jose replied with the following:

> "I believe this is not like, not really useful 'cause…kids like for me, I want to like get up on the board and show like ideas, but the kids are like just sitting there and watching the teacher do the math, and it looks like they actually wanna do like an activity that involves math and that they can actually show their expression of if this is like a way to do it or not."

In contrast, when Jose viewed a picture of students at a board drawing graphs on poster paper, he felt that they were engaged in useful mathematics:

> "I think this one's useful… 'cause, you can see the kids like, they're drawing on the board, so they're-they're showing like, what they think can be the answer to the question they might be answering. And they'll be showing the work and they can show the teacher like, is it-is this how I'll be able to figure out my answer to the question."

In both situations, Jose considered the structure of the activity students were engaged in ("watching the teacher do the math" versus "drawing on the board" and "showing the work") in order to decide whether the lesson was useful.

On the video response task, participants similarly considered aspects of the activity students were engaged in when deciding whether any useful mathematics was happening. As with all four card-sorting task responses, half of the video task responses focused on the usefulness of structures in which students were actively engaged and showing their thinking. For example, Liliana found one clip to contain useful mathematics because she could "see the work of the students" and "hear the student explaining their work and the teacher questioning them." In contrast, three of the video task responses (from two students) found more traditional activity structures to be useful. In one video clip Pauline thought it was useful "how the student is receiving information from the teacher," while Robert found it useful that the students "were paying attention and were taking notes."

A closer look: Method of interaction. Students who provided responses focused on the usefulness of particular methods of interaction (*n*=4) primarily found collaborative participation structures to be useful. Three of the four video task responses in this category emphasized the usefulness of students helping each other in mathematics. For example, after watching a video clip of two students working to solve a problem about the cost of Internet service, Pauline and Robert both considered the interaction involved when describing why they thought students learned something useful in the clip. Pauline wrote that the useful part was “how a student gets to teach another student,” while Robert noted that the mathematics lesson was useful “because it looks like the kid on the right is tutoring the kid on the left.” In both of these cases, the students made judgments about the usefulness of the lesson based not on the content or context of the problem (proportions, cost of Internet service) but on the way in which the students interacted to solve the problem (with one student “tutoring” or “teach[ing]” the other).

Similarly, the four card-sorting task responses in this category emphasized the usefulness of working with others to solve mathematics problems. When viewing a picture of a group of students building a bridge out of toothpicks and gumdrops, Victoria noted that working as a group can both challenge you and help with understanding:

> “You’re basically being challenged to put more effort into doing math with other people. Even if they’re better you try to do better. And if you make a mistake it will help you understand math more. And if you work with other people, um, maybe they can help you understand math more.”

In this example, Victoria focused on the usefulness of interacting “with other people,” rather than the usefulness of the specific activity students were engaged in or the mathematics content they were learning. When viewing a picture of a student writing on the board, Victoria also noted the potential usefulness of that student’s interaction with classmates:

> "Maybe the kids that are watching him write on the board, um, they can help him. Like, if he did something wrong, um, the kids who know will… can help him understand or the kids who don't know will see what he's doing wrong and will understand why he was doing it wrong if the kids who know or the teacher helps explain why it was wrong. So basically…what he's doing is useful to everybody almost and it helps the teacher improve and all the kids improve."

In this comment, Victoria highlighted the way in which interactions between the student, his/her classmates, and the teacher can be useful in itself – regardless of the mathematics content or problem context. Two other students responding to the same picture discussed only the student and teacher who were depicted in the image. These students emphasized the usefulness of having the teacher there to "help out the student" or provide suggestions while the student attempts to solve a problem. Only one of the eight responses coded as *method of interaction* did not highlight positive benefits of one's interaction with his/her peers or teacher.

These two ways of thinking about the usefulness of mathematics – in terms of the structure of activity and method of interaction – provide a new addition to existing conceptions of usefulness. Through the inclusion of new stimuli, such as pictures and videos of students engaging with mathematics, this research illuminates a previously unexplored way in which students think about usefulness in mathematics. While most responses on the aforementioned items were coded under either the applicability of content or features of the learning experience categories, some responses either could not be coded or fell into less common categories. Before moving on to discuss implications of this work, I will briefly address these other conceptions of usefulness.

Additional conceptions of usefulness. The responses not yet considered fell into several different categories. First, some responses were eliminated as not able to be coded because they were not specific enough for the coders to determine students' conceptions of usefulness. Second, some responses centered on tautological explanations that amounted to "It's useful because it's math." Setting those aside, as neither provides insight into the ways students think about the usefulness of mathematics, the remaining responses fell into four different categories: *value of learning new or important things, contribution to one's personal skills or development, affective,* and *other*.

Fifteen responses across all four of the aforementioned survey and interview items focused on the *value of learning new or important things.* Students providing such responses highlighted the fact that new or important knowledge was being learned, which made the task or material useful. For example, when viewing a picture of a student writing on a whiteboard while his teacher looks on from behind, Robert said the boy was doing useful math because "it looks like he's learning something new or he's trying out something."

Next, far less common were explanations about *contribution to one's personal skills or development.* Only five responses across all tasks focused on how mathematics can be useful for improving certain skills. When viewing a picture of students watching a teacher project something on the front board, Ethan stated, "I'd say this, for the most part is useful, because it-it enhances your listening skills, and it sharpens your eye coordination… and your focus too." Third, only three responses focused on *affective* reasons for why students viewed mathematics as useful or not useful. For example, one student reported that mathematics was the most useful subject "because it's my favorite subject and it's funner than the rest." Finally, a handful of responses were collapsed into an *other* code since there was only one instance of each type. Due to the infrequency with which such conceptions surfaced, they will not be discussed further in this chapter.

Discussion

Primary findings from this work cluster around three main themes. First, students find it important that they learn mathematics that is useful. While students in general also report perceiving mathematics as useful, there are some potential disconnects between stated perceptions of usefulness and perceptions of usefulness for one's own self. Second, there is important specificity to be added to the categories of usefulness that exist in the literature, particularly related to conceptions of usefulness for daily activities and one's future job/career. And finally, conceptions of usefulness related to features of the learning experience reflect students' perspectives but are not represented in the literature. In the sections below, I will elaborate on each theme and then consider the implications of this work.

Students' Perceptions of Usefulness and the Importance of Usefulness

One finding emerging from this work is that students placed great importance on learning mathematics that is useful. Although we might question whether their calls for usefulness symbolize some other frustration with or lack of interest in mathematics, two pieces of evidence suggest that isn't the case. First, students on average rated the importance on mathematics being useful as significantly more important than the importance of mathematics being fun or easy. This suggests that there is something about students' calls for usefulness that is distinct from their desire for enjoyment or simple success. Additionally, students more strongly agreed that they want mathematics to be challenging than to be easy, again suggesting that students are not simply expressing frustration with the difficulty of mathematics through their calls for usefulness. Second, of all responses coded for conceptions of usefulness, only three included explanations that were affective (e.g. It is not useful because it is boring). The infrequency of this type of response provides further evidence that students are making judgments about the usefulness of mathematics separate from their level of enjoyment of the subject. Students had real reasons for wanting mathematics to be useful and particular ideas about the ways in which it is and is not useful, which are important to further explore moving forward.

In addition to finding it important that they learn mathematics that is useful, students on average also viewed mathematics as useful. Perceptions of usefulness, as indicated by students' responses on a modified Fennema-Sherman scale, were fairly high in general (M=4.08/5.00). However, given the sociocultural lens I bring to this work, as well as the analyses conducted on students' conceptions of usefulness, several questions arise. First, are students viewing mathematics as useful for things that they care about? Related, are students viewing mathematics as useful for themselves or more broadly? The case study of Katie speaks to these two questions, as Katie has a strong sense of the usefulness of mathematics for particular careers, yet none of those are the careers she imagines for herself. And finally, what kinds of mathematics are students viewing as useful? While students might think that the subject of mathematics is useful because basic operations are used everywhere, they still might not see usefulness in the day-to-day activities and topics of seventh-grade mathematics. Thus, although self-reported perceptions of usefulness are high, these perceptions might not serve to motivate students as they work their way through middle school mathematics.

Adding Specificity to Existing Conceptions of Usefulness

A second theme emerging from these findings is that the existing categories of usefulness for everyday life and usefulness for job/career are not homogenous categories. When coded at a more granular level, there were notable differences in students' conceptions related to these two categories. First, while career-related items in existing research sometimes focus on usefulness for getting a job and sometimes focus on usefulness for performing one's job, students were much more likely to focus on the latter. Related, most students who focused on usefulness for performing a job did not have particular jobs in mind; rather, they made general statements about the usefulness of mathematics in various jobs. This lack of specificity might be a product of the relatively young age of these participants, as many did not know what job or career they planned to pursue. However, it might also be that the students have little knowledge of the specific duties performed in various jobs and instead take the word of adults who claim that mathematics is useful for one's future career. If students are, in fact, simply relying on the claims of others, it might be that their perceptions of usefulness related to career are not as motivating as they would be if they were strongly tied to students' own goals and values.

Looking back to Eccles and colleagues' expectancy-value model (Eccles & Wigfield, 2002), one's goals directly influence one's task values – including perceptions of usefulness – which in turn affect achievement-related choices and outcomes. Task values are also directly influenced by a child's affective reactions and memories, as well as expectations of success. However, there might be another lever directly affecting students' stated perceptions of usefulness. If students frequently hear from respected adults that mathematics will be useful in the future, they might come to echo their elders without searching for more specificity or having a strong sense of the ways in which it might be useful. While relationships with others can moderate goals in Eccles and colleagues' model, it is possible that these relationships influence perceptions of usefulness directly without impacting adolescents' goals. In such cases, stated perceptions of usefulness might be high even though there are weak or non-existent connections between students' own goals and their perceptions of the usefulness of mathematics for particular careers. This possibility will be important to examine in future research, as there might be implications for the degree to which perceived usefulness influences one's academic choices in such situations.

Another related possibility is that students genuinely view mathematics as useful for particular careers, yet they do not see themselves in those careers. In the example of Katie, we saw a student who held exactly this view. Again, the role of valued others might be relevant, as parents and teachers often tell students about the ways in which mathematics is useful for people in particular careers, such as engineers, architects, and cashiers. This focus might encourage students to develop a strong sense of mathematics as useful for careers, particularly if students place great value on careers; however, such a perception might not translate to a student viewing mathematics as useful for her own career. Again, it will be important in future work to examine the potential impact of such disconnects on one's motivation and performance in mathematics.

Second, within the category of usefulness for specific everyday activities, students focused most heavily on money-related activities. This emphasis provides a point of leverage for utility value interventions, as students might be most motivated by discussions that center on the usefulness of mathematics for handling and managing money. Additionally, students' emphasis on monetary activities suggests that they might have relevant out-of-school experiences to draw on that involve using or observing the use of mathematics when dealing with money.

However, given the few references to non-money-related activities, younger adolescents might also benefit from discussions about the range of activities they might need mathematics for in the future. If we consider using self-generated utility value interventions (e.g. Hulleman, 2007; Hulleman & Harackiewicz, 2009) with middle school students, they might struggle to write detailed descriptions of the ways in which mathematics is useful – particularly if the content they are considering cannot be easily connected to monetary activity. Rather, this age group might reap an especially large benefit from a combination of directly-communicated and self-generated utility value information, as proposed by Canning and Harackiewicz (2015). If direct communication is grounded in students' goals and values, such discussions may be particularly motivating and provide a foundation for students to write their own statements about the usefulness of mathematics. Additionally, helping students to recognize and articulate the various spaces where they use mathematics outside the classroom might help them to make stronger connections between the mathematics they participate in at school and their everyday mathematics experiences.

Pushing the Boundaries of Conceptions of Usefulness

The final, and perhaps most significant, theme to emerge from this work is that existing conceptions of usefulness in the literature do not capture the range of ways in which students think about usefulness. In particular, students discussed the usefulness of mathematics not only in terms of the applicability of its content (in everyday life, for specific daily activities, for one's future job/career, and for school/college) but also in regards to features of the learning experience. When asked to identify whether students in videos or pictures were engaged with useful mathematics, interview participants often considered the usefulness of the form of interaction or activity structure itself. Within those categories, students were most likely to find interactions to be useful when students were working together, teaching each other, or showing their thinking to a classmate or teacher. Although these conceptions of usefulness perhaps go beyond the mathematics, they reflect ways of thinking about usefulness that surfaced when students were asked about the usefulness of mathematics. Thus, students appear to be conceptualizing usefulness in a way that is not currently captured in the literature. As such, I argue that it is important to draw on students' own framing of usefulness and consider the category of features of the learning experience when studying utility value.

Additionally, while conceptions of usefulness that fall within this category are not explicitly considered in existing research, there is room to fit this way of thinking about usefulness within existing definitions. For example, Eccles & Wigfield (2002) described utility value as being "determined by how well a task relates to current and future goals" (p. 120). If we consider the goals of adolescents, Wentzel (1989, 1993) highlights middle school students' emphasis on not only academic but also social goals in the classroom, such as cooperation, dependability and responsibility. In fact, striving for socially appropriate goals strongly predicts academic achievement (Wentzel, 1989, 1993, 2004). Thus, students' consideration of the features of the learning environment – such as the form of interaction between students – when thinking about the usefulness of mathematics might be connected to their strong prosocial goals. If students possess such goals, it would make sense that they would consider whether the learning environment supports their achievement of those goals when assessing the useful of their learning.

Thus far, research on utility value has not considered the usefulness of features of one's learning experience; rather, researchers have focused on the usefulness of the subject's content, even if that focus has not typically been acknowledged. For example, Harackiewicz et al. (2012) stated that one "finds utility value in a task if he or she believes it is useful and relevant for other aspects of his or her life" (p. 1). While no specific aspect of "the task" is identified, existing interventions have focused specifically on the mathematics content of tasks. However, one can imagine considering the method of interaction as one component of the task that an individual might find relevant for other aspects of his or her life – be it future schooling contexts or interactions outside of the school setting. In fact, this research suggests that it might be especially important to consider this way of thinking about usefulness in middle school mathematics classrooms. Thus, we might consider adding specificity to the former definitions of utility value to say that a person finds utility value in a task if he or she believes that *either the content/material or form of participation/engagement* will be useful and relevant for other aspects of his or her life.

If we again return to Eccles and colleagues' expectancy-value model, one question we might pose is where valuing some forms of learning over others fits. Since valuing particular ways of learning seemed to directly influence students' perceived task value, we might inquire as to whether that feature is a component of a child's affective reactions and memories or goals and self-schemata. Do students value these particular forms of learning because they have had positive experiences that they remember from the past? Or do students view learning in particular ways as part of their self-schemata? Alternatively, perhaps a new addition to the expectancy-value model is needed. In future work, it will be important to explore potential mechanisms for valuing particular features or forms of learning to better understand where that influence on one's subjective task value fits.

Implications and Future Directions

Findings from this work have implications for both research on utility value and the design of learning environments that promote high perceptions of utility. I address each category of implications in turn below, also highlighting some future directions for this work based on those implications.

Implications for research on utility value. Contributions of this work have a variety of implications for future research on utility value. First, these findings highlight the importance of using mixed methodologies in utility value research. By including interview tasks and video response tasks along with rating scales and short answer survey questions, students were provided with multiple stimuli to provoke their ideas about usefulness. Using these various methods allowed new conceptions of usefulness to emerge. In particular, with only one exception, students discussed the usefulness of the interaction or participation structure when viewing pictures or video clips of students engaging with mathematics but not when answering survey questions about the usefulness of mathematics. While we might be able to imagine asking students about the usefulness of features of the learning experience using survey questions, that conception of usefulness initially surfaced only through the use of qualitative methods of data collection, suggesting that some conceptions might be missed when we focus solely on survey methods for gathering data. Certainly, using rating scales allows us to gather information from a larger sample of students; thus, I am not suggesting we move entirely to qualitative methods. However, it is important to supplement rating scales with open-ended questions and other techniques for eliciting student ideas to gain a more complete picture of students' conceptions and perceptions of usefulness. In particular, when designing utility value interventions, it might be helpful to begin by identifying how the focal participants think about utility, which can then inform the design of more targeted, effective interventions.

One possibility for an intervention that has an open-ended structure yet is also sensitive to time constraints involves the use of self-generated utility value prompts. Existing interventions that include self-generated utility value statements take a first step toward putting the conceptions of utility in the hands of the individual and allowing participants to consider whichever form of utility is most salient and important to them. For example, one existing intervention asked participants to write about the "potential relevance of this technique to your own life, or to the lives of college students in general" (Hulleman et al., 2010, p. 5). When responding to this question, participants were able to think about their own experiences and potential future uses of the material. Despite the benefits of such interventions, they still constrain respondents to considering usefulness in terms of the applicability of material. Moving forward I propose we design prompts that allow participants to consider both applicability of the content and features of the learning experience when generating utility value statements. For example, rather than prompting participants by asking them to consider how certain material might be useful in their own lives, we might consider broadening our prompts to ask, "How might this material, or the way in which you learn this material, be useful to you in your own life?" Alternatively, we might focus on benefits for mathematics learning in particular and ask students to consider how the ways in which they learn are useful for gaining competence in mathematics. Additionally, examples might be provided to encourage participants to think more broadly about how the experience of learning can be useful for other aspects of their lives.

In the future, we might also consider testing a new type of self-generated utility value intervention in which participants watch video clips of individuals performing the task they either already performed or will be performing. Rather than learning about the task through reading or direct instruction, watching others might allow participants to more strongly consider features of the learning experience. When generating utility value statements, participants might then be asked about the ways in which either the content or the form of learning might be useful for the person engaging in the practice. Reflecting on others' experiences performing a task or activity might help to further enhance perceived utility value.

A second set of implications of this research stems from the emergence of a new conception of usefulness related to features of the learning experience. Through applying a sociocultural lens to utility value, this work allowed students to express a way of thinking about usefulness that is not currently considered in the literature. Adopting an expanded definition of utility that includes this conception will have implications for the way we measure utility value and design interventions. Currently, measures of utility value primarily draw attention to conceptions of utility related to applicability of the content. While often not explicitly citing the content, scale items ask individuals to rate how useful mathematics will be for one's future career, for example, with items such as "Knowing mathematics will help me earn a living" (Fennema & Sherman, 1976). This phrasing of "knowing mathematics" places emphasis on the content, rather than the process of learning mathematics. However, one might also imagine the inclusion of items related to the process of learning, such as, "The ways I am able to participate in math class help me in other aspects of my life" or "The methods of instruction used in math class are useful." Such items might tap into perceptions of usefulness that have previously gone unmeasured but that might be especially important for particular groups of students and would allow us to more equitably measure perceived utility value.

It is worth noting, however, that such items might be more difficult for younger adolescents to respond to than older adolescents or adults due to the high degree of abstraction. Thus, some additional guidance might be important to include to help middle school students respond to such items. One option would be to include a sentence or two prior to each item to suggest examples of forms of participation or methods of instruction. For example, the second item stated above might be changed to "Teachers use different ways of teaching in math classrooms. Some ways of teaching are writing notes on the board for students to copy or giving students problems to work on together in groups. The ways of teaching used by my teacher in math class are useful." Another alternative would be to provide the initial statement followed by several scenarios to give students a sense of what methods of instruction they might consider. Finally, a third possibility is to create a digital version of the card-sorting task, which would allow information to be gathered from a larger number of students while still enabling students to consider the usefulness of features of the learning experience in a developmentally appropriate manner. Regardless of the particular way in which this is addressed, it is important to remember such developmental considerations when creating items for younger adolescents.

Another pathway to explore related to this new conception of usefulness is whether it is connected with mathematics specifically or applicable to many fields. While students' views regarding working with others might apply in other domains, ideas about the usefulness of particular representations, for example, might be more specific to mathematics. As a result, questions remain about whether students are considering the features of learning mathematics specifically, or ways of learning more broadly that are useful in other domains. Moving forward, it will be important to explore whether this new conception of usefulness is connected to the content of mathematics, as well as whether students' perceived utility of other subjects is influenced by features of the learning experience.

Finally, this work highlights the fact that there are multiple pathways through which students' perceptions of usefulness can be shaped. I propose that there might in turn be a range of implications depending on how students came to develop their perceptions of usefulness. Examining this possibility in future research will be important. Related, future work should explore how students come to believe mathematics is useful for particular purposes. This research highlighted students' strong sense of mathematics as useful for their careers; however, future research will be needed to more deeply explore how students come to have such beliefs, including which significant others most influence their perceptions of usefulness. As Martin's (2006) work highlights, parents often have vivid memories of their own mathematics experiences, as well as beliefs about mathematics that might be communicated to their children. The role of such stories and beliefs in students' perceptions of the usefulness of mathematics should be examined in future research. Additionally, looking across different communities and examining the relationship between careers for which students view mathematics as useful and the careers to which they have been exposed will be helpful in understanding the impact of students' exposure to careers on students' perceived utility of mathematics.

Implications for the design of learning environments. In addition to having implications for future research on utility value, this study's findings also have implications for classroom practice and the design of learning environments. First, while the results of this study related to students' conceptions of usefulness are not generalizable, the methods used to gather information about students' conceptions of usefulness can be taken up by teachers or districts. For example, these methods can be used to examine whether students are most focused on the applicability of content, or if they also emphasize the usefulness of particular methods of learning. Then when teachers attempt to frame mathematics content in terms of its usefulness, they can draw on the types of usefulness that matter the most to their own students.

Related, as previously described, this study was motivated in part by a teacher wanting to find ways to address her students' questions about the usefulness of mathematics, as well as students' broader lack of motivation in mathematics. Indeed, many mathematics teachers wonder about how to field such queries from their students. Drawing on this new conception of usefulness, we might consider not only highlighting the usefulness of mathematics content for jobs and everyday life but also the usefulness of particular ways of engaging with mathematics. Such a framing might be especially relevant in light of the mathematical practices encouraged by the Common Core State Standards. Many teachers have been increasingly urging their students to engage in thoughtful problem solving, explain their thinking to their peers, and critique the reasoning of their classmates. Such forms of interaction and structures of activity nicely align with some of these seventh-grade students' emphases on the usefulness of working cooperatively and showing their thinking. It might even be that students see usefulness in some of the Common Core's mathematical practices separate from any learning that occurs. Thus, highlighting the usefulness of particular methods of interaction and activity structures might provide another avenue through which students can see usefulness in the mathematics classroom.

Finally, broadening the definition of utility has implications for mathematics classroom practice. Rather than solely considering the usefulness of mathematics content, teachers might consider how to engage students in mathematics in useful ways. As adults, we can likely recall times when we learned useful things in ways that did not seem useful (an unnecessarily long, dry lecture on safety procedures, for example). While we might have come to accept the fact that information is sometimes learned in less useful ways, we have an opportunity to change that at the middle school level and to instead help students learn in ways they perceive as useful. Many current reforms in mathematics education have already been attempting to make such changes. Now we have additional reasons to work towards improving the methods we use to engage students, as those methods might influence students' perceptions of the usefulness of mathematics. In the next chapter, I examine some of the most common methods of interaction, norms, and expectations that the seventh-grade students in this study experienced in the mathematics classroom and compare those with the types of interactions and expectations that students reported experiencing in everyday activities involving mathematics.

4. Features of Student Engagement with Mathematics in Everyday Activities and in the Mathematics Classroom

"Why does everything have to be independent? Life is not independent. You have a partner! And then you have kids." Arturo, 7^{th} grade

Chapter 3 highlighted two primary ways in which students think about the usefulness of mathematics – in terms of applicability of the content and features of the learning experience. More specifically, students tended to view mathematics as useful when it could be applied in everyday life or in future jobs/careers, and when they were able to collaborate with others, engage in hands-on activities, or show their thinking. Given the former, it follows that students would see themselves as doing useful mathematics when they are engaged with mathematics in their everyday activities. In contrast, classroom mathematics might sometimes be viewed as useful and other times be viewed as not useful by students. In this chapter, I first explore students' everyday activities, considering the features of engaging with mathematics in those settings. Subsequently I examine typical features of students' engagement with classroom mathematics and compare those with the features that emerged while examining students' everyday activities. This comparison highlights the ways in which activity structures, interactions, and norms that students encounter in the classroom sometimes differ from their out-of-school experiences. In this chapter, I ask the following questions:

1) How do Ms. Sanchez's students engage with mathematics in everyday activities, and what are key features of their engagement with mathematics in those settings?

2) How do the features of mathematics learning in Ms. Sanchez's classrooms compare with the features of students' engagement with mathematics in everyday activities?

The chapter is organized as follows: I begin with a discussion of the methods and analyses that were used to explore these questions. Then I transition into the findings section, first describing the focal everyday activities that were explored and the mathematics that students reported using in those activities. Subsequently, I present the key features of students' engagement with those everyday activities, drawing on findings from student interviews. Finally, I move into a comparison of those features with salient features of the classroom learning experience that influence the ways students engage with mathematics. During that discussion, I also highlight students' comments regarding the authenticity of classroom mathematics, connecting their perceptions of authenticity with students' out-of-school experiences. I conclude with a discussion of the findings and consideration of implications of this work.

Methods

This chapter primarily draws on data from classroom observations, a student interest survey, and semi-structured interviews about out-of-school practices involving mathematics. Data from the first round of student and teacher interviews is briefly discussed, as well. Observations, first-round interviews, and administration of the student interest surveys took place in the three seventh-grade classrooms during the 2014-2015 school year. However, due to timing issues, second-round interviews (about everyday activities) could not be conducted until the 2015-2016 school year with Ms. Sanchez's sixth-grade students. To determine the similarity between the sixth- and seventh-grade students' experiences, baseline observations were conducted, and select questions from the student interest survey were administered. Once participation in similar activities was confirmed, semi-structured interviews were conducted with the sixth-grade students. Below I describe these processes in greater detail, highlighting the data collection and analysis techniques that were used for each.

Classroom Observations

Data collection. Three of the four seventh grade classes participating in the study were observed regularly over a period of five months. The classes were selected both to include variation in the level/track of classes and to comply with Ms. Sanchez's observation preferences. One of the classes was labeled an "honors" mathematics class, and the other two were considered "regular" mathematics classes (though Ms. Sanchez viewed one as much more advanced than the other.) In total, 60 classroom observations were conducted across the three classes between January and May of 2015 to examine student engagement, as well as salient features of the classroom context. Additionally, since Ms. Sanchez had mentioned that her students often questioned her about the usefulness of mathematics, observations were also used to document discussions of usefulness in the classroom. Jottings were taken during all class sessions, and fieldnotes were written up after each observation.

Five classroom observations of the participating sixth-grade class were also conducted during the first part of the 2015-2016 school year. The purpose of these observations was to document any significant differences in classroom context or student engagement from the seventh-grade classes. Since classes were twice as long as they were the prior year, observations also served to document how time was spent during the longer class. To supplement these observations, I also engaged Ms. Sanchez in several informal conversations about how her planning changed as a result of both the new class length and learnings from the prior year.

Jottings were taken during each class session, and fieldnotes were written up after each observation or conversation.

Data analysis. A coding scheme was developed to code all classroom observations. Prior research was first used to identify features of the classroom environment that might influence student engagement, as well as indicators of student engagement. In order to allow for new features and themes to emerge, a team of researchers also engaged in a round of open coding of the fieldnotes, noting both actions that indicated student engagement (or lack thereof) and features of the classroom environment that might have influenced engagement. Moving between the literature and open coding of fieldnotes, a coding scheme was developed to examine student engagement with mathematics in the classroom.

The coding scheme included three main categories: student interactions/actions, student orientations/perceptions, and features of the classroom context. Student interactions/actions was designed to capture the ways students engaged with mathematics in the classroom while the student orientations/perceptions category was developed to examine the ways students perceived and reacted to mathematics and features of the classroom environment. Meanwhile, the features of the classroom context category was used to identify salient features of the mathematics classroom that were related to the ways in which students engaged with mathematics.

Each category contained two or more levels of codes within it. Codes in the interactions/ actions category included helping behaviors (e.g. requesting help or offering help), question asking, discussion moves (e.g. justifying reasoning or critiquing another student's thinking), and general on-task and off-task behavior. Within the perceptions/orientations category, codes were grouped into the following sub-categories: perceptions of competence, relation to everyday life, orientation toward collaboration, orientation toward challenge, comparison to others, adherence or opposition to classroom rules/norms, orientation toward mistakes, and affective response. Finally, within the features of the classroom context category, fieldnotes were coded for the following: task (e.g. Do Now or note-taking), mathematics topic (e.g. ratios or commission and mark-up), participation structure (as stated by Ms. Sanchez; e.g. whole group discussion or independent work), observed participation structure (used when the participation structure observed differed from what Ms. Sanchez had stated), class period, and month of the year. The participation structure codes note the ways in which students were arranged while learning, such as in small groups or working independently. The complete coding scheme, including all four levels of codes within each category, can be found in Appendix C.

Codes within the features of the classroom context category were created and tested first. Two coders applied the coding scheme to several sets of fieldnotes, comparing their coding and then refining the coding scheme as needed throughout. Two new sets of fieldnotes, which constituted approximately 10% of the data, were then selected for interrater reliability coding. After coding, Cohen's Kappa was calculated, and a strong level of agreement was achieved (Cohen's Kappa = .85). Any segments that were coded differently were discussed by the coders, and a single code was agreed upon for each excerpt. One coder then coded the remainder of the data.

Following this process, the final two categories of codes were applied to the data. First, two coders coded a random selection of excerpts to identify differences in interpretation, codes that required additional clarification, and codes to be added. Several theoretically significant excerpts were then selected to further refine the coding scheme. After changes had been made, two new sets of fieldnotes from different points in the year (each comprised of observations of all three class periods) were selected for interrater reliability coding. Again, these fieldnotes constituted approximately 10% of the data. Following coding, Cohen's Kappa was calculated, and a satisfactory level of agreement was achieved (Cohen's Kappa = .76). Coders then discussed all excerpts that were coded differently until agreement on codes was reached. Each coder then took charge of one category of codes and applied those to the remainder of the data.

Interviews

Data collection. As described in the previous chapter, 12 students were interviewed across the three observed seventh-grade classes. Additional information about those interviews can be found in Chapter 3. For the purposes of these analyses, I will be drawing on one question that was asked of all students: “If you need help in math class, what do you generally do?”

An interview with Ms. Sanchez was also conducted near the end of the 2014-2015 school year. The interview lasted 83 minutes and focused primarily on Justine’s instructional goals, struggles as a first-year teacher, definition of mathematics, and definition of utility, as well as her perceptions of her students’ involvement outside the classroom, engagement with mathematics, and ideas regarding utility. In this chapter, I mainly draw on findings regarding Justine’s expectations for students in the classroom. Since classroom observations began in January and fieldnotes were unable to capture norms and expectations that were established in September, Justine’s reflection is used to gain insight into how she established classroom norms and expectations at the start of the year.

Data analysis. As described in chapter three, student responses to interview questions were coded to examine conceptions of usefulness, definitions of mathematics, and perceived benefits of mathematics. In this chapter, I draw on coded responses about what students do when they need help in mathematics class. Since there is only one teacher interview, Justine's responses are used to provide a rich qualitative description of her experiences as a first-year teacher and her perceptions of usefulness, mathematics, and her students. In particular, I highlight the ways Justine spoke about establishing expectations for her students in order to lay a foundation for understanding interactions in her classroom.

Interest Survey

Data collection. Forty students (21 female, 19 male; average age = 12.7 years old) across the three observed seventh-grade classes completed an interest survey. This sample represents just under two-thirds of all students in the three classes. All students were not able to respond to the survey for logistical reasons. Ms. Sanchez asked students to fill out the survey when they had completed other classwork, yet some students did not finish their work in time. She did give students the option of completing the survey at home (the link was posted on their course website) but few students did.

As with the utility survey, students completed the interest survey on their laptops. Participating students were first asked to identify hobbies or activities they participated in regularly by completing several short answer questions. The students were then provided a list of activities developed by the researcher and asked to select how frequently they participated in each – never, occasionally (less than once a month), regularly (at least once a month), or frequently (at least once a week). Activities included in the list were either activities the school offered for students to participate in, activities that students had referenced during classroom observations, or typical activities that adolescents participate in and that might involve mathematics. Students were then provided the list of activities they had reported participating in and were asked whether they thought each of the activities “does not involve math,” “might involve math,” or “involves math.” At the conclusion of the survey students also reported their gender, age, and race/ethnicity.

Twenty-four students (17 female, 7 male; average age = 11.25 years old) in the sixth-grade class also completed a condensed version of the interest survey. These 24 students represent all students who were in the class at the time of administration. The interest questions were administered in combination with questions from the utility survey one day in class. Students completed the survey on the computer in 10-15 minutes. In this survey, students were first asked whether they participated in six particular activities (the selection process for those activities is described below). Students then received the following prompt: “Some people think the activities they do involve math. For example, people who sew clothes sometimes say they use math to figure out how much fabric they need. Do you think any of the following activities involve math?” Students then chose “does not involve math,” “might involve math,” or “involves math” for each of the activities. Finally, students were given the option to write in any other activities they participated in that they perceived as involving mathematics.

Data analysis. The seventh-grade students’ survey responses were used as a basis for selecting focal activities to explore during semi-structured interviews with the sixth-grade students. Several criteria were used to select activities. First, at least 25% of the seventh-grade students who completed the survey must have reported participating in the activity occasionally, regularly, or frequently. This criterion was developed to ensure that chosen activities were participated in by a range of students and that a sufficient pool of students would be available for interviews about the activity. However, activities were not limited to those in which students most commonly participated, as I did not want to close off opportunities to examine activities that involved rich mathematical engagement simply because they were participated in by a smaller subset of students.

Second, the majority of students who reported participating in the activities must also have perceived those activities as involving mathematics. Since the goal of these surveys and interviews was to better understand how students engaged with mathematics outside the classroom, it was crucial to select activities that students perceived as involving mathematics. Again, however, I did not limit selection to activities that all students identified as involving mathematics, as some students might have engaged in different versions of the activities or might not have recognized the mathematics they were doing. In all activities that were selected, at least 75% of the students who participated in each activity viewed it as involving some level of mathematics.

Third, activities were selected based on the availability of potential observation sites in the area and the likelihood of being able to observe students engaging with mathematics while participating in the activities. Due to difficulties in both timing and gaining site approval, the observation component was removed from the project, and semi-structured interviews were used as a replacement for observations. However, at the time when focal activities were selected, observations were still part of the research design, so the criterion regarding potential observation sites influenced the selection of focal activities.

Finally, in addition to using the aforementioned criteria to select each individual activity, activities were selected in order to allow for variation in types of activity and participation across genders. The six activities that were selected for further exploration are the following: building/construction, cooking/baking, gardening, graphic design, jewelry making, and playing video games. These six activities were included in the sixth-grade students' survey to identify similarities in participation with the seventh-grade students. Student participation was tabulated, and again more than 25% of students participated in each activity, and the majority of those participating viewed each as involving mathematics. Thus, those activities served as the basis for semi-structured interviews, which are described below.

Semi-Structured Interviews

Data collection. Semi-structured interviews were conducted with nine of the 24 students in the sixth-grade class to better understand their participation in activities they perceived as involving mathematics. All students in the class were given the option to participate in an interview; any students who returned consent forms were interviewed. The students were fairly representative of the class as a whole, with a mean age of 11.2 years and student grades ranging from A+ to B. Comparable percentages of the sample and class identified as Mexican, Mexican-American, or Chican@, and comparable percentages reported living in houses and apartments. Additionally, the students reported participating in a mix of all of the focal activities to be considered in interviews. The one feature on which the sample differed from the class overall is gender. While 17 of the 24 students in the class identified as female, eight of the nine interview participants identified as female. Thus, results should be considered in light of this difference.

Responses from the condensed utility and interest survey were used as a starting point for interviews. In each interview, students were asked to consider a particular activity that they reported participating in and perceived as involving mathematics. Activities that students reported participating in most frequently were discussed first; if time allowed, activities students participated in less frequently but still perceived as involving mathematics were also discussed. For each activity, students were first asked to share when they started participating, how they got involved, how often they engaged with the activity, and where they performed the activity. Students were then asked about their purposes or goals for engaging in the activities. Subsequently, I asked students to imagine being at the location where they typically engaged with the activity. They described the space, including the tools and people most often involved, before discussing the actions they typically took and the mathematics they tended to use while engaging with the activity. Finally, students were asked to speak about why they participated in the activity and given an opportunity to add any additional information about their participation. If time allowed and students had noted other activities on the survey that they were involved in and perceived as involving mathematics, then the protocol was repeated another activity. The length of interviews ranged from 13-33 minutes, with most lasting between 20 and 25 minutes.

Data analysis. The goal of the semi-structured interviews was to develop an understanding of the ways students engaged with activities that involve mathematics outside the classroom. Thus, I drew on prior literature examining everyday mathematics and influences on adolescent engagement to identify key themes to explore. Topics were then selected for coding as a way to identify patterns of engagement within and across activities and students. Categories that were used in top-down coding included students' motivation and goals for participation, emotions, resources needed, people involved, arrangements of people and objects during participation, and mathematics involved. Two categories from Nasir and Hand (2008) were also used – access to the domain (or opportunities for learning about the practice and the skills necessary to engage in it) and opportunities to make unique contributions and feel valued (or "ways that students can incorporate aspects of themselves into the practice"; p. 148). All coding categories can be viewed in Table 4.1.

Table 4.1

Coding Categories for Semi-Structured Interviews

Coding Category	Description/Details of Category
Access to the domain	Borrowed from Nasir and Hand (2008). Includes opportunities for students to learn about both the practice at large and the skills necessary to engage in it.
Arrangement of people and objects	Includes the set-up of the space, as well as the way in which people are physically arranged while participating.
Mathematics involved	Includes any descriptions of ways in which mathematics is involved in participation. Responses were subsequently coded at a more granular level to identify the specific types of mathematics involved.
Motivation/goals for participation	Includes students' reasons for participating in the activities.
Opportunities to make a unique contribution and be valued	Borrowed from Nasir and Hand (2008). Includes students' descriptions of ways they can "incorporate aspects of themselves into the practice" (p. 148).
People involved	Includes references to any people involved in participation.
Resources/artifacts involved	Includes references to any resources needed for participation or key artifacts involved in participation.
Emotions/affective response	Includes any mention of students' affective response regarding participation or emotions experienced while participating.

After using this top-down coding process to identify relevant excerpts in each category across all interviews, responses were then examined for themes across the students. Several sub-categories were identified within the coded excerpts regarding the mathematics involved, motivation/goals for participation, and access to the domain, so responses in those categories were then coded at a more granular level. In the following sections, I highlight the findings that emerged from this coding process.

Findings

Findings are organized into three main sections. First, I provide an overview of the activities students reported participating in and the degree to which they viewed each as involving mathematics. In the second section, I explore themes from students' semi-structured interviews, highlighting both the types of mathematics students reported using in the aforementioned activities and key features of the out-of-school learning experience. Finally, I compare students' engagement with mathematics outside the classroom to engagement with mathematics in Ms. Sanchez's classroom. In particular, I explore methods of interaction and norms/expectations in the classroom, as well as the contexts included in problem-solving tasks. Below I begin with an examination of students' out-of-school activities.

Activities Involving Mathematics

The purpose of this section is to provide a broad sense of the activities students reported engaging in both to suggest potential areas of exploration for future work and to make clearer the body of data from which the six focal activities were selected. For these analyses, I draw on data from the interest survey that 40 students completed. Table 4.2 illustrates student participation in all activities that were included in the survey. Activities are listed in order of frequency of participation by students at any level – either occasionally (less than once a month), regularly (at least once a month), or frequently (at least once a week). As the table demonstrates, the activities participated in by the greatest number of students were cooking/baking, video games, basketball, and running. Students also used an "other" category to write in additional activities that were not included in this list. However, none of the activities (tennis, volleyball, and student council) were listed by more than four students; thus, they were not considered for further exploration.

Table 4.2

Participation and Perceptions of the Mathematics Involved in Out-of-School Activities – Grade 7

Activity	Number of students participating at any level	Number of students participating frequently (at least once per week)	Number of participating students who say the activity might or does involve mathematics
Cooking/Baking	38	13	36
Video games	36	24	30
Basketball	36	17	33
Running	33	16	27
Soccer	29	10	23
Baseball/Softball	27	6	24
Football	27	4	23
Bowling	25	2	21
Computer games or programming	23	13	21
Dance	21	10	15
Playing an instrument	19	5	10
Fantasy sports	18	7	14
Skateboarding	17	3	14
Graphic Design	17	5	13
Building/Construction	14	2	12
Gardening	14	2	12
Selling things	13	4	13
Chorus	12	8	8
Writing/composing music	11	4	8
Hockey/field hockey	10	1	9
Jewelry Making	10	3	9
Fashion Design	9	7	8
Speech and drama	6	4	4
Poms/cheer	6	2	5
Knitting	4	2	3
Makerspaces	3	1	2
Robotics	3	2	3
Yearbook	3	1	2

As previously mentioned, six activities from the overall list were chosen for further exploration: building/construction, cooking/baking, gardening, graphic design, jewelry making, and video games. Of the 40 students completing the survey, nearly all (*n*=38) reported cooking/baking at some level, while 13 reported cooking/baking frequently (at least once a week). Additionally, 36 of the 38 students believed that cooking/baking does or might involve mathematics. Nearly as many students reported playing video games at any level (*n*=36), with many of those students playing frequently (*n*=24). Furthermore, 30 of the 36 students who played video games believed it does or might involve mathematics. For the remaining four categories, 17 students reported engaging in graphic design, 14 students participated in building/construction, 14 took part in gardening, and 10 reported making jewelry. For each activity, more than 75% of the students viewed participation as involving – or perhaps involving – mathematics.

The six selected activities were then included in the survey administered to sixth-grade students. Table 4.3 illustrates sixth-grade students' levels of participation and perceptions of mathematics involvement in each of those activities. Of the 24 students completing the survey, between 11 and 22 students reported participating in each of the focal activities. Video games and cooking/baking were the only two activities that more than 4 students reported participating in frequently (at least once a week), with 12 students frequently cooking/baking and 13 frequently playing video games. Additionally, all of the students who engaged in cooking/baking, jewelry making, and building/construction at any level thought that participation involved or might involve mathematics. Of the 22 students who played video games, 17 reported that it does or might involve mathematics, while 14 of the 15 students who gardened and 12 of the 19 who engaged in graphic design reported the potential involvement of mathematics. These patterns of participation and views regarding the involvement of mathematics are comparable to what was seen with the seventh-grade students; thus, these six activities continued to serve as the focal activities for semi-structured interviews.

Table 4.3

Participation and Perceptions of the Mathematics Involved in Out-of-School Activities – Grade 6

Activity	Number of students participating at any level (occasionally, regularly, or frequently)	Number of students participating frequently (at least once a week)	Number of participating students who say the activity might or does involve mathematics
Video games	22	13	17
Cooking/Baking	19	12	19
Graphic Design	19	2	12
Gardening	15	3	14
Building/Construction	14	4	14
Jewelry Making	11	2	11

In addition to being questioned about the six focal activities, sixth-grade students also had the opportunity to write in additional activities that they participated in and perceived as involving mathematics. The following are the activities students reported with the number of students listing that activity in parentheses: art/drawing (4), hair stylist (3), fashion design/making clothes (2), make-up/nail artist (2), sports/basketball (2), music (2), dancing (2), laundry (1), time and speed (1), running (1), measuring (1), science EXT (1), and cutting wood for a play (1). Since some variety of fashion (hair, make-up, fashion design) was listed seven times, with one student providing an extended written description of the ways she uses mathematics when doing hair and make-up and designing clothes, fashion was considered as a seventh activity for student interviews.

Engagement with Everyday Mathematics

During interviews with nine students, the seven focal activities were discussed with the following frequencies: cooking/baking was discussed six times, building/construction was discussed five times, fashion was discussed four times (though often not as deeply, as this topic typically surfaced at the end of the interviews), video games and graphic design were each discussed by three students, and making jewelry and gardening were each discussed by two students. In this section, I look across all activities to explore how students reported using mathematics as they participated in these activities. Subsequently, I present key themes regarding aspects of students' participation that were central to the ways they engaged with mathematics in those settings. This process will help to highlight key features of students' engagement with mathematics outside the classroom.

Mathematics topics. As students discussed their uses of mathematics in out-of-school activities, three main mathematical topics emerged: measurement, counting and cardinality, and geometry. Although these topics might appear to be named at different grain sizes, the three categories arose from the ways in which students spoke about the mathematics they used, and they are all categories of content standards in the Common Core Standards. Below I examine the prevalence of references to each category, as well as the ways in which students reported each use of mathematics emerging. Then I briefly highlight several other mathematics topics that only a few students reported using. Table 4.4 shows an overview of the number of activities in which students reported using each topic, as well as the number of students who reported using it.

Table 4.4

Reported Use of Mathematics Topics in Everyday Activities

Mathematics Topic	Number of Activities in Which Topic Was Used	Number of Students Who Reported Using the Topic
Measurement	7	9
Counting & Cardinality	3	4
Geometry	2	2

Measurement. The mathematical topic that emerged most frequently in discussions of the seven focal activities was measurement. All nine students mentioned ways in which they used measurement while engaging in one or more out-of-school activities. Furthermore, measurement was cited as being used in each of the seven activities. When discussing measurement, students reported using at least five different measurement quantities: length, volume, mass, temperature, and time. Students also reported using a range of tools to assist with measurement, from rulers and measuring cups to their hands and coins. Below I provide several examples of the ways in which students reported using measurement in their activities.

When building, students reported measuring length, width, and height in ways that one might expect. For example, Analise reported using a measuring tape to measure the length of objects she builds for her brother: "When I'm building with just stuff that I have around the house, I have to measure how long it is and I would have to, and sometimes I would be building a box for [my brother], I would have to see how long it is or how far it's supposed to be across." Arianna similarly reported using a measuring tape or sometimes a ruler to build objects at home because her "dad mostly has all the tools for measuring."

Students who engaged with cooking reported using a variety of types of measurements, as well as both formal and informal tools for measuring. Some students, such as Kristal, reported using measuring cups in rather traditional ways while cooking: “I have to, like if I’m making pancakes or something, I’ll have to measure the flour, measure the milk, and like measure how many, how much of the little dough stuff goes into the pan to make the pancake.” Analise described her use of measurement similarly and then added the importance of measuring accurately: “If you don’t get the right amount you need, whatever you’re cooking can mess up and it’s not going to taste the way it’s supposed to be.” In a different realm, Reagan discussed her use of coins as points of reference to help her cut pieces of food the right width, a process she described as akin to measurement. Reagan reported, “[When I cook I’m] measuring things all the time and sometimes we have to cut the vegetables in a certain way. I mean we don’t actually like measure them, but we have to kind of like get them to a good size almost as if we’re measuring them.” When discussing what constituted a “good size,” Reagan recalled trying to meet her mother’s criterion of cutting vegetables “penny size or dime size.” Thus, Reagan used coins as a sort of informal tool, or reference point, to assist with measurement while cooking.

When describing their gardening practices, both Kristal and Cherell also reported using informal – as well as formal – tools to assist with measurement. For example, Kristal reported using a cup with a line drawn on it to measure the depth of the holes she digs: “You have to make sure the hole isn’t too deep and you have to put, you have to like, we usually take like a cup, like a normal cup, and fill it about halfway with soil, maybe less…. so like we’ll draw a line about halfway.” In contrast, Cherell discussed using a ruler when working with her uncle to measure the depth of their holes: “We just dig up like a hole, but not too deep for the plants… We usually use a ruler… We put it like up to the line and then we just leave the ruler there and then we start digging." In other examples, which will be discussed later, both girls reported using their hands to assist with measurement when spacing out their plants.

As a final example, Reagan recalled needing to measure length in several different situations while engaging in fashion design. First, she spoke about making a duct tape dress for her cousin:

> "I also like to do duct tape dresses, so I once did one with my cousin and I had to measure out how much of duct tape I wanted to use… She was there with me, so we took a garbage bag and we kind of measured it around her and we took the duct tape and also kind of measured it around her so we got an idea of how long we needed it to be and we just started cutting the same size pieces and just started putting them around her and making the dress."

Reagan also spoke about needing to know length measurements when she does hair and make-up. She recalled watching hair and makeup tutorials that tell her to take "an inch width of hair" or "a centimeter of white shadow…and like a quarter of an inch of black." While describing these examples, Reagan highlighted the importance of having a sense of what those measurements mean – or in her words, understanding "how much that is" – in order to be able to accurately replicate the hair or makeup style.

As described above, students reported seeing mathematics in their activities in the form of several different types of measurement. Additionally, students spoke about measuring in six different contexts and reported using different tools and strategies to carry out that measurement. After describing the other types of mathematics students saw in their activities, I will return to analyze some of the key themes and values that are important for understanding what it means for students to engage with mathematics outside the classroom.

Counting and Cardinality. The second most commonly cited use of mathematics in the seven out-of-school activities explored was counting. Four different students mentioned counting in three activities: cooking, jewelry, and video games. Two students provided one specific example of counting while cooking – counting how many eggs they need for their recipes. For example, Alexis said that she counts "how many eggs go into the batter," while Analise recalled, "Sometimes you need one or two eggs. You need to count how many eggs you have." No other examples of counting in the context of cooking were provided.

The two students who spoke about counting when making jewelry both spoke about counting their materials. Veronica, who makes bracelets on a loom, said that it's important to count "to make sure you have the right amount of bands on each peg or the whole thing falls apart." Alexis, who makes jewelry using beads, recalled counting for a different purpose. Rather than to keep her jewelry from falling apart, Alexis counted to make sure she had sufficient materials to keep her pattern going: "I kind of count to see how much there is… There's a thing, like a pattern I want… so I just to keep the pattern going, I want to have the exact number so I have four beads and four beads." Though for different purposes, both Alexis and Veronica reported counting as being a part of carrying out the activity of making jewelry.

Finally, both Jared and Veronica spoke about counting while playing video games. Jared reported counting squares to determine the width and height of his Minecraft shelters; thus, he used counting to carry out measurement. Though Veronica did not speak about measuring, she recalled that she "count[s] how long each [house] is to make sure they're all the same sizes." She also reported counting when she killed monsters in the game because “you have to hit a monster a few times before it can die.” Overall, Veronica stated, “I have to count a lot in Minecraft.”

Geometry. Two students spoke about seeing geometry in their everyday activities. In particular, Veronica saw geometry in both video games and building, and Analise saw geometry in building. When playing Minecraft, Veronica said that she uses “geometry and symmetry and stuff with squares and shapes.” She elaborated to say that “the world [in Minecraft is] basically just a square… Everything is pretty much squares. So I just think of squares everything.” Other than identifying the world as made up of squares, Veronica reported using geometry because “you kind of have to see how [the monsters] are shaped to know how to destroy them.” She also cited that it was important to her to ensure that the sides of the houses she builds are “parallel.” While Veronica’s discussion of geometry in Minecraft does not go much beyond a recognition of shapes and parallel sides, she sees geometry as a salient component of playing Minecraft.

Similarly, shapes were important to Veronica when she engaged in building. She again identified “geometry” and “symmetry” as being important in her process when she builds furniture for her American girl dolls. Again, the geometry component was largely about identification of shapes. For example, when she makes a table, Veronica said that she uses “geometry because I pick a shape.” She also cited identification of shapes as important for executing her plans: “I make sure it’s a square or rectangle and if it’s not I keep cutting and just make sure it’s all aligned right and it’s even, because I don’t want a parallelogram.”

Rather than focusing on shapes, Analise spoke about the importance of angles in her building. In particular, she discussed the need to calculate angles when her robotics club built a robot for a competition:

> "You had to figure out how long and like the ninety degree angle [for the robot] to shoot the ball or one hundred eighty degree angle of how to shoot it…Since [the goals] were on the corners, we would have to be on a, you know, seventy, ninety, almost a right angle, so we'd have to drive [the robot] and position it in the right way and make sure it's in the exact spot, otherwise it would just go and it would just miss the net. So we'd have to go around a ninety degree angle to get right where the low goal is and we'd have to be able to shoot it at a, I think, one hundred degree angle?"

In this case, determining the angle for their robot to shoot their balls at was a critical component of her team's building process. Furthermore, this example involves geometry being used in a more sophisticated way – to problem solve – rather than just as background knowledge. Similarly, when working at home, Analise saw both shapes and angles in her personal building activities: "Sometimes if I put [a box I'm building for my brother] in a corner I have to like, what shape should it be, like triangle… If I want to slide it into a corner, I would have to figure out what angle to put it into." Like Veronica, Analise saw the identification of shapes as part of her building process and then added the component of determining the necessary angles for her building project.

Other. Additional ways in which one or two students reported using mathematics in their activities included planning ahead, determining proportional relationships, adding/subtracting, and using fractions. When describing how she engages with building at home, Analise described the planning process as being one way in which she uses mathematics: "Well thinking of just like…planning ahead, thinking of where to put it, measuring how much, and just thinking of what I want to build. So basically all that planning." Similarly, when Reagan discussed making

Chex mix with her mother, she reported, “You have to like kind of stir the Chex mix every fifteen minutes for an hour and fifteen minutes. So you need to kind of plan out how many times you need to stir it. So I feel like there’s also math involved with that.”

Next, proportional relationships surfaced when Kristal spoke about the mathematics involved in changing recipe amounts to serve different numbers of people:

> “It involves a lot of math, cooking. Because you have to, like, if it’s a recipe for two and want to make it for six, I have to multiply everything by three, which isn’t the easiest, like to multiply like measurements and fractions by three. That’s actually what we’re learning right now but it’s not as easy as two times three… I usually do change the amount of the recipe. Like if I want to add, like sometimes if we have this big party and the recipe only feeds like sixteen, I’ll have to maybe multiply it by two or something.”

Although she did not refer to proportional relationships by name, Kristal found herself using her knowledge of proportional relationships to adjust recipes when her family has a “big party.” Third, Jared spoke about “adding and subtracting” when playing the video game Minecraft. He said that he needed to add and subtract to “keep track of the villages and how many [people] are dead and how many are alive” and “how many animals I killed to like use for food.” Finally, Julie saw fractions as important in the context of cooking, primarily because of the fractional measurements involved. She spoke about calculating measurements, such as “one third of a cup,” and then added, “So I use fractions, like fractions, they help me cook, too. Baking especially.” Julie did not provide further elaboration on the topic.

Summary. In the seven activities discussed, students reported seeing and using mathematics in a number of different ways. Many students reported measuring in their everyday activities, while a smaller subset of students recalled using mathematics in rather basic ways, such as counting and identifying shapes. When we look more closely at these processes, however, some interesting features of students' engagement emerge. In the next section, I highlight these features to elucidate the types of structures, norms, and interactions that students experienced when engaging with mathematics in everyday contexts.

Features of engaging with mathematics in everyday activities. The goal of elucidating structures, norms, and interactions in out-of-school activities involving mathematics is to enhance our understanding of features of students' engagement with mathematics they perceive as useful. Since students identified the seven aforementioned activities as spaces where they use mathematics in their lives, engagement with those activities is examined to better understand ways in which students engage with useful mathematics. Looking across student reports of engagement in the seven activities examined, four key features of students' experiences emerge. First, learning typically happens through apprenticeship with significant others or seeking guidance from experienced others. Second, norms of using estimation and/or trial and error often exist when engaging with mathematics in out-of-school settings. Third, students value the opportunities for creativity and self-expression that they experience when using mathematics in these everyday activities. And finally, students engage with the activities to accomplish meaningful interdependent goals. I will address each of these in turn below, highlighting specific examples of the ways in which each theme surfaced in students' interviews.

Learning from experienced others. Eight of the nine students interviewed spoke about engaging in activities with more experienced others. Most of these students discussed the contexts of cooking and building, though gardening, fashion, and video games were discussed once each, as well. The experienced others students spoke about were nearly always older family members, including mothers, fathers, aunts, uncles, grandmothers, and older brothers and sisters. Students recalled these experienced others playing a role both in their initial involvement with the activities and in their learning during participation.

The role of influential others in initial involvement. Three students discussed watching experienced others as the reason for their initial involvement in an activity. For example, Kristal reported that she began gardening with her mother when she was five years old. She recalled the moment her interest began:

> "When I was five I was, I saw a bag of dirt and I started poking at it because it's really squishy when like it's half empty, but it's still filled with dirt. So I started playing with it and then I saw my mom starting to put like seeds inside the ground and then I got like, 'Oh, what's that?'"

Kristal continued to discuss how she would stare at one tomato plant in particular and then started helping her mom to water it. Although the plant ended up dying, Kristal reflected, "After that, I kind of grew a liking to it and I just kept planting some with my mom."

Veronica was similarly introduced to playing Minecraft through watching and playing with her cousin and uncle. She recalled not being interested at first but then eventually deciding to play after continuing to watch her uncle:

> "I remember, I saw my cousin playing it and I was like, 'That game is so dumb. Why do you guys play it?' And then… I saw [my uncle] playing it more and then he asked me if I wanted to play and since I liked building things a lot I just kind of liked how they always came out."

Finally, Reagan reported getting into hair and make-up after watching her mother and older cousin. She recalled, “I just kind of got into it now because I see my mom doing it all the time. I just wanted to start doing that because my older cousin, she’s like a few months older than me, but she does a lot of hair and makeup.” Although none of the girls elaborated on what, if anything, they learned from these experienced others, they all acknowledged that watching their family members was the reason for their involvement in the activities.

Turning to experienced others for assistance. Other students more concretely discussed how experienced others helped them when they needed assistance. Some students discussed learning through watching others, receiving instruction from others, and seeking help on an as-needed basis. While Cherell discussed learning how to set their oven temperature through watching her mother, Analise recalled her mother explicitly teaching her how to cook particular meals: "Sometimes my mom would have me help her in the kitchen. So she’d tell me like how to cook pasta, chicken, you know, basic things that you would eat." Analise reported enjoying cooking with others “because if I don’t know how to cook something or if I can’t get it right, I always would have someone to help me out.” Since her mother had more experience than she did, she was able to turn to her mom for help anytime she needed it.

Analise similarly turned to more experienced others for help when participating in her robotics club. In the one example of a student citing a non-family member as the more experienced other, Analise discussed learning from her fellow team members. Initially, when Analise participated in tryouts for the robotics club, she recalled being lucky to have a partner who could help her out:

> "I think it was a good first experience because I never really worked on that type of crafty work with metal and things. But I had a partner who… helped me a lot because he knew how to. Because his dad helped him out with that, too. So he helped me out a lot. And he helped me with just screwing everything in and it was just a good experience I think."

After Analise made the team, she again learned from her fellow team members. When she encountered a problem she didn't know how to solve, she reported turning to others who had more experience than her: "I would always ask, because there's two people on my team who's experienced, so I would ask a lot, especially since it was my first year, I'd ask them, you know, 'How do you do this?' or, 'I don't know how to do this. Can you help me right away?'"

When at home, Analise also turned to more experienced others for help with building. She similarly recalled asking questions to her family members when she was in need of help:

> "Sometimes I would ask my mom and my nana. My nana is extremely smart. And my mom, she helps out a lot with my craft stuff because she helps me buy it, so if I run into something that I can't, I'd be like, 'Mom, can you help me? Nana, can you help me?' They would help me as much as they can. Sometimes my brother even helps with my Legos, so he would help me out with that, too. So my whole family."

As she reported doing in her robotics club, Analise turned to others who she viewed as "smart" or who had experience related to building when she needed help at home.

In the context of building, Julie and Arianna both reported turning to family members for help. Julie discussed making popsicle stick buildings using a booklet her father passed down to her from his childhood. She recalled how although he is often busy, he would give her advice on how to improve her buildings when he came home from work: "My dad comes home late, so I can't really show him since I got school, but if I been working on this and it's finished and he'll like see it and he'll tell me what would make it better and stuff. And yeah, he helps me." She also added that if she wants to make a particular type of building, "My dad will see how many sticks would probably be used for a house or skyscraper or anything." Since her father is the one with prior experience building popsicle stick structures, Julie turns to him for support with her own building.

When Arianna discussed seeking support in the context of building, she reflected both on her father's expertise in the domain and on her parents' strong mathematical ability. Arianna discussed how she "mostly enjoy(s)" building with her mother and father "because they were good at math as well and they always help me if I do something wrong." Thus, Arianna builds with these experienced others because she can rely on them to provide assistance when she needs it. She also recalled that building with her father allows her to learn new things and be able to make things that she wouldn't be able to make alone:

> "So I would want to like start learning how to like maybe cut wood and start making cool things with my dad, because my dad has that board that is—I don't know, it's a board that like cuts wood…And I'm still kind of very careful with this, like the wood, because like sometimes I get hurt and, you know, I need to pause for that. And I've always wanted to do that, so when I go to my dad's house, most of the times he teaches me and my sister how to do it."

Arianna added that her father works in construction, so "he knows all the measurements and knows all the tools." Since he has both the expertise and resources to be able to assist her in accomplishing new tasks and learning new things, Arianna turns to him for support with her building activities.

Shifts in roles assigned by experienced others. In addition to students explicitly seeking support from more experienced others, some students spoke about learning from others through being assigned particular roles that changed over time. For example, Kristal recalled that she began helping her parents cook when she was very little. At the start, she participated in rather simple ways, such as "helping my parents with the little salads and stuff." Over time, Kristal saw her parents make new recipes and started to become interested in getting more involved. She recalled thinking, "Oh I want to know how to make that," and then said she would "eventually learn."

Julie also talked about the different roles she has had while learning to cook. Julie initially learned to cook from her brother while she was volunteering at a café he worked at. She reported, "My brother doesn't want me to get hurt," so he started her out working at the front register rather than in the kitchen. However, during their free time, she would work with him to "cook wraps and parfaits, oatmeal, soup." Then after that, Julie started cooking at home. She began helping her mom with "measuring, cutting, and…peeling." She added that she uses a "safe knife" and "make(s) sure that somebody's there to see to help me out." Now, Julie often cooks on her own, making pasta with meat sauce, eggs, and her own brownies, among other things.

Finally, Reagan too spoke about the particular roles she is given when cooking with her mom. She discussed how she works to prepare vegetables on the counter while her mom is "either working on the stove or…if we have meat or chicken, she'll be usually marinating that." Since Reagan has not been cooking for very long, she is in charge of doing the simpler task of working with vegetables while her more experienced mother handles the meat and stove-related tasks. Reagan's mom will also give her instructions about how to cut the vegetables, such as making them "penny size or dime size." These instructions and particular roles that Reagan is assigned are helping her to become more familiar with various aspects of the process of cooking as she works towards becoming involved in more central ways.

Across the eight students who spoke about engaging in activities with more experienced others, several different ways of learning were discussed. While some students sought explicit help from their more experienced partners, others watched to learn or learned through direct instruction from their partners. Still other students were scaffolded into fully participating in the activity by first being provided with simpler tasks to accomplish and then gradually taking on more responsibility. Regardless of the method, however, the involvement of more experienced others to learn from was a key aspect of students' participation in activities that involve mathematics outside the classroom.

Use of estimation and trial and error. If we take a closer look at the mathematical practices that students reported engaging in as they participate in their activities with experienced others, two key themes arise: Students often use estimation when making calculations, and students frequently engage in a process of trial and error to find solutions. Of the nine students interviewed, eight spoke about using either estimation or trial and error when participating in their activities. I will first explore the theme of estimation and then examine students' use of trial and error.

Estimation in everyday activities. Estimation arose as a theme in students' discussions of cooking (4 students), fashion (1 student), gardening (1 student), and building (1 student). When discussing cooking, Julie recalled that she often estimates the amount of spice she adds to her dishes: "And I have to see if the spice is good enough so I'll have to like estimate how much it will be." Even when she uses recipes that specify amounts, Julie discussed the need to estimate: "It usually said how many times, uh, how much flour or how much spice you need. But I really, sometimes if I think it might need a little more, I'll actually just try to change the recipe. So if it says a half, I'll probably say, 'Maybe this needs a little less, or maybe one third of a cup.'" While the recipe specifies exactly how much spice to add, Julie reported using her own knowledge to estimate amounts if she thinks the recipe "needs a little less" or "a little more." Similarly, Cherell discussed the fact that although recipes provide exact measurements, she sometimes finds the need to "eyeball it" and see whether "you need more."

In other contexts, students also reported using estimation to accomplish mathematical tasks. When building, Arianna said that she often uses a ruler; however, she is sometimes able to estimate the distances in her head:

> "Sometimes I can just do it off my mind, because I can be like, 'Okay, that's an inch, that's another inch,' and I would just like go and I would already know sometimes, but if it's like really long, then I can't really remember the whole ruler, so I'd just be like, 'Okay, I need a ruler.'"

In this example, Arianna highlights when she uses a ruler and when she opts for estimating measures. She suggests that using a ruler is only necessary for longer measurements and that she can estimate the number of inches in her head for shorter distances.

Finally, Kristal provided two contexts in which she estimated measurements. First, she spoke about gardening and discussed the process of "making the hole." She recalled the process of removing the soil before adding their seeds: “Making the hole, what I usually do is I at least take out a little pile that’s about this big-ish. So like two inches around of soil.” Kristal does not use a ruler to measure her hole but estimates based on the fact that the pile should be “about this big-ish” and the hole should be “like two inches around.” Second, when doing hair, Kristal views taking the correct amount of hair as very important. However, she does not measure the amount of hair she takes but instead estimates:

> “I have to make sure I take the right amount of hair. If I take too much, it’s going to come out sloppy. Take too little, it’s going to come out really weird. So, um…I usually take, it’s kind of like eyeballing and making sure I have the right. So first I’ll see like, ‘Oh, okay, I’ll take this much.’”

Although Kristal believes it is crucial to “take the right amount of hair,” she views “eyeballing” the amount as a sufficient form of measurement. As an added precaution, however, Kristal also checks the amount she selected against the size of her hand: “I’ll always double check like it’s a little less than my hand… so I usually just take less than a handful.” This act brings up an additional theme related to estimation – the use of unofficial measuring tools or points of reference.

Rather than using rulers, three students reported using non-standard forms of measurement, such as their body parts or points of reference such as coins. In the latter category, Reagan spoke about cutting vegetables as either “penny size or dime size” while cooking. Using this point of reference for estimation began because Reagan’s mother compared the desired sizes of vegetables to the sizes of coins. Other students reported using body parts to measure size or distance, as Kristal does when she styles her cousin’s hair. Similarly, when discussing gardening, Cherell spoke about using her hands to measure distance: “You do it with your hands to see how long, how far apart to put them.” Cherell measures the distance between plants with her hands, rather than with a more official measuring tool, such as ruler. Finally, Kristal also stated that she uses her hand to space out her holes when gardening: “I’ll put my hand down, then that’s where I’ll start digging the hole. So that they’re not that far apart…but they’re not too close.” While Cherell started using her hand as a measurement tool because her uncle taught her to do that, Kristal reported getting the idea from her first grade teacher who taught her to use her finger to measure the space between words when writing.

Trial and error in everyday activities. In addition to using estimation frequently in their activities, students also recalled using a fair amount of trial and error. Six of the nine students provided examples of times they used trial and error, and all instances occurred during students’ discussions of building, gardening, or video games. It is worth noting that as with estimation, there was no prompting to inquire about trial and error; rather, students were asked about the ways in which they used mathematics in their activities, and examples of trial and error organically surfaced. I will describe several of those examples below.

While Jared spoke about using measurement when he builds structures both at home and in Minecraft, he also discussed a process of trial and error that is involved. Jared recalled using trial and error to select the dimensions of the houses he builds with Legos: “I think of [the length and width] before I build it and then I write it down on a piece of paper, then I do it like that.” After Jared builds the structure according to those dimensions, he decides whether the result is satisfying to him: “Then if I don’t like it, the number, and how it looks, I erase it and make a new number… So I can know how many Legos to put, like the width to put each of them.” Thus, while Jared engages in a sort of measurement by selecting the dimensions of his buildings, he does not select those as final dimensions but rather as dimensions to test through trial and error.

Similarly, Veronica described how trial and error was involved in measuring the dimensions of clothes for her dolls. She recalled that she initially uses a measuring tape to measure the size of the doll's waist. Then after she has created her clothing item, she tests out the fit on her doll: "I try to put them in, like put their legs inside the pants to make sure it fits and if not I'll cut it a little bit and hot glue it or something." In this process, Veronica expects that even though she did some initial measurements, she might have to make adjustments to her clothing items after she tests out their fit.

Analise offered a similar discussion regarding the role of trial and error in carrying out measurement and using geometry in robotics club. Figuring out the angle at which her group's robot would shoot involved testing out certain angles and then making changes as needed: "Well, because the net's like right here *(holding hand up)*, so it had to be not straight up, but also going pretty much up so it could get the ball, you'd have to have it like this *(arm up)* so it wouldn't go up all the way." When reflecting on whether their first angle was accurate, she recalled, "We did one that was a little lower and it didn't go in, so we changed it to one that looked exactly like this (*angled her arm)* and it went straight in." Thus, while Analise saw angles as important in her robotics work, using trial and error was a key component of calculating the correct angle at which their robot should shoot.

These excerpts highlight the key role that estimation and trial and error play in students' engagement with mathematics outside the classroom. Although they see measurement and geometry as important mathematical processes with which they are engaged, there is often no expectation that students will determine specific numbers that are wholly accurate. Instead, there is a norm of estimation and calculating only the level of precision that is needed in the given situation. Additionally, students tend to have an expectation that there will be opportunities to make revisions if necessary after testing out initial calculations.

Opportunities for creativity and self-expression. Another feature of students' experiences working with mathematics in their everyday activities is the opportunity for creativity and self-expression. Students spoke about that creativity both as a goal for their participation, as well as one of the aspects of their participation that they enjoy the most. Overall, six of the students interviewed spoke about enjoying the freedom to express themselves in one or more activities, and students spoke about self-expression in the context of cooking (3 students), building (3 students), graphic design (2 students), video games (1 student), and fashion (1 student). Below I highlight a general example of what self-expression and creativity looked like for a student who discussed cooking. Then I provide specific examples of students who saw creativity as the goal of their participation, students who reported enjoying participation because of that creativity, and students who exhibited pride in their participation as a result of their self-expression.

Julie, who stated that "cooking is part of our family," reported many different ways in which she feels free to express herself through her cooking. She discussed how she made her "own brownies…with Nutella, flour, and egg, and that's it." While Julie sometimes creates her own recipes, she also uses existing recipes but feels free to change them as needed: "Sometimes if I think it might need a little more, I'll actually just try to change the recipe. So if it says a half, I'll probably say, 'Maybe this needs a little less, or maybe one third of a cup.'" Additionally, Julie uses this flexibility to customize recipes for her family members. She even reports making multiple versions of the same dish so that she can incorporate her own knowledge of what different members of her family like:

> “I kind of adjust it because my family, we like different kinds of stuff. Well since they were all born like raised as together as a family, they kind of like the same thing, but I know some people are different. So if they like cinnamon, I’ll put like a little bit of cinnamon in the recipe. If they like less sweet, I’ll put like just a little bit of sugar. So I make like two batters."

The creativity involved in cooking is a central part of Julie’s participation in the activity. While discussing her engagement with cooking, Julie cited numerous ways in which she expresses herself and exhibits creativity. While other students provided similar examples, they also spoke about that creativity more concretely in terms of their goals, enjoyment, and pride in themselves.

Creativity as a goal of participation. One way students discussed the creativity involved in their activities was as a goal of their participation or a reason for participating. For example, Analise reflected on her goals and reasons for building things at home:

> "So my goal is to, I plan out something I want to draw, and I draw that out, and I plan out, and I try to make it look almost exactly like what I have or at least better. And then sometimes I would watch YouTube videos on how to make things and I would try to make it like that. And even sometimes on my own. So I always have a goal to make it look like that, but also my own twist."

Although Analise reported sometimes watching YouTube videos in attempt to replicate their creations, she added that she “always” wants to add her “own twist,” as well. Analise discussed a similar goal for her participation in cooking: "My purpose or goal for cooking is, the purpose it is because I get to, again, express myself with things I make, and also try different things." In a different context, Veronica saw creativity as an important goal for building houses while playing Minecraft: "I kind of try to make my own style of house, I don't really go by one style of house or tutorials on them." In each of these examples, the girls discussed their goals to create objects or food in ways that allow them to be creative and offer them the freedom to express themselves.

Creativity as an enjoyable aspect of participation. In other examples, students spoke explicitly about the enjoyment that comes from being creative. When Analise discussed her goal of adding her own twist when building, she also mentioned the enjoyment that comes from being creative: "I just like building because I express myself." Similarly, Veronica spoke about the enjoyment that comes from building her own houses, rather than following tutorials, when playing Minecraft: "I just like making my own way instead of using other people's ways." In a different context, Reagan described how creativity is central to her enjoyment of graphic design. She described a particular graphic design project she worked on, which involved creating a picture collage to hang in her locker: "Well I started looking up pictures that I really like and what I think would look good on my locker and just kind of make me happy." She also had the opportunity to "add colors to make everything, to make it more like our own and special." When reflecting on what she loves about the graphic design process, she stated, "It's just really fun to do and let my creativity just flow out onto papers and stuff like that." In each of these examples, the students spoke about freedom of expression and creativity as primary reasons that they enjoy the activities in which they participate.

Creativity as a source of pride. Finally, several students spoke about how the creativity involved in their participation has given them a sense of accomplishment or pride. While discussing building, Arianna spoke about the feeling she gets when she looks at an object she built herself: “Sometimes I just like to look at it because, you know, it looks cool. And it’s like, ‘Wow, I feel glad like I didn’t have to buy it or get someone to build it.’” Veronica also discussed the feeling that comes from viewing objects she built herself. In this case, she spoke about looking at the buildings she created while playing Minecraft: “Since I liked building things a lot I just kind of liked how they always came out. I was always proud of myself." Finally, Analise mentioned experiencing a similar sense of accomplishment after making a dish on her own without a recipe:

> “And my goal is to make the, like the best dish I can and actually come up with my own recipe for something. So not just have other recipes, actually come up with my own recipe and get a good like, I don’t know, like say, ‘That was delicious.’ And just make something completely all by myself."

Across all these examples, students cited their ability to express their own creativity as a key element of their participation – whether because of the sense of pride it gives them or the enjoyment they experience as a result.

Accomplishing meaningful interdependent goals. Another key element of students’ engagement in everyday practices is that involvement allows them to accomplish meaningful interdependent goals. In particular, eight of the nine students reported participating in at least one of the activities they discussed because they wanted to help others or do something for others. Students learned and used mathematics along the way, as needed, in order to be able to accomplish this goal. Five students spoke about helping others while cooking, two discussed building, and one each spoke about their interdependent goals in the context of jewelry making, gardening, and graphic design.

Helping others in everyday activities. The five students who discussed interdependent goals in the context of cooking reported cooking in order to help their families. For example, Julie provided the following reason for cooking: “Well I cook because sometimes my family works a lot…and my brothers are sick or they’re, ‘Oh can you make me this quickly?’…If my brothers are busy cleaning or something, they can ask me if I can make something with my baby brother." Cherell similarly reported helping out when other members of her family were too busy: “When my mom doesn’t have time to make food, I usually make some food. Well me and my sister make it.” Both girls saw cooking as a way to help out when other members of the family didn’t have time to cook. Rather than taking over cooking, Reagan spoke about cooking with her mom as a way of helping to get meals prepared in a reasonable amount of time: “I think the purpose is to help my mom out a lot...If I didn’t help her, then we wouldn’t have dinner until really late because there’s a lot of parts to it. So, yeah. I help her.” It is worth noting that students did not typically cite helping their families as the sole reason for cooking. In fact, some of the same students who stated the importance of cooking to help their families also reported enjoying cooking because of the self-expression involved.

In other contexts, students similarly spoke about participating in activities to help others. Kristal reported that she helps her mom with gardening so they can have fresh food for cooking, while Arianna said that she often helps her father with building. She recalled one particular occasion on which he asked for help, so she and her sister provided assistance:

> "I would help my dad build a lot of things. Like there was in the back, in the backyard, we had like a really big backyard and there was no gate, and we have a dog, so me, my dad, and my sister, we would, um, my dad wanted, needed help for the gate, so he would say, 'Grab the measuring tape, grab the tools, grab the nails,' and we would just help him with it, and it would be really fun."

In this situation, Arianna's father explicitly asked for help. However, in other cases Arianna reported building or engaging in graphic design because she voluntarily wanted to do things for others. For example, she recalled a time when she "wanted to make a pop out card" for her mother for Valentine's Day, so learned from her art teacher how to make it.

Analise and Alexis also spoke about engaging in building and jewelry making, respectively, because they wanted to do something for others. Analise recalled how she builds objects out of Legos for her brother: "I get these Lego sets where I can…build something for my little brother. Like he likes wrestling, so…I would build wrestling stuff for him." When Alexis discussed jewelry making, she said that she primarily engages in the activity when her younger cousins are around: "I normally do it with other—with my little cousins, because they're smaller. I just try to do something to entertain them." For both girls, doing something for their younger siblings or cousins was an important part of their participation. Finally, Veronica also spoke about building in terms of doing something for others. However, the others that Veronica wanted to do something for were her dolls. Although they are not living creatures, Veronica spoke about their feelings and her desire to keep them from being lonely:

> "[I want to] just like make them kind of have their own little life. Make sure they have everything that we have: a bathroom, a kitchen, their own rooms. They even have a

> Christmas tree... Because maybe they feel lonely being there, not being able to do stuff as humans do."

In this example, Veronica exhibited a similar interdependent goal of helping others even though she applied it to dolls, rather than real people. While some students' helping occurred voluntarily and others helped as a result of explicit requests or perhaps a sense of obligation, the act of helping others surfaced as a core aspect of participation across a variety of activities.

Reasons for wanting to do well in school. Given the focus on interdependence that emerged in students' everyday activities, it is worth highlighting students' responses to a set of survey items that were briefly described in Chapter 3. Students were asked to rate the importance of different reasons for wanting to do well in school on a scale of 1 to 5, where 1=Not at all important and 5 = Extremely important (modified from Stephens, Fryberg, Markus, Johnson, & Covarrubias, 2012). Students' mean ratings can be viewed in Table 4.5. Of eleven survey items, the three that related to family received the highest mean ratings of importance: "Make my family proud" received the highest rating (4.55/5.00, *SD*=.848); "Provide a better life for my own children" received the second highest rating (4.43/5.00, *SD*=.926); and "Help my family out after I graduate" received the third highest rating (4.34/5.00, *SD*=.887). These responses demonstrate not only a significant emphasis on interdependent reasons for success but also strong family values in particular. Additionally, they echo students' discussions in interviews regarding their goals for out-of-school activities and desire to help others.

Table 4.5

Importance of Reasons for Wanting to Do Well in School

Reason for wanting to do well in school	Mean (SD)	n
Make my family proud	4.55 (.848)	82

Provide a better life for my own children	4.43 (.926)	83
Help my family out after I graduate	4.34 (.887)	83
Show that people like me can do well	4.12 (.961)	82
Learn more about my interests	4.01 (1.000)	82
Expand my knowledge of the world	4.00 (1.018)	82
Explore new interests	3.98 (1.012)	83
Expand my understanding of the world	3.93 (1.022)	83
Gain respect from my friends	3.81 (1.076)	83
Give back to my community	3.66 (1.172)	83
Be a role model for people in my community	3.60 (1.249)	83

Note. 1=Not At All Important, 5=Extremely Important

Summary. As students spoke about the ways they use mathematics in their own activities outside the classroom, four key features of their experiences emerged: students learned from more experienced others, students participated to help others or do something for others – especially family members, students frequently used estimation and trial and error as they engaged with mathematics, and students appreciated the freedom of expression afforded by these activities. These themes highlight several points for comparison between students' engagement in everyday activities and their engagement with mathematics in the classroom. If students viewed authentic mathematics as involving trial and error, estimation, learning from more experienced others, and having goals of helping others and being creative, how do those views align with the ways Ms. Sanchez asked her students to engage with mathematics in the classroom? In the next section, I explore key features of students' engagement with classroom mathematics and compare those themes with the previously described features of students' engagement with mathematics outside the classroom.

Engagement with Classroom Mathematics Versus Everyday Mathematics

To facilitate a comparison of students' engagement with mathematics in different settings, I draw on Fredricks, Blumenfeld, & Paris's (2004) five features of the classroom environment that influence engagement (teacher support, peer support, classroom structure, autonomy support, and task characteristics). Looking across adolescents' engagement with mathematics in everyday practices versus the classroom, I identify three particular features to explore: 1) participation structures and methods of interaction, 2) norms and expectations, and 3) problem-solving task characteristics. Methods of interaction cuts across Fredericks et al's teacher support, peer support, and autonomy support categories and also speaks to students' discussions of learning from others in their everyday activities. Meanwhile, norms and expectations reflects the classroom structure category and also encompasses students' use of estimation and freedom of expression in their everyday activities. Finally, the category of problem-solving task characteristics echoes Fredericks et al.'s task characteristics category. While this category does not provide a direct comparison to students' engagement with everyday activities, characteristics of classroom tasks emerged as a key factor to which students attended in the classroom. Thus, I include that category to provide insight into students' perspectives on classroom tasks, as well as to serve as a potential basis for comparison to real world uses of mathematics.

Classroom fieldnotes were used to gain insight into the ways students engaged with – and were expected to engage with – mathematics in Ms. Sanchez's classrooms. In this section, I draw on coded fieldnotes from the 60 seventh-grade classroom observations to highlight salient features of the mathematics classroom context that fall into each of the three aforementioned categories. I then compare those features with aspects of students' engagement with everyday activities involving mathematics. When relevant, I also include students' perspectives on various features of classroom engagement to highlight different interpretations of those features. Table

4.6 provides an overview of key aspects of engagement in each setting that are discussed in the sections below.

Table 4.6

Features of Adolescents' Engagement with Everyday Versus Classroom Mathematics

Features of Engagement	Everyday Activities	Mathematics Classroom
Methods of Interaction	• Participation involves collaboration with others (especially family members) • Learning happens through interaction with experienced others	• Participation sometimes involves collaboration and sometimes involves working alone • Students express a desire to collaborate when told to work independently • Students seek help from both the teacher and advanced students
Norms and Expectations	• Estimation and trial and error are often used for mathematical calculations • Experienced others are available to help • Students have opportunities to express their own creativity • Participation accomplishes meaningful interdependent goals	• Precision and exactness in mathematical calculations are expected • Students are expected to seek help from classmates first, though many still seek help from the teacher • Students are expected to follow rules; there is limited opportunity for self-expression • A norm of quiet often exists
Problem-Solving Tasks	N/A	• Typical task contexts include mixing orange juice concentrate, calculating the cost of pizzas or oranges, calculating distance traveled, comparing prices, and determining sales tax • The authenticity of task contexts and numbers

	involved in tasks are questioned by students

Participation structures and methods of interaction. In this section, I examine the ways in which students participate and interactions are organized as students work with mathematics in and out of the classroom. I begin by briefly recapping common participation structures and methods of interaction in students' everyday activities, as reported in students' interviews. Then I compare those features with the participation structures and methods of interaction in the classroom context by drawing on fieldnotes of classroom observations. Finally, I highlight students' own perspectives on the participation structures and methods of interaction in the classroom with particular emphasis on students' views of the usefulness and/or authenticity of different ways of organizing activity and interacting in the math classroom.

Everyday activities. We can gain insight into the ways that participation is structured and interactions are organized in everyday activities by examining students' comments about who participates in their activities with them and how learning happens in those settings. During interviews, all students were asked whether they participate in their activities alone or with others and, if the latter, who else participates. All students but one, Jared, reported engaging in activities with family members. For example, Alexis bakes with her mother, grandmother, aunt, and sometimes brothers, and she makes jewelry with her younger cousins. Analise builds with her brother, cousin, mother, and nana, while she cooks with her mother and nana. The other students interviewed mentioned participating in activities that involve mathematics with their fathers, mothers, sisters, brothers, aunts, uncles, grandmothers, cousins, and occasionally friends. It is worth noting, however, that only two of the students (Jared and Reagan) recalled participating with friends, and one student reported getting feedback from – though not participating with – friends. The other six students only mentioned ways in which they engaged in these activities with family members.

Building on the theme of working with others, students spoke about learning as a process centered on interaction with others. As previously highlighted, students tended to learn through apprenticeship with significant others or seeking guidance from experienced others. Eight of the nine students reported learning new things in this manner, and nearly all of those students spoke about learning from older family members. Thus, not only do students engage with others while participating in their activities, but those significant others are also a crucial part of student learning in settings outside the classroom.

The classroom context. Now that we have a sense of how learning is arranged in students' everyday activities involving mathematics, we can consider the features of learning and engagement in Ms. Sanchez's mathematics classrooms. In the three observed seventh-grade classrooms, student work tended to be arranged in one of seven different formats: independent work, partner work, small group work, stations, teacher-led group work, whole class discussion, or whole class activity. Independent work consisted of moments when students were asked to work independently, while partner work and small group work involved completing class work in pairs or in groups of 3-5. Stations occurred when Ms. Sanchez split students up into several large groups (typically three) and provided different assignments for each group. Ms. Sanchez tended to work with one of the groups to provide guidance on a difficult topic or help to catch up students who had fallen behind. In eighth period, Ms. Sanchez's co-teacher also typically led a group during stations. Meanwhile, students who were not at a station led by a teacher worked either independently or with others at their table. Students sometimes remained in their stations during the entire duration of the period, while at other times Justine set a timer and groups rotated periodically.

Teacher-led group work mirrored the kinds of interactions that were witnessed when teachers worked with groups during stations. During teacher-led group work, a subset of the students in the class worked in a group with the teacher helping to guide them and provide support. Sometimes this lasted only a few minutes, as Ms. Sanchez walked the group through a particular aspect of an assignment, while at other times she pulled students aside to work with them for a more extended period of time. Finally, whole class discussion and whole class activity took place with all students in the class. Discussion was typically comprised of a teacher-led, whole group conversation in which Justine asked questions, and students volunteered ideas and provided answers. Since whole class discussions can look quite different in different classrooms, below I provide an excerpt from fieldnotes captured during observations to illustrate what typical whole class discussions looked like in Ms. Sanchez's classrooms. In this particular excerpt, Justine asked a student if she could share his work with the class. She wrote his method on the board and then engaged the class in a discussion about it:

> Justine writes 10/4 and 8/3 on the board. Then below each she writes 10/4 x 30/30 = 300/120 and 8/3 x 40/40 = 320/120. "Can you tell us your thinking?" she then asks the boy. He explains that they first found the multiple of all the numbers, gesturing towards the 10/4 and 8/3. He says that was 120 so he put it in the denominator. Then he did 120 divided by 4 to get 30. "Thirty is the what?" Justine asks, and the boy responds that it's the scale factor. Then he says he did the same thing to the numerator, so he got 300.

> "This is called the what ratio?" Justine asks. He says "equivalent." Then he says that he did the same thing to the other side and made an equivalent ratio. "What does this information even mean?" Justine asks the class. The class is quiet for a moment, so Justine asks, "Ten is what?" Students say the number of people, so she labels 10 "people." Then she asks what the four is, and students say pizzas. Now that ratio says "ten people to four pizzas." Justine then asks, "Three hundred represents what?" Again the students say people, so she writes people next to 300. Then she finishes by asking what the 120 represents, and they say pizza. (February 12, 2015)

During whole class discussions such as the one described above, Justine typically stood at the board directing conversation, while students sat in their seats or sometimes on the floor and responded to Justine's questions. In contrast, whole class activities included an active element that allowed students to engage in a way other than just speaking. Only two whole class activities occurred during observations: One involved students timing themselves to measure their walking rates, and the other was a gallery walk in which students traveled around the classroom in small groups to provide feedback and take notes on posters created by their classmates.

As jottings were recorded continuously throughout class sessions, and fieldnotes are simply expanded versions of those jottings, examining the percentage of fieldnotes that involve students in various participation structures can shed light on how students were typically arranged in Ms. Sanchez's classroom. Looking across the fieldnotes for all three seventh-grade classes, students were most frequently involved in whole class discussion (29% of the time), followed by partner work (21%), small group work (17%), and independent work (12%). Approximately 9% of the fieldnotes captured students working in stations, 7% of the time students were engaged in a mix of independent and small group work, 4% of the time was spent in teacher-led groups, and less than 1% of class time involved whole class activities.

Since the participation structure that was observed was sometimes different than the participation structure stated by Justine, codes were also applied to capture the observed participation structure when differences were noted. Across all 60 sets of fieldnotes, there were 28 instances[6] of different observed participation structures, which amounted to only about 3% of the typed fieldnotes. However, if we look inside that 3%, a pattern emerges indicating students' desire to collaborate. Nearly 63% of the time when the observed participation structure was different from the stated participation structure, students were observed working with partners or small groups. Thus, when stated expectations were broken, it was typically when students were supposed to be working independently but wanted to work with others. Less than 1% of the time, students were observed working independently when the teacher had requested group work. The remaining 36% of the time when differences were noted, the observed participation structure was whole class discussion. In nearly all of those instances (94%), students had been told to work independently, but Justine then went against her own instructions and engaged students in a whole class discussion. Thus, examining these instances in which observed participation structures differed from stated participation structures reveals a pattern of both students wanting to work with others and Justine providing additional whole class support when students were initially directed to work independently.

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[6]These instances varied in length, as the boundaries of participation structure codes matched the duration of the given arrangement. In other words, a single independent work code was applied to the entire section of fieldnotes that described students engaging in independent work. If the arrangement changed, even if temporarily, that code ended, and a new code was applied.

Looking more closely at what happens during different participation structures elucidates the types of interactions that happen when students are arranged in particular ways. Given students' emphasis on seeking help from more experienced others in everyday activities, one type of interaction that is of particular interest is students' helping behaviors. Before exploring helping and help-seeking in the classroom, however, I would like to turn to students' interview responses regarding their help-seeking behaviors. All 12 interview participants were asked what they do when they need help in mathematics class. Ten students reported that they go to their teacher Ms. Sanchez, and eight students reported that they often ask other classmates for help. While it seems obvious that students would ask their teacher for help, as she is clearly an experienced other, Carrie – one of the two students who reported not typically seeking out Ms. Sanchez – stated the following reason for not asking her teacher questions: "It's kind of a little bit harder 'cause she's learned this a long time ago, and she already understands it and everything." Instead, Carrie asks one of her classmates for help "because we're both learning the material [so] they can explain it better because they've been there where they don't understand it." In these quotes, Carrie highlights the fact that it is especially beneficial to seek help from others who are more experienced but not so far removed from the learning process that they might not understand how best to support current learners.

Students who reported seeking out help from their classmates typically identified those classmates as "advanced" or as students who "know what's going on." Ethan recalled that he will ask for help from another student if there is someone who is "zooming through his or her work," while Victoria reported that she will seek help from her "friend who looks like or seems like they are more advanced and they understand more." Similarly, Robert reported regularly approaching a friend at his table for help because “he's really smart. He always finds a way to help me and he understands what I mean most of the time.”

Examining fieldnotes of classroom interactions provides further insight into students’ help-seeking behaviors. One primary way in which students reached out to each other for support was through questioning. Across all 60 sets of fieldnotes, 236 instances were captured of students posing inquiries to other students. Forty-three percent of these questions surfaced during partner work, and 28% were asked during small group work. Meanwhile, 12% were posed during stations, 11% during independent work, and 4% during whole class discussion. In contrast, nearly one-third of the 207 total inquiries directed at the teacher (31%) occurred during whole class discussions. Twenty percent of inquiries to the teacher were posed during partner work, 17% were asked during small group work, 15% were posed when students engaged in a mix of small group and individual, and 10% occurred during independent work. Thus, while students were captured asking nearly as many questions to their teacher as they did to their classmates, the times at which questions were posed to various experienced others differed.

In addition to highlighting instances of student questioning, codes captured moments when students explicitly sought out help from other students. In order to establish boundaries for coding helping behaviors, helping was defined as either responding to an explicit request for help from another student or providing direction to another student. While there were many occasions when students conversed about a problem or provided suggestions for solution paths, those fell outside the boundaries of helping for this work. One drawback of this narrower definition is that many interactions between students – such as Robert and the friend he turns to for guidance – are not included as helping behavior, as they involved a back-and-forth between students rather than being guided by one student. However, such helping interactions are often captured in the analyses of student questioning described above, as these exchanges tended to start with a student inquiry.

Examining students' helping and help-seeking behaviors provides additional insight into how and when helping unfolds in the classroom. Across all fieldnotes, thirty-two instances of students seeking help from other students were captured. As with student questioning, help-seeking occurred most frequently during partner work (36%) and small group work (35%), followed by independent work (17%) and stations (7%). Examining the other side of help-seeking, there were 84 captured excerpts of students helping other students. As might be expected, helping behaviors also occurred most frequently during partner work (42% of the helping excerpts) and small group work (35%), followed by independent work (11%). These excerpts ranged from short exchanges with just a few comments to longer exchanges in which one student provided direction or guidance to another throughout a significant portion of the class period. In addition, excerpts differed according to whether the help provided was voluntary or was prescribed by the teacher. In 68 of the 84 excerpts, students voluntarily helped their peers; meanwhile, in 16 of the excerpts, Justine asked a particular student to help his or her classmates. Additionally, there were 10 captured instances of students offering their help to other students without receiving any requests for help. Four of these instances occurred during small group work, while three each were observed during partner work and independent work.

Finally, in addition to observing helping behaviors, we can examine student contributions in class to further understand the ways students engage with mathematics when interactions are arranged in different ways. Four types of contributions that were coded for are critiquing another student's reasoning (*N*=79), debating points of view (*N*=10), justifying reasoning (*N*=113), and showing one's thinking (*N*=22). Critiquing another student's reasoning – an action promoted by the Common Core State Standards in Mathematics – occurred 58% of the time during partner work and small group work, and 32% of the time during whole class discussions. Justifying reasoning, on the other hand, occurred most frequently in whole class discussions (62% of instances), followed by partner and small group work (22%). Debating points of view occurred about equally in whole class discussions (52%) and during partner and small group work (48%), while showing one's thinking surfaced most frequently in teacher-led group work (43%), followed by partner and small group work (31%). As is to be expected, all of these practices occurred least frequently during independent work. While I cannot compare these behaviors to students' actions in everyday activities since I only have reports of students' participation, the behaviors seem important to examine in the context of classroom interaction, as interacting with others enabled students to participate in these ways.

Comparing the classroom with everyday activities. A comparison of the participation structures and methods of interaction in the mathematics classroom versus in students' everyday activities reveals three main themes: First, while students participated in nearly all of their everyday activities with partners or small groups of others, students spent only 38% of their class time working with partners or small groups. Twenty-nine percent of the remaining time was spent in whole group discussion, which did involve interacting with others but did not typically involve opportunities for students to ask questions of each other or help each other. Additionally, students spent approximately 12% of their time working independently, an action which was reported by only one student during discussions of everyday activities. Even among the three students who played video games, which we might think of as involving solitary activity, only one student reported playing alone.

Second, partner and small group work promoted student helping and help-seeking behaviors more than any other participation structure. While students frequently asked questions of their teacher during whole group discussion, they very rarely asked questions of each other in that setting. Certainly, this was particular to the classes observed – as is the case with all of these themes – since one can imagine a teacher facilitating productive student discussion in a whole class setting. Although there is not a direct comparison of this finding to students' engagement with everyday activities, adolescents tended to report working with one or two others when participating in their everyday activities and seeking help from those others whenever needed.

Finally, students found it beneficial to be working with others both in and out of the classroom. In Ms. Sanchez's classroom, students most frequently opposed stated participation structures when they were asked to work independently but wanted to work with others. Furthermore, student interviewees reported specific benefits of asking classmates for help, such as that classmates can be especially helpful in providing understandable explanations to their peers. Similarly, in everyday activities students sought out experienced others for help and reported participating with others regularly. Additional student perspectives on participation structures and methods of interaction that surfaced in classroom observations will be discussed below.

Students' perspectives on classroom participation structures and methods of interaction. Two main perception codes speak to students' views on the participation structures and methods of interaction used in the mathematics classroom. First, excerpts were coded in which students explicitly opposed or supported independent work or collaboration. Second, student comments on the authenticity and usefulness of mathematics were captured, two of which pertain to the authenticity of participation structures. These excerpts are provided not because of the frequency with which they surfaced but to provide insight into the various ways in which students perceived expectations regarding interaction in Ms. Sanchez's classroom.

Upon analyzing students' remarks related to collaboration and independent work, a theme again emerges of students preferring collaboration to independent work. Across all sets of fieldnotes, students were captured opposing collaboration or requesting independent work on only six occasions. Furthermore, in some of those instances students were not opposing collaboration at large but rather collaboration with particular classmates. In contrast, 16 requests for collaboration were captured, as well as seven comments that expressed opposition to independent work. Requests for collaboration included questions such as "Can we work with our tables?" and "Can we work with partners?" Meanwhile, opposition to independent work occurred when students either commented that they did not want to work independently or questioned why they had to work independently, as in the following example: "Upon hearing the noise, Justine comes over and tells them to work independently. 'My god, why we always gotta work independently?' the boy asks." (February 26, 2015)

In addition to these requests for collaboration and opposition to independent work, on three occasions students were captured stating the important of collaboration. For example, on January 23, 2015, Carrie was working with her table when some members of her group started to get sidetracked. After about a minute, Carrie loudly and sternly announced to her classmates that they all needed to "focus and pay attention and help each other out… because some of us don't get it." Similarly, on January 27, 2015, a different student was captured telling his tablemates, "We need to work together if we're going to get through this."

Next, two classroom excerpts were captured in which students expressed their views on the authenticity or usefulness of ways of engaging with mathematics and with others in the classroom. In the first excerpt, a student was working on a set of problems that required him to draw pictures of coins in bags as a way of thinking about how to solve equations. The student was able to correctly perform the process in his head, so he opted to not draw the pictures. Justine's co-teacher Tina approached him to let him know that drawing the pictures was required, and the student resisted her request:

> Tina tells him that she needs him to start drawing the pictures. "We never need to explain it, we just need the answers," he responds. Then he starts mumbling and swaying back and forth, continuing to talk to no one in particular other than himself. He mutters, "You never go to a cashier and say, 'Here's eighteen dollars. This is the way I got it.'" (May 18, 2015)

In this excerpt, the student provided a view that the activity he was being asked to engage in was not authentic to what he would do in the real world. Whereas some students in Justine's classes saw the usefulness of showing their thinking – such as José, as described in the last chapter – this student found the practice to be inauthentic. One possible reason for this difference is the student's level of comfort or knowledge with the topic at hand. While José found showing one's thinking to be an effective way of enhancing understanding for oneself and others, the student quoted above already knew how to solve the problem without showing his work. Thus, he did not see the task requirements as useful. Related, in everyday contexts one would likely not be asked to show his thinking if he already understood how to approach a problem, as the boy highlighted. Thus, while José's showing thinking might be useful in everyday contexts if one was aiming to learn something new, an adolescent would typically not be asked to show work in an everyday context after he already reached a solution.

In a second excerpt, a student compared the way activity is arranged in the mathematics classroom with the way activity is arranged in everyday life. After Justine asked her class to work independently, the boy highlighted the disconnect between being asked to work independently in math class and constantly engaging with others in everyday life: "'Why does everything have to be independent?' a boy sitting near me asks. 'Life is not independent. You have a partner!' A few seconds later he adds more quietly, 'And then you have kids.'" (April 17, 2015) Again, this student saw a lack of authenticity in the way activity is organized in the mathematics classroom. He felt that he was often asked to work independently even though "life is not independent." In both of these excerpts, students took issue with the structure of activity and methods of interaction in the mathematics classroom specifically because they did not align with what those students saw and experienced in their everyday lives. Findings from student interviews provide additional evidence of those experiences, as students nearly always reported working with others and never recalled having to illustrate their solution methods for others.

Norms and expectations. A second way to explore what it means to engage with mathematics in the classroom versus in everyday activities is to examine the norms and expectations that are established for the structure of activity and interaction. In the sections below, I highlight the norms and expectations that surfaced in discussions of everyday activities, followed by the norms and expectations that were observed in the mathematics classroom.

Finally, I again end by comparing the two settings and drawing attention to students' views of the authenticity and usefulness of classroom norms and expectations.

Everyday activities. In analyses of everyday activities, several themes emerged that speak to the norms and expectations of engagement in those activities. In terms of the mathematics students engaged with while participating, students reported experiences that suggest norms of using estimation and trial and error. Rather than being expected to calculate exact answers, students were allowed – and even encouraged – to estimate results. Additionally, even when working with experienced others, students reported using trial and error to test out measurements and then learn from their mistakes.

Related, when students needed help with their activities, they reached out to experienced others for assistance. None of the students recalled being expected to figure things out on their own when they encountered difficulty in their everyday activities. Rather, students consistently spoke about turning to others for assistance, whether those others were older siblings, parents, grandparents, or other relatives. The frequency with which such actions were discussed by students indicates that relying on others for help is a norm in their everyday activities.

An additional norm that emerged related to self-expression and creativity. Students were not expected to follow rigid rules when participating in their everyday activities. Instead, they were given the freedom to express themselves and exercise their own creativity during participation. This expectation of being creative emerged when students spoke about altering recipes, deciding what to cook, and coming up with their own ideas about what to build.

Finally, students reported engaging in everyday activities for meaningful interdependent reasons. While this might not be a norm that guides students' actions as they participate in their activities, students' overall participation was often inspired by their desire to help others, become more connected with others, or express themselves in meaningful ways. Thus, students' experiences translated to a different type of expectation in their everyday activities, in which doing mathematics was part of a larger goal to achieve meaningful interdependent outcomes.

The classroom context. In the mathematics classroom, a different set of norms and expectations emerged. Examining both Ms. Sanchez's interview responses and students' perceptions of classroom norms provides insight into the expectations and norms that existed in Ms. Sanchez's mathematics classrooms. First, during her interview, Ms. Sanchez recalled that she "did a horrible job with expectations." She reported that she was a bit "afraid" at the start of the year and didn't "have really any guidelines." She said that the main thing she told the students was "We make mistakes, we learn from them, we work together, we solve math problems." She recalled that she also mentioned that she was okay with "loud" or "long" conversations as long as they were about math because she "cannot expect [everyone] to be quiet." When reflecting on how things went over the course of the year, Ms. Sanchez reported, "I feel like in the beginning of the year it was better plus I feel like many of my kids were actually doing more math and focusing on doing and caring more, but towards the end it's kind of getting lost." This comment sheds light on some potential mismatches between Ms. Sanchez's reported expectations and what was observed in the classroom during the latter part of the school year.

One way in which norms and expectations of the classroom were captured was through student comments, as recorded in fieldnotes. Moments in which students either opposed or enforced classroom norms were coded for, and those coded excerpts were subsequently examined to identify the specific norms that students either opposed or enforced. Examining student comments in addition to the teacher's reported expectations sheds light on students' own perceptions of classroom norms and expectations. Additionally, Ms. Sanchez's actions will be incorporated when appropriate to illustrate some of these perspectives in action. Three main types of norms that emerged from analyses will be discussed: a norm of precision, exactness, and following guidelines without room for creativity; an expectation of relying on others; and an expectation of quiet.

First, as one might expect, engagement with mathematics in the classroom involved a norm of exactness and precision, in terms of both calculating correct answers and following procedures as explained by the teacher. Students frequently turned to their tablemates or raised their hands to have Justine check whether their answers were correct. There were no observed instances when an estimate or a response that was close to the actual answer was viewed as an acceptable answer. Similarly, Justine expected precision in the language students used to discuss mathematics. She often asked students to recall that they had calculated a "scale factor" or an "equivalent fraction" when students were not using that language. Additionally, she regularly encouraged students to use the word "to" and the terms "numerator" and "denominator" when discussing ratios, as in the following example:

> Justine asks a student if he can give an example of a ratio. After he provides an example, Justine reminds the students to use the proper language and emphasizes the word "to." She repeats his answer of "30 over 60" as "30 TO 160." Then Justine asks the student how he got that answer; another student offers that you multiply by two. Justine multiplies the numerator by two and then asks, "Only to the numerator, right?" Many of the students respond by saying "no" and then shout out that she needs to do it to "the

bottom," too. Justine replies, "Don't say bottom –it's the denominator!" (January 22, 2015)

As this example illustrates, students were expected to not only perform operations correctly but also to use the correct language when explaining their actions.

Similarly, students were expected to follow rules as specified. On one occasion a student reported to Justine that she did her homework on the computer and wanted to make sure that was okay. Justine responded that it needed to be in her notebook to which the girl groaned. On a different occasion, Justine had asked students to work on problems in class and to highlight their answers in pink. A conversation quickly began about whether students could choose their own colors. Some students asked, "Why pink?" or "Why not red?" or called out that they used a different color on their homework. As the conversation continued, a boy loudly announced to the rest of the class that they needed to use pink "because she said it and that's that." (April 20, 2015) From this student's perspective, the teacher's directions must be followed as stated, which Justine then reinforced, even though other students wanted the freedom to choose their own colors.

Other rules that limited students' freedom and required them to follow their teacher's guidelines exactly related to where to sit and when to use computers. Justine frequently gave students orders about whether their computers should be open, closed, or "at a forty-five degree angle." Additionally, students were assigned seats, and if they attempted to move seats, they were often reprimanded. Sometimes students were even asked to sit in particular places at a table, as in the following example:

> Justine tells the two boys sitting at the side of the table against the wall that they need to move to the other side so she can see their screens. "You can trust us," one said… As she's leaving, Justine says again that they need to move to the other side of the table and that this is "the last time" she's going to tell them. The boys groan but lift their computers and notebooks and move to the other side. (April 27, 2015)

Although some students were able to get away with discretely changing seats or convincing Justine to let them sit in a place other than where they were assigned, Justine typically enforced a norm that students sit where they were told to sit, as in the above example.

Second, as Justine stated in her interview, there was an initial expectation that students should rely on each other for help before asking her. Justine's enforcement of this expectation was observed on January 26, 2015, when Carlos tried to ask a question, and Justine replied, "I want you guys to rely on each other first, and then I'll tell you." While Justine made similar comments during the first half of the observations, those comments dissipated towards the end of the year, and she more frequently responded to students' queries. Additionally, even as students questioned each other, many students asked questions to Justine, as well. As reported above, students posed nearly as many questions to Justine as to their classmates over the course of the 60 observations. Additionally, there were 183 captured instances of students requesting help from their teacher in the form of approaching the teacher, calling her over, raising their hands, or explicitly stating a need for help. Certainly, many additional instances likely occurred that were not captured in the fieldnotes. However, what is noteworthy is that only 32 instances of students requesting help or feedback from a classmate were captured. This difference in help-seeking further highlights the discrepancy between Ms. Sanchez's stated expectation of asking others for help before turning to her and the classroom norm that was enacted on a daily basis.

Finally, although Justine reported establishing the expectation that students would be talking and that loud conversation was acceptable, an expectation of quiet surfaced throughout the year. This expectation was typically enacted when students worked on their Do Nows (quick problems at the start of the class) and during independent work. At other times Justine asked students to be quiet while reading or when small groups working together got too loud. Across all fieldnotes, the word "quiet" – or some variation thereof – surfaced 497 times. This prevalence suggests a norm of quiet in the class, even if students were allowed to be loud during certain types of work.

The following fieldnotes excerpt provides an example of this expectation of quiet being enacted in the classroom: "Justine asks them to do their work quietly, but quite a few students are talking. As Justine wanders around the room, she asks the students to be quiet a couple times. Then she says, 'I still hear side conversations' and most of the room finally gets quiet." (January 22, 2015) Similarly, Justine sometimes quieted students by shushing them, as in the following example: "She continues, 'It seems many of you are not reading or are not listening' and then makes a shhhhh sound." (January 22, 2015) Thus, while Justine reported stating at the start of the year that speaking loudly was acceptable, by the latter half of the year, there were many moments during which that norm was explicitly opposed.

In addition to Justine frequently enforcing a norm of quiet, students were observed enforcing this norm on several occasions. On January 23, 2015, Justine's co-teacher asked a table of students to be quiet and then walked away. As the group got louder again, one of the students scolded the rest: "About a minute after Tina leaves the table, one boy starts loudly talking and another reprimands him and says, 'Didn't she JUST say be quiet?' Similarly, that same day, a different boy told a girl sitting near him to be quiet, and she asked why. The boy whispered, "Cause we're going to get in trouble!" and pointed at Justine's co-teacher.

Overall, several different – and sometimes conflicting – norms existed in the mathematics classroom. Justine reported establishing norms of making and learning from mistakes, working together, turning to classmates for help, and speaking loudly about mathematics in the classroom. While working with others and turning to classmates for help were observed – though many students continued to rely on the teacher – students were very infrequently observed viewing mistakes as a learning opportunity. In fact, there was only one captured instance of that happening, as well as one instance of a student perceiving a mistake as a thing to be avoided. Additionally, while loud conversations did often occur, a norm of quiet also existed in the classroom. Students were sometimes asked to work quietly with partners or groups, and students were always expected to remain quiet during both independent work, which constituted nearly 12% of fieldnotes, and while working on the Do Now at the start of each class. Finally, in addition to providing rules regarding speaking volume, Justine also specified where students should sit and whether computers should be used, guidelines which students were expected to follow. If students went against Justine and/or Tina's stated guidelines, they tended to be either verbally reprimanded or to receive a school "foul" in the form of a pink slip.

Comparing the classroom with everyday activities. Comparing norms and expectations for engagement in everyday activities involving mathematics with expectations for engagement in the mathematics classroom points to areas of both alignment and mismatch. In terms of alignment, there is some overlap regarding expectations of relying on others for help. While students turned to others for help in both settings, the nature of that help-seeking differed. In everyday activities, students turned to those who they believed to be experienced others and who they thought would be helpful. While students tended to do the same in the classroom, students differed in who they identified as experienced others. Students who wanted to turn to the teacher for help faced the established expectation that they seek help from their classmates before their teacher. This expectation limited the individuals from whom students could seek help, a practice which was not reported in everyday activities. Thus, while students were able to ask for assistance in both settings, restrictions on help-seeking sometimes existed in the classroom but not in everyday life.

Several other points of mismatch arise between norms of engagement in the classroom versus everyday mathematics. While students often reported having the freedom to express themselves and be creative in their everyday activities, no norm of self-expression was observed or noted in the classroom. Rather, Justine was observed expecting specificity and precision from students – both in terms of their answers to problems and their following of directions. This expectation lies in stark contrast to the norms of creativity, estimation, and trial-and-error that occurred in students' everyday activities. Additionally, while students were able to use mathematics to accomplish meaningful goals in their everyday activities, goals were generally imposed on students by their teacher in the mathematics classroom. Although analyses specific to this topic were not conducted, Justine always decided what work students would be doing and what assignments they needed to turn in, as is typical in middle school classrooms. Students were occasionally given the freedom to choose particular aspects of an assignment, but their overall goal was generally to complete of a set of problems by a particular date as specified by their teacher.

Finally, while there were many instances of loud, animated student discussion in the classroom, a norm of quiet was frequently enacted, as well. Whereas students never reported needing to cook, build, or garden silently, students were regularly asked to work quietly in the mathematics classroom. Students' perspectives on these norms and expectations will be discussed below.

Student perspectives on classroom norms and expectations. When examining the excerpts coded for students' perceptions of usefulness or authenticity, several examples emerged related to classroom norms and expectations. Again, while there is not enough evidence to highlight a clear pattern, it is worth examining some of these student comments to gain additional insight into student perspectives and the various ways students are thinking about what norms and expectations are associated with useful, authentic mathematics.

In the prior section, I highlighted student perspectives on various participation structures in the classroom, which also speak to students' views of norms around relying on others and working quietly. In that section, I drew attention to the fact that students tended to request collaboration and go against instructions to work with others. These actions suggest that students preferred norms of collaborating with others and discussing ideas, rather than working quietly.

Several students also spoke out against the expectation that students should do problems that are assigned to them without having any larger, more meaningful goal that completing those problems helps them to accomplish. While Justine typically assigned problems either from the school's curriculum or sometimes that she wrote, students were dissatisfied when they did not feel they could apply those problems in their lives. For example, on March 4, 2015, Justine was working with her eighth period class to solve the problem 6/2=4/p. As they reached the solution of p=4/3 (the first time students had seen a fraction within a ratio), two students highlighted the fact that they saw no larger purpose for such a problem beyond the classroom:

> "How is this helping life?" one boy asks loudly. Another student echoes, "Yea, how is this helping life?" The boy who had asked the original question looks at this student as he talks and then turns to Justine and asks again, "But how is this helping life?" Justine says, "There are different applications where this will help you." He questions, "Four over four thirds?" She says, "Well maybe not with the fractions." He smiles and says, "I thought so," nodding his head.

Through the boys' questioning, they highlighted a desire to be engaging in tasks with more meaningful goals that are "helping life." They were not content with the norm of solving problems that don't apply to their lives and pushed Justine to explain how a fraction within a ratio might contribute to some larger purpose in life in the future.

During an eighth period class on May 14, 2015, a group of students engaged in a similar exchange with Justine. This time they were working in a teacher-led group on a problem Justine had created. The problem stated that Ms. Sanchez wanted to buy t-shirts for all of her students. In order to do so, she needed to create two tables to compare t-shirt prices between the "Mathematician store" and the "Science store." As Justine began to lead the group in making the first table, the following exchange occurred:

> "If I asked you how is this gonna help in life, are you going to give me the same answer as yesterday?" a boy asks. (This is a boy who Justine says is always asking about how

> useful different things are or how they will help him in life.) Justine says her answer will be similar. "Okay, so how is this gonna help in life?" he asks. Justine says they'll talk about that in a few minutes; right now they need to solve the problem. Students write for a minute or so. As Justine starts to walk around the table, the boy again asks, "Okay, so how is this gonna help me in life?" Justine tells him that yesterday he was using a calculator to solve similar problems, but today they are trying to understand why it works. The boy again asks in a monotone voice, "How is this going to help in life?" She tells him that he's "understanding the process." A girl at the end of the table then chimes in, saying, "But you still haven't answered the question! When are we going to use equations *(finger quotes)* in life?" Justine says it will be used in the field of science and mathematics. "But you're talking to a girl and a kid that doesn't like mathematics!" the girl replies, gesturing towards herself as she says "girl" and the boy as she says "kid." The group suddenly starts to get louder, as some students begin talking and others laugh. One boy shouts, "Name a job! Name a job!"

This exchange again begins with a boy who has a desire to see how the mathematics they're doing will "help in life." This boy and the students that chime in after him have an expectation that they will learn material that they can "use…in life." However, as is evident in these two excerpts, students sometimes cannot see the connection between classroom mathematics and everyday life, causing them to speak out against the norm of doing work without questioning its larger purpose.

On the topic of precision, two student comments highlight interesting perspectives on the authenticity and usefulness of precision. First, during a whole class discussion on February 26, 2015, students were volunteering equations to represent the relationship between gallons of gas and miles traveled if one gallon allows you to travel 30 miles. Several students volunteered equations that Justine then discussed with the class, deciding that each was incorrect. At the end of this sequence, a boy loudly asks, “How do we know it gives exact miles? Is someone gonna waste that much time?” With this question, the boy highlighted a perspective related to the authenticity of precision. By questioning whether someone would actually “waste that much time” to develop an equation that “gives exact miles,” the boy suggested that taking the time to create such an equation would not be worthwhile. We can infer from this that the boy believes an estimate would be sufficient, and there is no real need to create an equation that gives exact miles as that would not be a sensible way to spend one’s time. This view of the authenticity of precision ends up interfering with the student’s understanding of equations and their purpose.

In a second excerpt, a student was called on to read a problem out loud for the class. The student read the problem, which ended with the question, “What information does Gus get from solving each proportion?” After reading this last sentence, the boy added, “Because he has nothing else to do.” While at first glance this might seem like an innocuous comment from a middle school student trying to be funny, it actually tells us something important about this student’s view of the problem he was asked to complete. The problem describes a scenario in which an imagined person, Gus, sets up two proportions to determine which store has the better price for groceries. The task draws on an experience that is likely familiar to students – shopping at grocery stores – presumably to give students a “real world” problem and/or help them connect with the material they are learning. However, the student who read the problem saw a lack of authenticity in it, despite the fact that he might have been able to relate to the grocery shopping experience. The boy's comment "because he has nothing else to do" suggests that he views it unlikely that Gus would actually take the time to be so precise as to write out proportions and solve them to calculate the differences in cost. Rather, he believed that Gus's time could be spent more wisely. Thus, this student's comment highlights an issue with the authenticity of the "real world" context that was laid out in the problem, as the boy did not view the given scenario as likely to occur in the world. On that topic, I will now shift to exploring the context and numbers involved in school mathematics problem-solving tasks, as well as students' perceptions of the authenticity and usefulness of those tasks.

Problem-solving task characteristics. In this section, I will discuss students' views of the authenticity of the problem-solving tasks they complete in the classroom. There are two aspects of problem-solving tasks about which students make judgments regarding authenticity: the problem context and the numbers involved. Rather than describe everyday activities as I did in prior sections, I will begin by describing some of the problem contexts students worked with throughout the year. Then I will delve into students' views of the authenticity of problem-solving tasks, including both the contexts and numbers involved. Since students' comments about problem contexts naturally connect to the everyday life experiences presented in those contexts, there will be no explicit comparison to students' everyday activities in this section.

Problem contexts. In the five months during which Justine's classes were observed, students primarily worked on the following topics: ratios and proportions, rate of change, linear relationships, variables and equations, and commission and mark-up (including sales tax). Across these topics, students completed a mix of problems without contexts (such as 6/2=4/p) and problems with contexts. Here I will highlight some of the contexts students worked with on problem-solving tasks.

During the first two weeks of observations, students primarily worked on problems that focused on mixing concentrate with water to make juice. Over the following two weeks, students moved into a series of problems involving pizza. Some of these problems asked students to compare prices – both between different restaurants and for pick-up as compared to delivery – while others involved the sharing of pizzas among different numbers of people. Students also worked on problems that involved calculating the cost of different numbers of oranges and calculating distance traveled per amount of gasoline used. Throughout this time and moving forward, students periodically worked on problems comparing grocery prices and t-shirt prices, as well.

During the last two months of school, students worked on problems asking them to calculate sales tax on items given different tax percentages. Students also worked on tasks that involved calculating the rate of change in various domains including the following: travel, money deposited into a savings account, and money earned for doing chores. Additionally, students completed tasks centered on determining walking rates for a walk-a-thon and also participated in a short whole class activity to figure out their own walking rates. Finally, near the end of the year, I observed students working with problems involving mystery pouches and coins that were used to help students learn equations. In these problems, students read an equivalency between certain numbers of pouches, coins, and combinations of both and then had to determine how many coins must be in each pouch. While students certainly completed other types of problems with different contexts throughout the year, these were the main types of problem contexts observed during visits to LMS.

Students' perspectives on mathematics classroom problem-solving tasks. Through coding fieldnotes for students' perceptions of authenticity and usefulness, two themes emerged related to problem-solving tasks. First, students highlighted aspects of task contexts that they viewed as inauthentic, and one student even conveyed an expectation of inauthenticity on problem-solving tasks. Second, students wondered about the authenticity of numbers involved in tasks and sometimes conveyed an expectation that authentic numbers would be used. Across both categories, 13 such comments surfaced, providing a window into students' perspectives on classroom mathematics problem-solving tasks.

Problem contexts. Students made three types of comments related to the authenticity of problem-solving task contexts. First, some students highlighted aspects of contexts that seemed inauthentic to them. On one occasion, students watched a short video of someone buying juice made from concentrate as an introduction to problems on mixing concentrate and water. At the end of the video, a customer took some juice, tried it, didn't like it, and walked away. After the customer left, a student exclaimed (and then was echoed by other students), "He didn't pay her!" While this comment might not be consequential for the mathematics students subsequently engaged in, it highlights students' attention to whether problem contexts are authentic. Similarly, in a problem about another teacher at LMS sharing cookies with her students, one student remarked, "Miss Jones wouldn't do that."

In other circumstances, students questioned aspects of problem contexts that didn't align with their understandings of parallel real world contexts. For example, on February 27, 2015, a student in seventh period read out loud a task that asked students to figure out how far someone could drive on both one gallon of gas and zero gallons of gas. After reading the question, he replied, “It makes no sense. How are you going to drive if you have no gallons?” After a brief pause he added, “Maybe it’s just like a lawnmower,” but then upon reflection continued, “Actually they’re nothing alike.” In this situation, the boy viewed part of the question he was asked as inauthentic. He thought it made “no sense” that he was asked to calculate how far someone could drive on zero gallons of gas because they can’t drive at all. This belief caused the boy to spend time trying to figure out how the question might make sense, considering whether it might parallel some other familiar situation. In the end, he decided that it didn’t and was presumably left thinking that the task was not authentic.

During two other classes, students questioned the authenticity of calculating a walking rate given that people change speeds and take breaks. On March 2, 2015, after completing a brief problem about Caesar Chavez’s unit rate per day during a 25-day walk from Delano to Sacramento, one student asked, “What if he takes a break?” Similarly, on April 13, 2015, students timed themselves walking in the hallway to calculate their own walking rates. After Justine modeled what they would be doing, a boy commented loudly but seemingly to himself, “I don’t think that’s statistically—well that is statistics but—she could like slow down and speed up and slow down.” Both of these comments signify the students’ attention to the authenticity of what they were asked to do. Both students questioned whether the calculated rates were accurate given that the task did not take into account potential changes in walking speeds. While this was a discussion the class could have engaged in, the influence of breaks or changes in speed on walking rates was not addressed. Thus, the boys’ attention remained focused on the lack of authenticity in the problems, as the contexts did not seem to account for changes in speed that the boys viewed as likely.

Finally, given students' attention to the authenticity of problem contexts in the prior examples, it is perhaps not surprising that some students expected mathematics problem contexts to be inauthentic. Such a belief influenced Robert's ability to identify an incorrect answer in his work: As students completed a problem on mark-up in small groups, Justine called over some students who had questions, including Robert, to work with her. During the conversation, Robert asked a question that I was unable to hear. Justine then paused and replied, "You thought the car costs three hundred dollars?" Robert responded, "I don't know" and said that he thought it was "another weird question like the orange juice ones[7] that didn't make any sense." (March 4, 2015) This comment highlights Robert's expectation that mathematics problem-solving tasks will not "make any sense" and thus won't reflect his real world knowledge. His view is consistent with the perspectives of prior students who also saw problem-solving tasks as missing key elements that would be present in comparable real world scenarios.

Numbers involved in problem-solving tasks. Another way in which students attended to the authenticity of problem-solving tasks was through consideration of the numbers involved in tasks. Students sometimes questioned the numbers involved, while at other times they took the numbers to be authentic representations of what one might find in the world. First, students sometimes inquired about whether numbers involved in tasks accurately reflected real world numbers. For example, on April 13, 2015, a boy was working on a problem about the cost of

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[7] The "orange juice ones" refers to the ratio problems students completed that focused on mixing concentrate with water to make juice.

orange juice. He signaled to Justine to come over and then quietly asked her whether the number in the problem was real and you would actually pay $6 for orange juice. In a similar situation when students were learning about sales tax and working on a problem with 8.5% sales tax, a boy asked Justine's co-teacher, "So is that what it actually is?" He added, "In real life?" She replied to him that it depends where you are and that sales tax is different in different places. He then went on to ask, "So what is it like here? Is it eight point five here?" In both of these excerpts, students were interested in knowing whether the numbers in their problem-solving tasks were authentic to the cost and sales tax, respectively, that they would encounter in the world.

On other occasions, students expressed surprise at the numbers used in problems. Sometimes this surprise consisted of just a brief passing comment. For example, when reading a problem about a TV for sale, one student exclaimed "wow" when he read the size of the TV and the price at which it was being offered. At other times, however, students' surprise was more extended. On February 2, 2015, as students worked on a problem about the price of oranges, the following exchange was observed:

> As the boy walks back towards the table closest to me, he says, "Oranges are really cheap." Then the boys talk about what kind of oranges they are likely to be, and one suggests that it could be the "tiny Cutie ones"…Then Justine comes over and asks the boy for his explanation. One of the other boys says, "Oranges are cheap." Justine tells him that's not an explanation, but the boy who was supposed to explain says, "It is. It figures a lot of things out." As he continues on, one of the other boys interrupts to ask why they're talking about the oranges when "we should go get them"…Justine asks the

> boys what is “the relationship with the amount of money you get and the number of oranges you get.” One boy yells out times five. Another boy says, “Yea, oranges are really cheap. They’re point two.” Then he continues, saying that five times point two equals one, so they’re really cheap. “You can buy 5 for a dollar,” he says.

In this extended exchange, students were fixated on the fact that oranges were “really cheap.” They took the number in the problem to be authentic and reacted accordingly. Despite the fact that Justine attempted to engage students in a discussion about the scale factor, the students remained focused on the low price of oranges. This interaction illustrates the power of numbers in problem-solving tasks to grab and maintain students’ attention.

Finally, on two occasions students were captured either opposing or expressing skepticism at the numbers included in problem-solving tasks. While working on a problem that asked about the cost to buy one hundred oranges, a boy questioned, “Who would want to buy a hundred oranges?” (March 2, 2015) During a different class, students worked on problems Justine had created about the cost of various video games. The following exchange was captured:

> At the back right table, a boy comments about the NCAA game that it costs “like ten dollars,” not $39, which is the price stated in the assignment. A boy at the back center table says no and comments that you can get it “for like two dollars.” He says that you can just go to Gamestop and find it in a particular bin.

In both of these examples, students expressed some level of opposition to the numbers included in problems. As with the prior examples, students’ attention was drawn to the specific numbers involved; however, in this case the students did not believe the numbers seemed accurate. Regardless of this subtle difference, all examples in this section highlight a certain attention that students paid to the numbers involved in problems and whether they accurately reflected students' experiences in the world.

Discussion

Several key areas of findings emerge from this work. First, students reported using mathematics in many of their everyday activities. Although the level of mathematics they use is often somewhat basic, students reported features of engaging with mathematics in everyday activities that will be important to consider moving forward. Second, features of classroom environments encouraged different types of engagement with mathematics than features of students' everyday activities. And finally, students attended to the authenticity of the mathematics content they learned in class, the contexts of problem-solving tasks, and the ways in which they were asked to act and interact in the classroom. In the sections below, I elaborate on each of these themes and then consider the implications of this work for both future research and classroom practice.

Mathematics in Everyday Activities

Student responses in surveys and interviews revealed that students see themselves using mathematics in many of their everyday activities. For every activity listed on the interest survey, more than half of the students – and often nearly all students – who reported participating in each activity also reported that they believed it might or does involve mathematics. Thus, students appeared to see many connections between mathematics and their everyday activities.

Within those activities, one question worth exploring is what types of mathematics students use during participation. Through semi-structured interviews with nine students on seven activities (building, cooking, fashion, gardening, graphic design, jewelry making, and playing video games), three main mathematical topics emerged that were being used by students –counting and cardinality, measurement, and geometry. Counting is the most basic of the three topics and is only mentioned in the kindergarten Common Core Standards. Although students reported counting while playing video games, cooking, and making jewelry, these uses are unlikely to be particularly advantageous for teachers to draw upon in middle school mathematics classrooms. Rather, it will be productive for future research to examine whether early elementary students use counting in these or other activities as a way to help students strengthen their foundational knowledge related to counting and cardinality.

Measurement was the most commonly reported topic of mathematics that students used in their everyday activities, from measuring length and width while building to measuring volume while cooking. Again, the category of measurement is only included in the elementary-level Common Core Standards. However, unlike counting and cardinality, measurement is a focus through fifth grade. As with counting, it would be worthwhile to explore at what age students begin to engage in these types of measurement in their everyday activities. Additionally, although the standards only focus on measurement through fifth grade, aspects of students' measurement can still provide an entry point for several middle school mathematics topics. For example, students who reported measuring while baking have a familiarity with fractions that might be helpful to draw upon when working with fraction operations in sixth and seventh grade. Additionally, students' engagement with measurement while gardening or doing graphic design might be used as a starting point for exploring area, surface area, and volume in the middle school grades.

Finally, though only two students saw themselves using geometry – when determining angles and identifying shapes while both building and playing video games – these uses offer the potential for engagement with more complex mathematics. In middle school, students explore the topics of area, surface area, volume, congruence, similarity, scale drawings, and solving for unknown angles. Although students themselves did not report using geometry for these particular purposes, it is worth observing students' engagement with these activities in the future to see whether entry points emerge for connecting with school mathematics topics.

Overall, while students reported seeing mathematics in many of their everyday activities, it is noteworthy that most of these examples fell into only two categories of mathematical topics – measurement and counting. Though it might not be surprising that measurement and counting surfaced in many activities, it is perhaps more surprising that other topics did not surface with greater frequency. For example, despite that fact that most of the nine students reported cooking, only one mentioned the involvement of fractions, and only one spoke about using ratios when changing recipe amounts. Certainly, it might be that case that observing students engaging in these activities would reveal additional, more complex uses of mathematics. In other words, students might not recognize many of the more sophisticated ways in which they use mathematics in their lives. Thus, Vygotsky's zone of proximal development, which describes the space between what a learner can do on his own and what a learner can do with the help of others (Vygotsky, 1978a; Vygotsky, 1978b) might apply here. It might be the case that teachers can build on student reports of the ways in which students use mathematics in their lives to then scaffold them to identify additional, more complex uses of mathematics. As students learn new concepts, teachers can work to help them locate those concepts in their everyday activities, as with the example of ratios in cooking. This practice might in turn help students to see additional usefulness in the new concepts they learn.

While including observations of everyday activities in future research would be beneficial, I argue that it is still important to consider students' perceptions of the ways in which they use mathematics, as those perceptions likely have implications for their perceived utility of mathematics. In this research, students primarily focused on ways in which they used two, somewhat basic mathematics topics (counting and measurement) in their everyday activities. Although students in the study overwhelmingly reported viewing mathematics as useful, neither of those topics were a focus in Ms. Sanchez's sixth- and seventh-grade classrooms. Rather, students spent a great deal of time learning about fractions, ratios and proportions, and expressions and equations, none of which were discussed by more than one student in interviews.

This disconnect raises questions regarding the relationship between students' perceptions of the usefulness of mathematics as a subject and their perceptions of the usefulness of specific mathematics topics and tasks in the moment. Students might perceive the subject of mathematics to be useful because they see counting, measurement, addition, subtraction, and other basic mathematics processes in many places in the world. However, in a given moment as they learn in class about variable expressions, for example, that larger perception of usefulness might not serve to motivate students if they do not see the usefulness of variable expressions in particular. Thus, highlighting the types of mathematics students see in their everyday activities may shed additional light on the ways in which they perceive mathematics to be useful and provide insight into students' queries about usefulness in the mathematics classroom.

Engagement with Mathematics in Different Contexts

Second, findings highlight some key differences in the ways that students engage with mathematics in everyday activities versus in the classroom. In their everyday activities, students reported regularly participating with others and seeking help from experienced others when needed. Students also reported participating in their activities to help others or to connect with others, as well as to express their own creativity. In contrast, classroom norms and expectations typically involved following the teacher's rules as specified, related to seating assignments, voice volume, and class work. Students had little room to express creativity and were generally not given the opportunity to work towards larger goals other than completing specific assignments to avoid penalty. Additionally, while students did have substantial opportunity to interact with their peers, students also spent a significant amount of time working independently and being asked to work quietly. Furthermore, students were sometimes constrained in regards to which experienced others they could approach, as Justine specified that they were to ask their classmates questions before they asked her.

One aspect of these features of learning in everyday activities that is worth considering is whether they are specific to mathematics or apply to learning and interaction more broadly. With the exception of students' use of estimation and trial and error, I do not intend to claim that these other features (seeking help from experienced others, expressing one's creativity, and working towards meaningful interdependent goals) apply only to mathematics. In fact, it is likely the case that they do not. As the student quoted at the start of the chapter asked in Ms. Sanchez's class, "Why does everything have to be independent? Life is not independent. You have a partner! And then you have kids." This student made a connection between mathematics in the classroom and interdependent goals and interactions in life at large, suggesting that his focus on interdependence goes beyond everyday interactions around mathematics. Regardless, however, these features of the learning experience are features that co-occurred with Ms. Sanchez's students' engagement with mathematics in their everyday lives. Thus, when her students consider what it means to use mathematics in the world, the associations they make are likely accompanied by these features of engagement. As a result, even if the features are not specific to mathematics, they are important for understanding students' experiences with mathematics that they perceive as real and useful.

Specific to mathematics, students also experienced different norms surrounding precision in everyday activities as compared to the mathematics classroom. In their activities, students reported frequent use of estimation and trial-and-error to solve mathematical problems they encountered, which is consistent with prior research highlighting students' use of estimation outside the classroom (Nasir, 2000). To examine students' processes in greater depth and make comparisons to strategies used in prior research on everyday mathematics (e.g. Lave, Murtaugh, & de la Rocha, 1984; Saxe, 1988; Scribner, 1984), observations of students' participation would need to be conducted. Some comparisons can be made, however, between students' estimation in everyday activities and classroom norms surrounding precision. In the classroom, students were typically expected to calculate exact answers using particular procedures they learned and to explain their methods using precise mathematical language. While one of the eight mathematical practices of CCSSM states that students should "attend to precision," that practice also states that students should express answers with "a degree of precision appropriate for the problem context" (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). At times, the degree of precision that students thought was appropriate differed from what was requested from their teacher, which resulted in students questioning the requirement of exactness when they thought an estimate would suffice.

In highlighting these disconnects, I do not intend to claim that all of these mathematics classroom practices should be changed. Certainly, there is great benefit to having students use precise mathematical language, for example. However, noting the differences between features of mathematics use in everyday practices versus in the classroom is important to understand the perspectives that students might bring to the classroom. If students experience different norms and expectations in these settings, they might come to view classroom mathematics practices as inauthentic. As a result, they might have difficulty complying with classroom norms that conflict with the norms they experience when engaging with mathematics in their day-to-day lives. For example, students might struggle to understand why it is worthwhile to develop equations for calculating miles traveled using various amounts of gasoline when they can easily calculate estimates in their heads. One way to address this issue might be to more carefully select problem contexts in which estimates would not suffice and in which precision is required. Such problems might also be administered to students along with problems for which estimating would be appropriate. This variation could lead to a rich discussion about contexts in which estimation is preferable and contexts in which precision is required. Another possibility is to explicitly discuss with students the differences between classroom norms and those found in other contexts, and to regularly highlight the reasons for those norms and expectations. If teachers help students to see why precision is crucial and useful in many mathematical contexts, they might be less likely to oppose the level of precision that is required in the classroom.

Finally, there are some norms and expectations for classroom mathematical practice that might be worth changing. For example, reconsidering participation structures in Justine's classrooms might be beneficial, as students not only emphasized the value of working with others but also reported doing so in nearly all of their everyday activities. Since students most frequently engaged in questioning each other and seeking help from each other during partner and small group work, those structures might be especially productive for allowing students to engage with mathematics in useful, meaningful ways. This suggestion also lines up with the prior chapter's findings in which students saw usefulness in cooperating and working with others. While Ms. Sanchez's students did frequently work with partners and in small groups, they also spent a fair amount of time working independently or engaging in whole group discussions in which the teacher was the main source of authority. One possibility for shifting students' focus away from seeing the teacher as the main experienced other to drawing on classmates more frequently is to re-think what a whole class discussion looks like in a way that allows for more student-to-student interaction.

Student Perceptions of Authenticity and Usefulness

Finally, this disconnect between ways of engaging with mathematics might contribute to students' perceptions of the authenticity and usefulness of classroom mathematics. One way in which students emphasized the importance of authenticity was in terms of specific problem-solving tasks. In particular, students attended to the authenticity of both the contexts of problem-solving tasks and the numbers involved in those tasks. Thus, while mathematics classrooms often involve a norm of engaging with both authentic and inauthentic problems – as can be seen by looking through mathematics curricula – that norm might stand in opposition to students' perspectives. The seventh-grade students in this study highlighted problems that they viewed as inauthentic and questioned whether particular numbers in problems were authentic. This focus connects with the prior chapter's discussion of the importance students placed on mathematics being useful. Since students found it crucial that they learn useful mathematics, it is not surprising that they wanted to know whether specific numbers were authentic or that they compared problem contexts to their experiences in the world.

Another way in which students emphasized authenticity was when thinking about the structure of activity and forms of interaction in the mathematics classroom. As previously discussed, some students questioned the need to work independently when people typically interact with others in their everyday lives. This questioning aligns with prior research on the importance of peer relationships and students' emphasis on prosocial goals in middle school, as discussed previously (Wentzel, 1989, 1993, 2004). Also, one student questioned the authenticity of needing to show his thinking, stating that he would never explain to a cashier how he figured out the amount of money he needed to pay her. While participation structures might be worth rethinking, as discussed in the last section, justifying one's thinking is certainly a practice we want to continue encouraging in mathematics classrooms. However, it is important to know when students do not see the benefit or usefulness of a practice, as that can have negative implications for their interest and motivation. Thus, hearing a comment such as the one from this student highlights a need to help students understand why the practice of showing one's thinking is useful and to expose students to contexts in which the practice is authentic.

Implications

This work has important implications for both research and practice. First I highlight several contributions to the literature and directions for future research. Then I address implications for the design of learning environments and classroom practice.

Contributions to the literature and future directions for research. When considering implications for research, this work highlights the importance of considering features of engagement when studying students' perceptions of usefulness. Decisions about whether a subject or task is useful might lie not only in the content students are engaging with but also in the ways in which students engage with that content. This finding adds to the last chapter's discussion regarding the different ways in which students conceive of usefulness. In addition to sometimes thinking about usefulness in terms of features of the learning experience, students also reported interacting with mathematics in different ways in their everyday activities than they do in the classroom. Such reports shed light on possible reasons for students conceiving of usefulness in terms of features of the learning experience. If students see this disconnect, they might be more likely to view classroom practices as not useful. In contrast, if classroom practices align with students' practices in everyday activities, they might be more likely to view such practices as useful. Considerations regarding the usefulness of ways of engaging with mathematics will be particularly important when designing utility value interventions, as designs should not only focus on the usefulness of content but also on the usefulness of modes of delivery and interaction.

In addition to thinking about different ways of conceiving of usefulness, we must also attend to the different levels at which students might make judgements about the usefulness of mathematics. For example, one might have different opinions regarding the usefulness of the subject of mathematics, the usefulness of a particular mathematics topic, and the usefulness of a specific mathematics task. This disconnect regarding the grain size with which we discuss perceptions of usefulness has important implications for research. Many scale items ask students about their perceptions of the usefulness of the subject of mathematics as a whole, as does the Fennema-Sherman scale (Fennema & Sherman, 1976). However, such stated perceptions might be less effective in driving students' academic-related choices and outcomes, as Eccles and colleagues' expectancy-value model suggests, if students do not perceive the specific tasks and topics they are learning to be useful. Thus, it seems important to explore these different levels at which students make decisions about usefulness and their impact on course-related outcomes.

Moving forward, I propose several additional areas of research to explore. First, one limitation of this work is that eight of the nine interviewed students identified as female. It is possible that features of engagement with everyday activities, such as goals of participation and people involved, differ among male and female students; thus, additional work is needed to examine male students' perspectives. It is worth noting, however, that despite the greater number of female students involved in interviews, the focal activities were fairly representative of activities in which the male students participated. All seven male students in Ms. Sanchez's class played video games, six reported engaging with graphic design, four reported building, and three engaged with cooking; no male students reported making jewelry or engaging in fashion design. Thus, while the frequency of students participating in each activity differed by gender, both male and female students seemed to participate in most of the activities. Related, students' emphases on pursuing interdependent goals and seeking outlets for their creativity emerged during discussions with the female participants but not the single male interviewee. As a result, it will be important to expand this work to new populations to examine whether the same themes emerge with male students.

In future research it would also be worthwhile to examine whether everyday practices are enacted in similar ways in other communities and/or across age cohorts. For example, given the emphasis on interdependent rather than independent motives in working class and Latin@ communities (Esparza & Sanchez, 2008; Sabogal, Marín, Otero-Sabogal, Marín, & Perez-Stable, 1987; Stephens, Fryberg, & Markus, 2012; Stephens et al., 2012; Valdés, 1996), it is possible that students' focus on pursuing meaningful interdependent goals was particular to the focal community and would not be observed with similar frequency in communities with different demographics. Thus, expanding this work to a range of communities will allow us to examine how norms of engagement with everyday mathematics might differ in different contexts. Another interesting area for future research is examining the developmental nature of students' emphases on authenticity. Given these seventh-grade students' focus on the authenticity of classroom mathematics, it is important to explore whether attending to authenticity is especially common among early adolescents, or if the same focus emerges with older and younger students. It would also be worthwhile to explore whether students' focus on authenticity differs depending on their levels of expertise and intrinsic interest in the subject.

Finally, while this work contributes to our understanding of students' perspectives of the mathematics involved in their everyday activities, some uses of mathematics might be missed if they are not recognized by students. In this work, many students reported cooking but only one student made the connection between changing recipes and proportions (which she said they had just begun learning in mathematics class). In line with Vygotsky's concept of a zone of proximal development (Vygotsky, 1978a; Vygotsky, 1978b), it might be the case that students were unable to make that connection to more advanced mathematics on their own but could benefit from adult guidance to highlight the association. Thus, it would be productive for future research to observe students engaging in everyday activities to identify spaces where students engage with mathematics in rich ways that they do not yet recognize. Similar work has been done in other domains, such as Lee's (2001) research on cultural modeling in which youth engaged with everyday practices that provided a powerful foundation for literary analysis, yet they did not understand the disciplinary significance of their practices.

Implications for design and classroom practice. Lastly, findings from this research suggest several implications for design and classroom practice. At the broadest level, this work highlights the importance of developing an understanding of students' everyday mathematics experiences. In addition to learning about the mathematics content students engage with, examining everyday activities can shed light on the practices and norms involved in participation. Gaining insight into these features of engaging with mathematics in everyday life will allow teachers to see points of disconnect between norms, expectations, and participation structures found in classrooms versus in everyday activities. Teachers then have the opportunity to either change those norms or to frame classroom norms in terms of authentic everyday practices. For example, Ms. Sanchez might consider highlighting for students some people who show their thinking in everyday life and the ways in which showing their mathematical thinking is particularly important. In addition, she might consider reducing the amount of time students spend working independently, or restructuring whole class discussions to encourage more student-student engagement.

Next, students' focus on authenticity raises the question of how we can work to enhance perceptions of authenticity in the mathematics classroom. One way, as discussed above, is to arrange learning in authentic ways, such as encouraging students to work with others. However, issues regarding the authenticity of problem-solving tasks still remain. Certainly, we can work to draw on real world contexts to make problem-solving tasks more authentic. Even for task contexts with which students have familiarity, additional work might be done to ensure that the tasks students are asked to accomplish are themselves authentic to the real world contexts. If not, the context is likely to still be viewed as inauthentic by students. This work aligns with Taylor's (2012) discussion of his Multi-approach Engagement Framework, in which student engagement is strongest when tasks are both personally authentic and strongly connected mathematically (identified as "Mathematics In Practice"). Since students reported using mathematics in rather basic ways in this study, it might be that observations are needed to identify uses of more complex middle school mathematics in students' everyday lives, and that adults need to help students make these connections. Alternatively, we can explore the possibility that students would be equally engaged by mathematics that they view as authentic (perhaps because they observed significant others engaging with it) even if they have not personally engaged with mathematics in those ways. That possibility will be explored in the next chapter.

Finally, one component of strongly connected mathematics that has received little attention in the literature is the authenticity of numbers involved in tasks. While there has been a focus on ensuring that students engage with authentic mathematical practices, findings from this work suggest that it is equally important to choose authentic numbers for mathematical tasks. Teachers often manipulate numbers in problems to allow students to engage with particular mathematical ideas. However, I argue that the mathematical opportunities afforded by such manipulations must be weighed against the authenticity of the numbers and students' likely perceptions of that authenticity. One way we might address this issue is to consider allowing students to select their own numbers for problem-solving tasks. Certainly, this would likely constrain the mathematical opportunities available to students, as teachers would not be specifying particular numbers with which students would work. However, prior work has illustrated several benefits of allowing students to select numbers from a range of options, such as the autonomy it provides students and the opportunity for students to identify appropriate levels of challenge for themselves (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Christenson & Wager, 2012; Drake et al., 2015; Tyminski, Land, Drake, Zambak, & Simpson, 2014). In addition to these benefits, I argue that allowing students to select their own numbers might also remove the focus on whether task numbers are authentic and instead provide additional authenticity in the form of students' own prior knowledge and experiences. This possibility will be considered in the next chapter, as I discuss the design of problem-solving tasks that students are likely to view as useful.

5. Designing High Utility Problem-Solving Tasks

In order to design interventions and materials that promote perceptions of utility, it is crucial to begin with an understanding of the conceptions of usefulness of students who will be using those materials. In the prior two chapters, I presented several ways in which students in this study conceived of usefulness, as well as features of their engagement with useful mathematics outside the classroom. Having gained these insights, I was then able to engage in ecologically valid design to develop problem-solving tasks that drew directly on students' perspectives and experiences.

This approach presents a contrast to many existing utility value interventions in which the design of materials intended to promote perceptions of utility is not explicitly discussed. Without design principles or guidelines for the development of materials, it is unclear whose goals, values, and prior experiences are guiding the design of interventions. In contrast, this chapter carefully describes the approach taken to design two mathematical problem-solving tasks that were intended to promote perceptions of usefulness among students. I begin by referring back to Eccles and colleagues' expectancy value model to discuss the theoretical foundation for this design. Subsequently, I highlight the broad design principles that guided the task development process, as well as details of those principles' instantiations in my work with Legacy Middle School students. After completing a discussion of the design process, I then present the two tasks that were designed, as well as the methodology of the study that was conducted to examine those tasks in use. Finally, I present findings regarding students' performance, engagement, and perceptions of usefulness. I conclude by highlighting key themes from findings, challenges encountered and lessons learned during the design and implementation process, and implications for future design.

Theoretical Basis for Design

Eccles and colleagues' expectancy-value model was used as a starting point for thinking about the factors that influence students' perceptions of usefulness and thus seem important to consider when designing high utility tasks. According to the model, utility value is directly influenced by two factors: 1) one's affective reactions and memories and 2) one's goals and general self-schemata. One's affective reactions and memories are directly influenced by one's interpretations of her previous achievement-related experiences, while goals and self-schemata are affected by the cultural milieu, as well as one's perceptions of task demands and socializers' beliefs and expectations, among other things.

In an effort to design tasks that students were most likely to perceive as useful, I drew from these influences on utility that are specified by the expectancy-value model and also on findings from Chapters 3 and 4. Related to the expectancy-value model, findings from prior chapters highlight several possible points of leverage related to influences on perceptions of usefulness. First, students conveyed goals of wanting to help their families and/or connect with their families in out-of-school activities. In those activities, students enjoyed their participation and saw mathematics being used in important ways to achieve interdependent goals. In contrast, while students reported that they wanted to do well in school to be able to provide for their future families and give back to their own families, there were no observed connections made between those interdependent goals and classroom mathematics. Thus, incorporating an interdependent focus into the goals of classroom mathematics – such as by considering ways one can use mathematics to help one's family or to accomplish shared goals with others – might be one point of leverage moving forward to help students see usefulness in the material they are learning.

Adolescents' conceptions of usefulness also provide insight into how to design tasks that students perceive as useful. Students' responses about why they see mathematics as useful or not useful and whether they expect to use mathematics in the future provide a different sort of insight into students' goals and values. For example, when a student focuses on the usefulness of mathematics in terms of its applicability to his future career, that focus might signal the importance of career to that student, as well as a goal to be successful in his future career. Similarly, students who emphasize the usefulness of mathematics for various money-related activities, such as budgeting and doing taxes, likely place significant emphasis on their ability to manage money effectively in the future. While students might still need additional support in learning about the breadth of ways in which mathematics is used in these domains, examining students' conceptions of usefulness provides a window into their goals, as well as the ways those goals influence their thinking about the usefulness of particular tasks or topics.

Separate from findings that can be clearly mapped onto the expectancy-value model, two additional bodies of findings can be used to design tasks that promote perceptions of usefulness. First, students considered features of the learning experience (such as the method of interaction and structure of activity) when making judgments about the usefulness of particular tasks or learning situations. It might be that these features not only influence students' perceptions of usefulness but also support identity-related needs for feeling efficacious and being connected to others. Thus, it is important to also consider features of learning environments when designing tasks or learning situations for students.

Second, students frequently made connections between their mathematics learning in the classroom and their everyday experiences. Students saw themselves doing mathematics in many places in their lives, experiences that likely not only affected students' views of their own competence but also their views of what it means to do useful mathematics. Students' ideas about features of the learning experience that are useful versus not useful might also be connected with these experiences of doing mathematics in everyday life. Thus, problem-solving tasks designed to promote perceptions of usefulness among students should consider students' prior experiences engaging with mathematics in everyday life.

Taking an approach of starting with students' own conceptions of usefulness, goals and values, and prior experiences highlights several considerations for the design of useful problem-solving tasks. Some of these considerations align with the relationships highlighted by the expectancy-value model, while others represent potential additions (which will be considered further in the discussion section). Drawing on these various factors that influence perceptions of usefulness, I now present the design principles that were used to guide the development of high utility problem-solving tasks.

The Design Process

Design Principles

Four main design principles guided the development of high utility problem-solving tasks:

1. Select contexts that students perceive as useful and with which students have prior experience.

2. Align problem contexts with mathematics content to ensure that the mathematics content in tasks is authentic to those contexts.
3. Select numbers that both facilitate exploration of the focal mathematics topic and are authentic to the chosen contexts.
4. Incorporate features of engagement that students perceive as useful.

Below I describe each of the principles, as well as details of their instantiation in the LMS design process.

Design Principle #1. The first design principle is as follows: *Select contexts that students perceive as useful and with which students have prior experience.* The first component of this principle is a key component of the ecologically valid design of high utility problem-solving tasks. In order to develop tasks that students perceive as useful, contexts must be selected to align with students' own conceptions of usefulness. Based on findings regarding LMS students' conceptions of usefulness related to the applicability of content, the following instantiation of this broader principle guided the design of tasks for this study: *Include a focus on money, career, and/or supporting one's current or future family.* These contexts were selected because they were mentioned by many students as contexts that they valued and/or viewed as involving mathematics.

The second component of this principle is strongly related to the first. As prior research on culturally relevant mathematics instruction has illustrated, drawing on students' own experiences and strengths can serve as a resource in the classroom and help students connect with the materials they are learning (e.g. Brenner, 1998; Civil, 2007; Ladson-Billings, 1995; Tate, 1995). Thus, findings from student interviews and classroom connections to everyday practices were used to select contexts with which students had prior experience.

Design Principle #2. The second design principle builds on the first, focusing on the problem contexts that are selected for tasks: *Align problem contexts with mathematics content to ensure that the mathematics content in tasks is authentic to those contexts.* This design principle draws on students' emphases on authenticity during classroom observations. As Taylor's (2012) work highlights, contexts are often "mathematized," or used for problem-solving tasks in which the mathematics does not represent mathematics with which one would actually engage in that context. Students' commentary regarding the authenticity of classroom tasks suggests that it might be particularly important with this age group to ensure that the mathematics content in problems is authentic to the selected contexts. Thus, in this study, tasks are designed to reflect as accurately as possible ways in which mathematics content is used in the real world.

While Taylor's (2012) "Mathematics in Practice" approach – in which teachers draw on authentic ways that students use mathematics outside the classroom – might be the ideal approach, some mathematics content might not be used by students in their everyday lives. In that case, I propose that drawing on authentic real world practices with which students have some knowledge or familiarity might serve the same purpose. This type of approach is similar to Taylor's "Vicarious Mathematics Engagement" in which teachers expose students to others who use mathematics in order to increase the real world relevance of tasks. However, in the examples provided by Taylor, teachers did not appear to explicitly consider whether the contexts were meaningful to students or relevant to their own lives. In this work, I consider contexts that students are likely to find meaningful, as they build on students' prior knowledge and are aligned with students' prior experiences. In particular, I focus on contexts that students likely have already been exposed to, perhaps through watching significant others, and/or that students might expect themselves to engage with in the future. I argue that this prior familiarity with and value of the contexts might provide added authenticity, moving the approach beyond typical vicarious engagement.

Design Principle #3. The third design principle connects strongly with the selection of problem contexts: *Select numbers that are authentic to the chosen contexts*. Again, as students frequently highlighted the authenticity (or lack thereof) of numbers included in problem-solving tasks, it is important to select numbers that are authentic to contexts. This practice should increase the likelihood that students view the tasks as accurate representations of real world situations and, thus, as useful tasks with which to engage. For Legacy Middle School students, I used a particular version of this design principle: *Allow students to select their own numbers to ensure that they view the numbers as authentic to the contexts.* The contexts selected appeared ripe for allowing students to choose the numbers they would work with, as estimation was a key part of both tasks. Additionally, allowing students to choose their own numbers is an effective mathematical practice, as it can increase access to the material since students are able to select numbers that provide an appropriate level of challenge for them (Aguirre et al., 2012; Drake et al., 2015; Tyminski et al., 2014). Thus, students were asked to select their own estimates in the two designed tasks to both increase access to the mathematics and allow for greater authenticity, as well as to create room for self-expression and encourage an appropriate level of precision (to be discussed shortly).

I must note that there are also potential drawbacks to letting students choose their own numbers. Since the numbers chosen sometimes affect the mathematical concepts that can be explored, this practice will not be effective in all situations. For example, if a teacher wants students to explore fractions in particular, then letting students select any number to work with might not be a productive approach. Rather, the teacher might want to include some constraints, perhaps by providing students with a range of number options, as Drake et al. (2015) do. One important consideration, however, is whether students view those number options as authentic. In such cases, the teacher might need to do some initial work to validate the authenticity of the selected numbers and/or to illustrate to students when such numbers will be used in real life in the given contexts.

Design Principle #4: The fourth design principle draws on findings regarding students' conceptions of usefulness in Chapter 3: *Incorporate mathematical practices and features of engagement that students perceive as useful.* Rather than focusing solely on selecting useful problem contexts and finding ways to illustrate the usefulness of content, problem-solving tasks should incorporate practices that students perceive as useful. With Legacy Middle School students, these practices included both particular ways of interacting with others and modes of expression when engaging with mathematics. Specifically, four instantiations of this broader principle were used to guide the design of problem-solving tasks for LMS students:

3a. Provide opportunities for working together and drawing on classmates' knowledge and experiences.

3b. Encourage students to show their thinking.

3c. Require the degree of precision that is authentic to the given context.

3d. Incorporate room for student creativity and self-expression.

Principles 3a and 3b draw on findings related to students' conceptions of usefulness and directly tie into CCSSM's mathematical practices. Principle 3c also aligns with the mathematical practices and stems from students' engagement with useful mathematics in everyday practices. Finally, principle 3d was also developed based on students' reports of key features of engaging with everyday practices involving mathematics.

Problem-Solving Tasks

Using the aforementioned design principles, two problem-solving tasks were developed (see Appendix D and Appendix E). In the first task, students were asked to imagine their lives in 20 years and think about how they would calculate their monthly and yearly grocery budget. Students first decided how many people they expected to be cooking for and then estimated their weekly cost of groceries before using that amount to determine monthly and yearly costs. In the second task, students were asked to consider a situation in which they wanted to make extra money for their families and so decided to start a business with some of their friends. Each group was asked to select an item to sell, as well as the cost of that item. Based on that cost, students then determined how many items they needed to sell to make various amounts of money.

The goal of design was to create tasks that students would perceive as useful and that would enhance student engagement with the material. For logistical reasons, the mathematics content to be included in the tasks was selected first. Given the timing in the school year, Justine's preferences, and its applicability to real world contexts, algebraic expressions and equations was selected as the focal topic. Task contexts were then selected, drawing on students' conceptions of usefulness and everyday experiences. Budgeting for one's future family and starting a business to make money to help one's family were selected to incorporate students' strong emphasis on family, prior experience with money-related family activities, and conceptions of the usefulness of mathematics for money-related activities. Problems focused on different points in time (now versus the future) to explore whether considerations of current or future relevance influenced students' judgments on the usefulness of tasks.

In order to align with students' conceptions of the usefulness of working with others, tasks were not only completed in groups but designed specifically with the goal of creating groupworthy tasks (Featherstone, 2011). In other words, tasks were created in such a way that the participation of all group members was required. While it was common for students in Ms. Sanchez's class to work in groups, the tasks with which they worked could typically be completed independently. In contrast, these tasks were designed specifically to draw on the affordances of working with others.

As previously described, both tasks asked students to estimate costs – a weekly budget in task A and the sale price of a product for task B. Including this element aligned with several of the design principles. First, it allowed for an appropriate level of precision. For example, when thinking about task A, adults would typically estimate the cost of groceries per week or month rather than calculating the exact cost of every item purchased in a month. Given students' closeness with their families and familiarity with their families' uses of money, I suspected that students would have prior experiences with purchasing groceries to draw on – just as adults would – when making these estimates. Second, allowing students to select their own numbers was intended to provide greater authenticity, as students were able to select numbers that they viewed as reasonable for the particular situation. In task A, each student was asked to select a different estimate that was based specifically on the number of family members they expected to be cooking for in the future. Related, familiar contexts were selected for tasks so that students would have prior knowledge to draw on when engaging with the tasks. Thus, asking students to determine estimates provided an opportunity for them to use and share their own knowledge with their groupmates. Finally, choosing their own estimates allowed for some degree of creativity and self-expression on the part of students. Particularly in task B, groups were able to select both the product they wanted to sell and the price at which they would sell that product.

It is worth noting that Ms. Sanchez and I did recognize potential drawbacks of this approach, such as that numbers selected by students might vary considerably and that students might not have the prior knowledge to develop reasonable estimates. (One unanticipated result, which will be discussed later, was the time students spent determining estimates.) As a result, we considered providing students with some guidance to determine estimates in task A. In particular, we considered showing students data from the U.S. Census on families' grocery spending habits. However, we decided that some students might not see themselves reflected in the data provided and also that some students might be tempted to simply select the exact amounts provided in the data, rather than considering the sizes of the families they expected to have. Additionally, we were interested to see what prior knowledge students drew on in making their estimates and whether they would be able to determine reasonable estimates. Thus, for the first iteration of this design, no resources were provided to students to assist with the process of estimation.

The last component that was included in tasks based on the aforementioned design principles centers on the mathematical practices involved in engagement. Throughout the problems, students were asked to show their thinking, as showing thinking was cited by some students as a useful practice. However, given that not all types of showing thinking were viewed as useful by students, attempts were made to include requests to show thinking at times that might actually further students' thinking, rather than feel like busy work or unnecessary add-ons. Additionally, most questions in the tasks were posed in such a way to allow students to use multiple strategies for problem-solving. Subsequently, students were sometimes asked to compare strategies or answers with their peers, or to draw on the expertise of their peers, in order to facilitate productive interaction with group members. Below I describe the methods of the study in which these tasks were embedded, including the selection of curriculum tasks, forms of assessment, participants involved, and strategies for data analysis.

Methods

A study was designed to examine how students engaged with and perceived the two designed tasks versus typical curriculum tasks. Key components of this study included the selection of problem-solving tasks and the development of a survey to measure student perceptions of the task. After discussing these components of the design, I will describe the participants involved and details of task implementation, as well as forms of data analysis used.

Overall Study Design

This study was designed to examine the effect of two main task features on students' perceptions of usefulness – task type and interaction structure. Three types of tasks were included in the study – decontextualized, contextualized, and personalized. Decontextualized and contextualized tasks were selected from a curriculum that was used by LMS. Decontextualized tasks included numbers and variables but no context, while contextualized tasks (i.e. word problems) included numbers embedded in written scenarios. Personalized tasks were the previously described tasks that I designed to align with students' conceptions of usefulness, values, and engagement with mathematics in everyday contexts.

The two interaction structures considered were independent work and group work. Tasks to be completed as independent work are referred to as "Do Now" tasks, as that language is consistent with the language Justine used in her classroom. In the initial study design, students were intended to complete two Do Now tasks, one decontextualized and one contextualized. (Ultimately, students ended up completing two tasks of each type, the reasons for which I will describe below.) During group work, students completed four different tasks. One task was contextualized, one was decontextualized, and two were personalized. Students completed both personalized tasks in groups because working with others was a key feature of students' perceptions of engagement with useful mathematics, and these tasks were designed to be viewed as useful by the students. Table 5.1 depicts the problem type and participation structure of each of the six tasks, the selection of which will be described below.

Table 5.1

Problem-Solving Tasks

	Level of Interaction	
Type of Problem	Independent Work (no interaction)	Group Work (3-5 students per group)
Decontextualized (from curriculum)	*Task 1:* Use ideas you've learned about solving equations to solve the following equation: $4t + 12 = 76$. *Task 2:* Use ideas you've learned about solving equations to solve the following equation: $3x+14=86$	Use ideas you've learned about solving equations to solve the following equation: $87 + 2c = 105$. Show your group's work below.
Contextualized (from curriculum)	*Task 1:* Single admissions at Wild World Amusement Park cost $21. a. Write an equation to answer	Ocean Bike Tours wants to provide bandanas for each person who goes on their tour. The cost of the

	this question: How many single admissions were sold on a day the park had income of \$9,450 from single admissions? Use *n* to represent the number of single admissions the park sold on that day. b. Solve the equation. Explain how you found your answer. *Task 2:* Souvenir t-shirts at Wellesley Stadium cost \$16. a. Write an equation to answer this question: How many souvenir t-shirts were sold on a day the stadium had an income of \$6,736 from t-shirt sales? Use *n* to represent the number of t-shirts sold on that day. b. Solve the equation. Explain how you found your answer.	bandanas is \$95 for the design plus \$1 per bandana. a. Write an equation to answer this question: What is the total cost of the bandanas if Ocean Bike Tours buys 21 bandanas? b. Solve the equation. Explain how you found your answer.
Personalized (designed by author)	N/A	Personalized Problem-Solving Task #1 (see Appendix D) Personalized Problem-Solving Task #2 (see Appendix E)

Selection of Tasks

As previously described, the selection of tasks began with the identification of the mathematics content on which the tasks would focus – algebraic expressions and equations. I spoke with Justine to determine what the students had already learned in terms of expressions and equations up to that point in the year including what size and type of numbers they had worked with, what operations they had worked with, and how many variables they were used to seeing in equations. Given their prior experiences and difficulties, she suggested that focusing on one-step equations for word problems and two-step equations for decontextualized problems (both involving only one variable at a time) would be appropriate and comparable.[8]

The next step was to identify four tasks from the curriculum used by the school to serve as foundations for the contextualized and decontextualized tasks. Upon speaking with Justine, she informed me that her sixth-grade team did not consistently use the curriculum that her seventh-grade team had used during the prior year (*Connected Mathematics 3*; CMP3). Instead, they used a combination of materials they created, materials they were developed by other teachers and purchased by the school through an online platform, and materials from CMP3. Since created and purchased materials were not available ahead of time, problems were still modeled off of the CMP3 curriculum materials.

First, two decontextualized problems were selected from the curriculum. Both problems were two-step equations with single-digit coefficients and double- or triple-digit constants (e.g. $4t + 12 = 76$). Both required a combination of subtraction and division to solve. The language used in the curriculum to introduce the problems was also used for the problems in the study.

Subsequently, two contextualized problems were selected to match in difficulty with the decontextualized problems. Justine reported that students struggled with word problems and that one-step word problems in which students had to write their own equations should be comparable in difficulty to two-step decontextualized problems. Thus, two problems were selected in which students were required to write and solve equations and then explain how they

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[8] Due to logistical constraints, I was unable to empirically test Justine's claim. However, students did in fact view all problems as comparably difficult. In future research, field testing should be done to ensure that all tasks are a comparable level of difficulty before they are implemented.

found their answers. The first problem required students to write an equation that involved a coefficient but no constant (24n=9,450) and required division to solve. The second problem required students to write an equation that involved a constant but no coefficient (95+21=n) and required addition to solve. The wording of the contextualized problems was changed slightly to ensure that the language used was consistent across both problems, and numbers in one of the problems were changed in attempt to make the difficulty comparable.

Finally, at this point the previously described personalized tasks were designed. The same procedures were followed for the selection of mathematics content, including both processes required and numbers involved. To align with the contextualized problems, one problem involved equations in which the variable was already isolated on its own side of the equation, while the other involved variables with coefficients. In particular, the first personalized problem asked students to write equations that required only multiplication to solve (e.g. 15x4=n), while the second task required students to write equations that involved variables with coefficients (e.g. 5n=100). In contrast to the contextualized tasks, personalized task contexts were selected based on findings from prior analyses, as detailed earlier, rather than from standard curriculum materials.

All six tasks were reviewed by several experts in the field for consistency in difficulty (e.g. placement of variables and operations involved) and appropriate language use. Revisions were subsequently made, and then tasks were reviewed by Ms. Sanchez for the same factors. Additionally, Ms. Sanchez recommended changes to increase the similarity between the formatting of the tasks and the formatting with which students were most familiar from prior tasks. Using these suggestions, a final round of changes was made to prepare tasks for administration. One decontextualized and one contextualized task were then selected for independent work. The remaining contextualized task and one personalized task were grouped in a packet labeled “Group Work A,” while the remaining decontextualized task and the second personalized task were placed into a packet labeled “Group Work B.” Separate group recording sheets were printed out onto which students were asked to record their group’s responses to the questions.

Student Survey

A survey was designed to administer to students after completion of the tasks. There were three goals for the survey: First, students’ perceptions of the overall usefulness of each task were measured to identify whether students did, in fact, view the personalized tasks as more useful than the curriculum tasks. These ratings were also used to compare perceptions of the usefulness of tasks completed using different participation structures. Second, students’ perceptions of the difficulty and interest level of each task were measured in order to determine a) whether there were unintended key differences in difficulty across the tasks and b) whether students’ interest level was correlated with their perceptions of usefulness. To measure usefulness, interest, and difficulty, students were asked the following questions:

How interesting did you find this problem?

How difficult did you think this problem was?

How useful does this problem seem to you?

For each of these three questions, students were asked to select one of five responses – not at all, slightly, somewhat, very, or extremely. Finally, student surveys were used to gather qualitative information about students’ engagement with and perceptions of usefulness of the various tasks.

Four different short answer questions were designed to provide different prompts for students to think about the usefulness of the tasks. Two questions asked students to compare the three tasks they had completed that day, while the other two questions asked students specifically about their perceptions of the personalized task. The comparison prompts included the following:

> Look at all 3 problems on the other side of this page. Which problem did you think was the most useful? Why?
>
> Look at all 3 problems on the other side of this page. Which problem were you most engaged in while working? In other words, which problem made you feel the most focused and interested in your work? Why?

The first prompt was intended to gain insight into students' perceptions of usefulness of the personalized task in relation to the contextualized and decontextualized tasks. Meanwhile, the second prompt was meant to explore students' engagement on the personalized task in relation to the other two tasks. After each of these prompts, students were also asked whether there was anything else they wanted to share about how they felt about any of the problems.

The next two prompts focused specifically on the personalized tasks. One of these prompts asked about the personalized task broadly speaking:

> Do you think you will ever have to deal with a problem like [*personalized task]* in your own life? Why or why not?

This prompt was designed to examine whether students saw the personalized tasks as applicable to their lives. The second task asked about students' ratings of the usefulness of the personalized task. After selecting their usefulness ratings, students were prompted with the following:

> Please explain your last answer. Why do you think *[personalized task]* was useful – or

not useful – to do?

This prompt was intended to broaden ways of thinking about usefulness to not only focus on the applicability of content (as the prior prompt did) but also allow students to consider other aspects of the task that were useful or not useful.

Participants

The sixth-grade class that participated in semi-structured interviews and observations during Year 2 of this research was the focal class for this study. Only one class was used, as this pilot study is the first step in a larger design process and is meant to inform task design for future iterations of this work. Examining one class deeply allows for the identification of initial strengths and weaknesses of the design. These findings will then be used to make changes to the tasks before presenting them to additional students.

Twenty-eight students split into seven groups participated in this study. Groups ranged from three to five students in size, with the exception of the second day on which tasks were administered when Ms. Sanchez asked two students to work independently. The nine students who participated in semi-structured interviews formed two of the seven groups. Kristal, Cherell, Julie, and Veronica worked in one group, while Reagan, Jared, Arianna, Alexis, and Analise comprised a second group. Those groups' work sessions were audio-recorded, and recordings were transcribed for later analysis. The former group will be referred to as Kristal's group, while the latter will be referred to as Reagan's group.

Task Implementation

Tasks were implemented over the course of two days in March and April, 2016. On the first day, students completed two Do Now problems (one decontextualized and one contextualized) independently and then completed one contextualized problem and one personalized problem in their groups. Students spent approximately 10 minutes working on the Do Now tasks and 40 minutes on group work. Both independent problems were administered at the start of class as Justine initially intended for students to complete all six of the problems in one day. However, it was the last day of school before spring break, as well as pajama day and Ms. Sanchez's birthday, so students were more distracted than usual. Thus, Justine and I together decided to administer only Group Work A (one contextualized and one personalized problem) and to save Group Work B (one decontextualized and one personalized problem) for after spring break.

Since the survey was designed such that one set of group problems was paired with one Do Now for evaluation, each student ended up evaluating only one of the two Do Now problems they had completed on the first day. Thus, during the second administration of tasks, students were again asked to complete two Do Now problems (different tasks, though of the same type as the first set), and again approximately half of the class rated each task[9]. Due to this change in administration, some students rated decontextualized Do Now tasks twice, while others rated contextualized Do Now tasks twice. Only 14 students rated both types of Do Now problems. Changes to analysis that were made based on this unexpected distribution of ratings will be discussed below.

[9] The same procedures as discussed in the methods section guided the selection of these two additional Do Now tasks. The new tasks were designed based on the previously selected curriculum tasks, and comparable numbers were selected for inclusion. Contexts were changed only slightly, and mathematical procedures required to solve the problems remained the same.

During the second administration, students completed a decontextualized problem and the second personalized problem in their small groups (considered Problem Set B, along with the two Do Now tasks). The tasks included in Problem Set A and Problem Set B can be viewed in Table 5.2. Students again spent approximately 10 minutes working independently on the Do Now tasks and then approximately 30 minutes working in groups on Problem Set B. While students were initially assigned to work with the same groups they had worked with during the first administration of tasks, Justine asked two students to work independently part way through, as she was disappointed in their behavior and did not think they were taking the tasks seriously. Towards the latter part of the thirty minutes, many students began to get distracted and antsy, and some started wandering around the classroom. Thus, Justine ended work time earlier than originally planned.

Table 5.2

Groupings of Tasks in Problem Sets

	Problem Set A	Problem Set B
Do Now (independent tasks)	*Decontextualized:* Use ideas you've learned about solving equations to solve the following equation: $4t + 12 = 76$. *Contextualized:* Single admissions at Wild World Amusement Park cost $21. a. Write an equation to answer this question: How many single admissions were sold on a day the park had income of $9,450 from single admissions? Use *n* to represent the number of single admissions the park sold on that day. b. Solve the equation. Explain how	*Decontextualized:* Use ideas you've learned about solving equations to solve the following equation: 3x+14=86 *Contextualized:* Souvenir t-shirts at Wellesley Stadium cost $16. a. Write an equation to answer this question: How many souvenir t-shirts were sold on a day the stadium had an income of $6,736 from t-shirt sales? Use *n* to represent the number of t-shirts sold on that day. b. Solve the equation. Explain how you found your answer.

	you found your answer.	
Group Tasks	*Contextualized:* Ocean Bike Tours wants to provide bandanas for each person who goes on their tour. The cost of the bandanas is $95 for the design plus $1 per bandana. a. Write an equation to answer this question: What is the total cost of the bandanas if Ocean Bike Tours buys 21 bandanas? b. Solve the equation. Explain how you found your answer. *Personalized Task #1* (see Appendix D)	*Decontextualized:* Use ideas you've learned about solving equations to solve the following equation: $87 + 2c = 105$. Show your group's work below. *Personalized Task #2* (see Appendix E

At the end of each administration, every student received a survey asking them to rate the three tasks (one Do Now and the two group tasks) on interest, usefulness, and difficulty. Each student was also provided with one of the four short-answer prompts described above. Thus, four different versions of the survey were created for each administration. To facilitate comparisons between the different types of tasks, two versions (one with a comparative prompt and one with a non-comparative prompt) included the decontextualized Do Now problem, while the other two versions included the contextualized Do Now problem. Surveys were administered such that approximately one-fourth of the class (seven to eight students) completed each question during each administration and comparison questions were asked equally across different combinations of problems.

Several additional aspects of administration are worth noting before moving on. First, one student in each group was allowed access to their computer so that they could use their computer calculators if need be. Second, all aspects of administration were the same during the first and second sets of group work with one exception. Tasks in Problem Set A were written on a single worksheet, and an additional recording sheet was provided on which students were asked to show their work and write their answers. However, during the first administration, Justine and I both noticed that students seemed overwhelmed by the amount of consecutive text on the problem sheet. Thus, for Problem Set B, we decided to incorporate work space into the tasks as needed so that students only focused on one step in a problem at a time. Third, while I monitored the two focal groups and provided assistance only regarding task guidelines, Justine wandered around the room and helped other groups when they had questions. It is unclear exactly how much or what form of help she provided, as her work with those groups was not documented. Finally, it is worth reiterating that students were rather distracted on both days of administration. As a result, it took Justine longer than usual at the start of class to settle everyone down and get students started on their work, which resulted in less work time than planned. Additionally, Justine ended group work earlier than I had originally thought she would, presumably because of students' lack of focus. Given all of these factors, most groups did not finish their work, which makes performance difficult to assess for personalized tasks.

Data Analysis

Four primary types of data analysis were completed in this study, in addition to broader reflections on the overall design and implementation process. First, students' survey ratings were analyzed using quantitative methods. Mean ratings of interest, usefulness, and difficulty were calculated for each task, and paired t-tests were used to calculate differences in ratings across tasks. For all four group tasks, one rating was provided by each student (with the exception of one student who left two responses blank). However, for the Do Now tasks, some students did not have the opportunity to rate either the decontextualized or the contextualized Do Now task. Thus, mean ratings were calculated using only the portion of students who had the opportunity to rate each task. Additionally, only students' first ratings of a particular task type were included, so if a student rated the decontextualized Do Now task during both administrations, that student's second rating was dropped. Due to this complexity, regression analyses were not able to be conducted as planned, as only 14 students provided ratings for all six tasks. In future work, similar ratings will be collected with a larger number of students so that regression analyses can be used to determine the effects of task type and participation structure on students' perceptions of usefulness.

Second, short-answer survey responses were analyzed using qualitative methods. For each question type, responses were coded to identify key themes. As there were a limited number of responses to each prompt (13-15), themes were used largely to provide additional insight into students' ratings of the usefulness of each task. Responses to prompts were also used to evaluate the effectiveness of different prompts at gathering valuable information, as well as to propose new areas of exploration for future work focused on students' conceptions of usefulness.

Third, student performance on each problem-solving task was analyzed. Responses to all four independent Do Now tasks were coded as correct, incorrect/no answer, or partially correct. Responses that were incorrect were grouped with responses for which students had no answer, as it was sometimes unclear whether the work shown was meant to be an answer. Responses coded as partially correct involved correct procedures but incorrect final answers. Students' mean success rates were then calculated for each decontextualized and contextualized Do Now task (four tasks total).

For group work, decontextualized and contextualized tasks were coded using the same three categories – correct, incorrect/no answer, or partially correct – and mean success rates were again calculated. However, for personalized tasks, correctness categories could not be applied in the same way, as there were many parts of the tasks, so partial correctness was common. Additionally, since personalized problem-solving tasks were completed last, most groups did not have the opportunity to complete all parts of the tasks. Thus, many questions were left blank simply because of lack of time. Due to these complexities, I instead examined students' responses for overall themes, including correctness of equations and calculations and correctness of strategies, as well as groups' degrees of completion of each task.

Finally, student engagement on group problem-solving tasks was analyzed using qualitative methods. A total of four audio recordings (two on each day of administration) were captured from the two groups of students who provided consent to participate. Audio recordings were transcribed, and open coding was used to identify possible codes related to student engagement. Many codes overlapped with the coding scheme created for Chapter 4 analyses; thus, the same coding scheme was used with just a few additions. New codes included "Going beyond the problem" and "Narrowing the scope of the problem," as well as codes for group and task type that were meant to facilitate an examination of patterns both within and across groups and tasks.

Findings

Findings are organized around three main themes: perceptions of usefulness, problem-solving success, and engagement. To lay the foundation for examining the first theme, perceptions of usefulness, I begin by highlighting students' perceptions of the relevance of tasks to their lives, as well as their interest in and perceptions of difficulty of the tasks. Then I dive into examining students' perspectives on the usefulness of the tasks, highlighting differences observed across the three types of tasks and two participation structures. Second, I examine students' problem-solving success on the four Do Now tasks and four group tasks. Finally, I describe key themes that emerged related to students' engagement with the personalized problem-solving tasks as compared to the decontextualized and contextualized curriculum tasks.

Student Perceptions of the Relevance and Difficulty of Tasks

Since one of the goals of task design was for students to see the tasks as relevant to their lives, I begin by examining whether that goal was achieved. One of the four prompts to which students responded after completing each set of tasks was "Do you think you will ever have to deal with a problem like [*Personalized Task A or B]* in your own life?" In total, thirteen students responded to the prompt. Of the seven students who discussed Personalized Task A, three expected to deal with a similar problem in their lives, two thought they might deal with a similar problem, and two misunderstood the question. When students wrote about why they expected to deal with similar problems, they mainly commented on expectations for their own futures (e.g. "I will have to buy food when I'm older for people and I would like to estimate to see how much I would spend"). Similarly, of the six students who discussed Personalized Task B, five expected to deal with a similar problem in their own lives, and one thought he might deal with a similar problem. This time when students wrote about their futures, they highlighted the fact that they might need to sell items to make money and that they wanted to support their families. For example, one student wrote "I might want to sell/make things to get money," while another reported, "I might…use this problem in real life because you can learn how much money you need to support your family." One student also wrote about expecting to deal with a similar problem "because I will have to figure out my designing company." Overall, students thought that they might or would deal with problems similar to both personalized group tasks in the future.

Another goal of task design was to select and create tasks that were perceived as comparably difficult by students. After completing each set of problems, students were asked to rate each task on its difficulty by providing a rating from 1 to 5, where 1 represented not at all difficult, and 5 represented extremely difficult. As can be viewed in Table 5.3, on average students viewed all six tasks as between slightly and somewhat difficult. Paired t-tests results indicate that there are no significant differences in difficulty ratings across the six tasks. Thus, the goal of selecting tasks that students viewed as comparable in difficulty was achieved, and students' perceptions of usefulness should not be influenced in any significant way by differences in perceived difficulty of the tasks.

Table 5.3

Student Perceptions of the Difficulty of Tasks

Criterion	Task	N	Mean	SD
Difficulty	IND-DECON	22	2.32	.894
	IND-CON	20	2.20	1.005
	GRP-DECON	27	2.26	1.318
	GRP-CON	27	2.44	1.050
	GRP-PERS-A	27	2.52	.975
	GRP-PERS-B	28	2.46	1.138

Note: IND-DECON = Decontextualized Do Now (independent), IND-CON = Contextualized Do Now (independent), GRP-DECON = Decontextualized Group Task, GRP-CON = Contextualized Group Task, GRP-PERS-A=Personalized Group Task A, GRP-PERS-B=Personalized Group Task B.

Perceptions and Conceptions of Usefulness

In this section, I examine students' perceptions of usefulness of the six problem-solving tasks. I begin by comparing students' ratings of the usefulness and interest of the six tasks, as well as highlighting perceptions of usefulness expressed through short-answer responses. Subsequently, I examine the conceptions of usefulness that students applied when assessing both the usefulness of the personalized tasks and the most useful tasks in each problem set. Overall, students viewed the personalized tasks as the most useful, with Personalized Task A being viewed as considerably more useful than all other tasks. Evidence for this finding will be described below.

Perceived interest and usefulness of tasks. As previously described, students were asked to rate the interest and usefulness of each task they completed, providing ratings from 1 to 5, where 1 represents not at all interesting or useful, and 5 represents extremely interesting or useful. Mean student ratings and standard deviations can be found in Table 5.4. For interest ratings, students had similar levels of interest in all problem-solving tasks, with the exception of Personalized Group Task A (GRP-PERS-A). Students rated GRP-PERS-A as significantly more interesting than all five of the other tasks ($p<.01$), regardless of task type or participation structure. As there is a strong correlation between interest and perceived usefulness, it is unsurprising that students also viewed GRP-PERS-A as significantly more useful than all other tasks. (The only other significant difference in usefulness ratings was between GRP-PERS-B and GRP-DECON; $p<.05$.)

Table 5.4

Student Perceptions of the Interest and Usefulness of Tasks

Criterion	Task	N	Mean	SD
Interest	IND-DECON	22	2.36	1.093
	IND-CON	20	2.60	1.353

	GRP-DECON	28	2.14	1.079
	GRP-CON	27	2.56	1.251
	GRP-PERS-A	27	3.44	1.340
	GRP-PERS-B	28	2.79	1.475
Usefulness	IND-DECON	22	2.11	.950
	IND-CON	20	2.40	.940
	GRP-DECON	27	2.48	1.189
	GRP-CON	27	2.85	1.064
	GRP-PERS-A	27	3.29	1.211
	GRP-PERS-B	28	2.75	1.481

Note: IND-DECON = Decontextualized Do Now (independent), IND-CON = Contextualized Do Now (independent), GRP-DECON = Decontextualized Group Task, GRP-CON = Contextualized Group Task, GRP-PERS-A=Personalized Group Task A, GRP-PERS-B=Personalized Group Task B.

Table 5.5 highlights paired t-test results examining similarities and differences in students' usefulness ratings across all tasks. As the table illustrates, ratings of usefulness were statistically significantly higher for GRP-PERS-A than for all other tasks with the exception of the Contextualized Group Task (GRP-CON). Ratings of usefulness were also statistically significantly higher for GRP-CON than for the Contextualized Do Now task (IND-CON), $t(18)=2.379$, $p<.05$, suggesting that working with others enhanced students' perceptions of usefulness of contextualized tasks. Finally, ratings of usefulness were also higher for GRP-CON than for the Decontextualized Do Now (IND-DECON), $t(21)=3.114$, $p<.01$, suggesting a combined effect of problem context and participation structure.

Table 5.5

Paired Sample T-Test Results for Student Usefulness Ratings

Tasks Being Compared	t	df	Significance (2-tailed)
IND-DECON * IND-CON	-.520	13	.612
IND-DECON * GRP-CON	-3.114	21	.005**
IND-DECON * GRP-DECON	-1.348	21	.192
IND-DECON * GRP-PERS-A	-4.294	21	.000**
IND-DECON * GRP-PERS-B	-1.704	21	.103
IND-CON * GRP-CON	-2.379	18	.029*

IND-CON * GRP-DECON	-.271	18	.790
IND-CON * GRP-PERS-A	-4.025	18	.001**
IND-CON * GRP-PERS-B	-1.876	19	.076
GRP-CON * GRP-DECON	1.098	25	.283
GRP-CON * GRP-PERS-A	-1.868	26	.073
GRP-CON * GRP-PERS-B	.547	26	.589
GRP-DECON * GRP-PERS-A	-2.545	25	.017*
GRP-DECON * GRP-PERS-B	-1.192	26	.244
GRP-PERS-A * GRP-PERS-B	2.193	26	.037*

* Correlation is significant at the 0.05 level (2-tailed).
** Correlation is significant at the 0.01 level (2-tailed).
Note: IND-DECON = Decontextualized Do Now (independent), IND-CON = Contextualized Do Now (independent), GRP-DECON = Decontextualized Group Task, GRP-CON = Contextualized Group Task, GRP-PERS-A=Personalized Group Task A, GRP-PERS-B=Personalized Group Task B.

Overall, these comparisons can be used to consider the role of participation structure and problem type in students' usefulness ratings. According to t-test results, working in groups was related to higher perceptions of usefulness for contextualized problem-solving tasks but not for decontextualized problem-solving tasks. Problem type (decontextualized, contextualized, or personalized) did not appear to play a role in students' perceived usefulness ratings for independent tasks and was only connected with differences in perceived usefulness ratings for GRP-PERS-A compared to GRP-DECON. In other words, students viewed the personalized group task as more useful than the decontextualized group task, though the same result did not occur for GRP-PERS-B. Considered together, participation structure and problem type had a significant effect on students' usefulness ratings of GRP-PERS-A, with students viewing that personalized group task as more useful than both Do Now tasks.

Students' perceptions of the usefulness of the six tasks is also confirmed by their responses to the question, "Which problem did you think was the most useful? Why?" Seven students replied to the aforementioned prompt for Problem Set A, while eight students replied to the prompt for Problem Set B. Table 5.6 illustrates students' responses for each problem set.[10] Overall, students rated the personalized tasks as the most useful tasks in both problem sets, with Personalized Task A again being rated as even more useful than Personalized Task B.

Table 5.6

Student Perceptions of Most Useful Task in Each Problem Set

	Number of Students Rating Each Task as Most Useful				
	IND-DECON	IND-CON	GRP-CON	PGT	None
Problem Set A	0	0	0	6	1
Problem Set B	2	0	2	4	1

Conceptions of usefulness applied to tasks. In addition to examining students' perceptions of usefulness of the tasks, we can explore the conceptions of usefulness that students applied to make decisions about their perceived usefulness of the tasks. Two prompts asked students to reflect on their perceptions of usefulness, providing insight into the criteria students used to make those judgements about usefulness. Fifteen students responded to the question, "Which problem did you think was the most useful? Why?" while 13 students responded to the prompt, "Why do you think *[personalized task A or B]* was useful – or not useful – to do?" The coding scheme used in Chapter 3 was applied to examine the conceptions of usefulness that guided students' responses on these two questions.

Of the 28 total responses across the two prompts, 10 referenced conceptions of utility focused on the applicability of content. Specifically, eight students wrote about the usefulness of

[10]One student felt that the Decontextualized Do Now and Decontextualized Group Task in Problem Set B were equally useful; thus, two of the responses in the second row of results in Table 5.6 represent one student.

Personalized Group Task A for helping in their future and for thinking about issues related to family and money (e.g. “I think the last problem was very useful because if I have a family, to be precise, 4 family members, I would know, on an estimate, of how much I would spend”). Meanwhile, only two students wrote about the usefulness of Personalized Group Task B in terms of applicability of content; one of those students wrote about the usefulness of the problem for his/her future career (“if I ever wanted to start a business then I could refer kinda back to this”), while the other wrote about broad applicability (“if I ever do something like that I need to know how to do the math and what to do”). It is worth mentioning, however, that some of the ten students seemed to focus more on the applicability of the problem context, rather than the mathematics content.

Students who focused more on the relationship between the problem context and their lives highlighted the relevance of that context without mentioning how they might apply the particular mathematics content in the future. For example, while one student who thought about applicability of content wrote “I will need to know this later in life,” another wrote “It gives an idea of what it’s going to be when you have a family.” The latter comment does not mention how the student might apply the mathematics in the future; rather, it focuses on the context of the problem and the connection between that context and the student’s own future family. Certainly, the applicability of the content might be part of that student’s thinking, as the mathematics was central in giving students a sense of “what it’s going to be” when they have families in the future. However, what is notable is that the response foregrounds the usefulness of the problem context for thinking about the future now, rather than the applicability of the mathematics content for doing similar calculations in the future.

Related, there were three students whose comments seemed to focus squarely on the applicability of the context and did not include any consideration of the content. Two of these students wrote that Personalized Group Task B was the most useful of the second set of problems. One explained that it was the most useful "because it had to do more with life," while the other wrote "because it talks about your family." Both of these students considered the context of the problems, rather than the mathematics content when deciding which problems were most useful. Similarly, the third student felt that Personalized Group Task A was the most useful of that set of problems "because it was relatable, and… problems that you can relate to your life are the most useful to you." Again, this student did not highlight the applicability of the content in the problem but rather that the task felt very "relatable" and that there was a connection between the task context and the student's own life.

The second most common type of response to the usefulness questions focused on the challenge of the tasks, which was not a category that emerged in Chapter 3. Of the six students who provided such responses, three found challenging tasks to be the most useful and easy tasks to be the least useful (e.g. "It was not useful because it was easy for me to do" and "Problem A [was the most useful] because it challenges your group"). Two students highlighted that problems were most useful when they were easy to understand and not too complicated. For example, one student thought Personalized Group Task A was not useful "because I think it confuses the reader," while another student thought that the Do Now was the most useful task in the second set of problems because "it was the least complicated." Finally, one student appeared to think that easier problems were more useful because she "knew how to find the answer."

Next, three responses focused on the usefulness of features of the learning experience. One student thought that Personalized Group Task B was the most useful problem in the second set because “it help[s] us work together,” while another thought that problem was useful “because you need to think about different strategies on how to solve a problem.” Rather than focusing on how the content can be applied or on the level of challenge of the problem, these students focused on the activity and participation structures, highlighting that “work[ing] together” and using “different strategies” are useful. A third student presented a new category within features of the learning experience – the type of task. This student reported that Personalized Group Task B was not useful “because [these problems] are just warm ups.” In this answer, the student seemed to judge the usefulness of the problems by the fact that they were not consequential tasks but rather “just warm ups.”

Finally, the remaining six responses were spread across five categories, some of which emerged in the Chapter 3 discussion of less common conceptions of usefulness. For example, one student replied that a problem was not useful “because it’s math,” while another student responded that a task was not useful because it wasn’t introducing new material (“I already learned it”). A different student provided an affective response, deciding that a task was not useful because “I don’t care about it,” while a fourth student seemed to focus on the authenticity of the task as the reason for it being useful (“it might [be] right that [I] might spend that much money”). It is worth mentioning that some of the applicability of content responses also hinted at the usefulness of authenticity. For example, in one response focused on usefulness for one’s family in the future, a student wrote, “It told me how many kids I want and how much money imma pay for food." Embedded in such responses regarding usefulness for one's future is a sense that the problem provided the students with authentic information about their own futures.

Problem-Solving Success on Tasks

Next, I examine students' problem-solving success on each task. Table 5.7 illustrates overall correctness on Do Now tasks. Despite the fact that students on average viewed Do Now tasks as only moderately difficult (and comparable in difficulty to the group tasks), students experienced a considerable amount of struggle when solving these tasks. Between three and six students from the entire class answered each question correctly, with another two to seven students providing partially correct answers. In contrast, between 15 and 23 students provided incorrect answers on each problem.

Table 5.7

Student Success on Do Now Tasks

	Correct	Partially Correct	Incorrect/No Answer
Decontextualized Do Now #1	5	2	20
Decontextualized Do Now #2	3	2	23
Contextualized Do Now #1	4	4	19
Contextualized Do Now #2	6	7	15

Overall correctness for contextualized and decontextualized group tasks can be viewed in Table 5.8. For the contextualized group task, five of the seven groups answered the question correctly, and two groups answered it incorrectly. It is important to note that the five groups who answered the question correctly were the groups that Justine helped as she wandered around. In contrast, I monitored the other two groups, which included the consented students, and provided little to no guidance on the problems; both of those groups answered the question incorrectly.

Table 5.8

Student Success Rates on Decontextualized and Contextualized Group Tasks

	Correct	Partially Correct	Incorrect/No Answer
Decontextualized Group Task	4	0	5
Contextualized Group Task	5	0	2

For the decontextualized group task, four of the seven groups provided correct answers (including the two focal groups this time), while the other three groups answered the question incorrectly. Additionally, the two students who Justine asked to work independently answered the question incorrectly or left it blank. Thus, across all contextualized and decontextualized tasks, students struggled, though they provided a higher percentage of correct answers when working with groups. The degree to which the teacher's support influenced this result – particularly for the contextualized task – is unclear.

Finally, as previously noted, several aspects of students' performance on personalized tasks will be highlighted, rather than mean success rates. For Personalized Group Task A, most groups did not get as far as calculating the cost of groceries per year. Of the three groups that did, two multiplied the weekly cost by 52, while the other multiplied the monthly cost by 12 to get the annual cost. For the first set of equations in which students calculated the monthly cost of groceries for their families, two groups wrote and solved equations correctly. One group provided mostly correct equations and answers, while another recorded correct answers but did not write equations. Two other groups wrote incorrect equations (one attempted to write equations to model how they reached their weekly estimates, rather than how they would calculate monthly estimates), while the last group did not get as far as writing equations. Thus, overall, group performance was mixed on this problem, with about half of the groups writing and solving equations accurately.

For Personalized Group Task B, a greater number of groups provided correct equations and solutions – perhaps due to specific features of the problem or perhaps due to the later date of administration. Of the nine groups/individual students who completed the task, four groups provided correct answers, strategies, and equations for all parts of the problem (not including the challenge). Two other groups provided correct answers and showed correct strategies but did not get as far as writing equations. Another group provided correct answers and partially correct equations, while one of the individual students wrote incorrect equations for both parts and misinterpreted the second part of the question. Finally, one group spent nearly the entire time calculating the cost of ingredients needed to make their lemonade in order to figure out how much to charge per cup, so they did not get beyond estimating the cost. Overall, students experienced greater success on task B than task A and greater success on both personalized tasks than either Do Now task (though not the decontextualized and contextualized group tasks).

Engagement in Group Problem-Solving Tasks

In this final section, I examine students' engagement on the problem-solving tasks. First, I highlight students' perceptions of their own engagement, as indicated by responses to one of the four short-answer prompts. Subsequently, I examine themes that emerged in focal students' engagement while they worked on the group tasks. In particular, I highlight their use of everyday knowledge, perceptions of authenticity, interest in going beyond task requirements, and engagement with mathematical practices.

Students' perceptions of engagement. After completing each set of problems, fourteen students provided responses to the question, "Which problem were you most engaged in while working? In other words, which problem made you feel the most focused and interested in your work? Why?" For Problem Set A, six students replied to this prompt, and all reported that they viewed Personalized Task A as the most engaging. Students felt most engaged while working on that task for a variety of reasons. While some students wrote about their enjoyment of the task ("I enjoyed making a budget" and "it was fun to do but I was interested in it"), others wrote about their feelings of competence on the task ("[Personalized Group Task A] is what I felt good in") or the connection between the task and their lives ("it made me think more out of what I'm used to" and it "made me focused on my future").

For Problem Set B, eight students responded to the prompt, three of whom viewed Personalized Group Task B as the most engaging and one of whom viewed the decontextualized Do Now as most engaging. Four of the students either misunderstood the problem or did not provide responses about which task was most engaging. Of the three students who viewed GRP-PERS-B as the most engaging, one reported being "mostly interested…because I love to make things, and especially, help my family out." Another felt engaged "because there was some sort of story along with the problem," while the third felt that "everyone was involved and it's hands-on learning also." The student who thought the decontextualized Do Now was most engaging reported that it "got me interested because I love inversing." Thus, students provided a mix of responses related to the mathematics content, the problem context, and features of the learning experience.

Students' responses across both problem sets again echo findings related to students' perceptions of the usefulness of the tasks. Students reported being most engaged in the personalized tasks, though again, Personalized Task A was viewed as even more engaging than Personalized Task B. While students referenced feelings of enjoyment, competence, and connection to the problem for Personalized Task A, they mainly focused on alignment with various values in Personalized Task B. In the next section, I examine themes regarding the focal students' engagement with both of these tasks, as well as with the contextualized and decontextualized group tasks.

Observed engagement of focal groups. Several interesting themes emerged from coded transcripts of students' discussions while working on the group tasks. First, students were most likely to make connections to everyday life and draw on their own personal experiences when they were working on personalized problem-solving tasks. Second, while students made comments regarding the authenticity of the mathematics in both contextualized and personalized tasks, students commented on the authenticity of aspects of the *problem context* on contextualized tasks but tended to question the authenticity of *others' contributions* on the personalized tasks. Third, students were more likely to go beyond the requirements of the task in the personalized group tasks, and especially in GRP-PERS-B. And fourth, students' engagement with mathematical practices did not differ in any systematic way across the tasks. Below I discuss each of these findings in turn. For reference during the following sections, the approximate amounts of time groups spent working on the four problem-solving tasks can be found in Table 5.9.

Table 5.9

Time Spent on Group Problem-Solving Tasks

	GRP-CON	GRP-PER-A	GRP-DECON	GRP-PERS-B
Group A Time	12 minutes	27 minutes	4 minutes	25 minutes
Group B Time	5 minutes	34 minutes	5 minutes	24 minutes

Connections to personal experiences in everyday life. First, students were far more likely to make connections to their own personal experiences while working on the personalized tasks than while working on the contextualized and decontextualized tasks. Connections to personal experiences were made 16 times during GRP-PERS-A, nine times during GRP-PERS-B, and zero times during the contextualized and decontextualized tasks. Connections made were both mathematical and non-mathematical. Considering non-mathematical connections, students most frequently referenced experiences with their families. For example, when working on GRP-PERS-A, Cherell commented, "My kids are going to help me when they grow up, like my mom did." Similarly, Reagan considered her own family when she decided how many people would likely be in her family: "Four people. Like my family at home." Although these connections are not mathematical, the context of the problems prompted students to make associations between the tasks and aspects of their own personal lives outside the mathematics classroom.

Mathematically, students often referenced personal experiences with money – and especially shopping – when solving the personalized problems. For example, while working to estimate the cost of groceries per week, Reagan and Analise shared the amounts their parents spend when they go shopping. Reagan began by recalling, "My mom, when she goes to Costco, which is like once every two weeks, she spends like three hundred dollars." Analise then replied, "Oh, yeah. My mom doesn't spend that much. She goes once every month to get a huge haul. Like every month…she would spend like two hundred." Then when discussing how they came up with their estimates, Analise commented, "So wouldn't it be the only reason we got these estimates is because we have our knowledge of like family members buy[ing] food." In this exchange, the girls called on their knowledge of the amounts their mothers spend while shopping to determine their own estimates. No similar references to students' personal lives were made during the contextualized and decontextualized tasks.

Perceptions of the authenticity of tasks. Next, students commented on or reacted to the authenticity of tasks when working on both contextualized and personalized problems. A total of thirty-five comments were split approximately evenly across those three tasks. For the contextualized task, nearly all student comments regarding authenticity were reactions to the cost of the bandanas. One reason for the multitude of comments on this topic was students' lack of understanding of the problem context. Many of the focal students did not understand that the $95 stated in the problem referred to the cost for someone to design the bandanas, not the actual cost of the bandanas (which was $1). Due to this lack of understanding, students made comments such as, "They are quite expensive," and "They have really expensive bandanas." Reagan even stated, "I mean, what bandanas sold for ninety-six dollars?" At the completion of the problem, students made similar comments, such as that the company "made a lot of money off of these bandanas."

In contrast, on the personalized problems, students brought their own knowledge to bear on their classmates' ideas to comment on whether their classmates' contributions and estimates were authentic. For example, when deciding on the cost of the cupcakes they would sell for GRP-PERS-B, several members of Reagan's group expressed that amounts their classmates proposed were "too expensive." Similarly, students judged the authenticity of each other's estimates for GRP-PERS-B. After Analise wrote down $1,200 as her estimate, Arianna asked, "Are you rich? Because that's a lot of money just to spend on a week. One thousand two hundred on food. A week." Then Reagan added, "Yea, I don't think she—I don't think anybody spends that much in a week." As this example illustrates, students questioned the authenticity of their classmates' contributions while working on the personalized tasks, bringing their own knowledge to bear to assess whether their classmates' estimates aligned with their own everyday experiences.

Pursuing questions beyond task requirements. Third, students were most likely to go beyond the requirements of the problem when engaging in the personalized tasks. In fact, no student comments that went beyond the requirements of the tasks were captured while students worked on the contextualized and decontextualized tasks. However, students went beyond the requirements of the task on 22 occasions while working on GRP-PERS-B and 19 times while working on GRP-PERS-A. There were again two primary ways in which students went beyond requirements – mathematical and non-mathematical. For each task, eight of the instances involved students going beyond the problem mathematically, while the remaining 14 instances for GRP-PERS-B and 11 instances for GRP-PERS-A were non-mathematical examples of going beyond the problem.

An example of going beyond the task mathematically occurred when Analise's group worked on GRP-PERS-A. After Reagan reported a possible weekly estimate, Analise then used that estimate to consider what her monthly and yearly cost might be (prior to reaching that part of the problem). Analise commented, “Well then, once every week is going to be nine hundred per month… So there’s twelve months… Nine hundred times twelve is how much?” Similarly, after there was some confusion about whether Analise planned to spend $1,200 per month or per week, Reagan called out, “Let’s see how much she would pay if she paid that much in a week… She’d be spending three thousand six hundred per month.” (At this point, several of the students thought there were three weeks in a month.) In both of these examples, students voluntarily went beyond the requirements of the task to mathematically explore questions that they had not been asked to answer. (Although both questions were included later in the packet, students had not yet reached that point and simply felt compelled to pursue those questions on their own.)

Several examples of going beyond the problem in a non-mathematical way occurred with Kristal’s group as they worked on GRP-PERS-B. Once the group decided that their business would be focused on fashion design, they started to discuss many details of the business that were not required for the problem. The girls first decided what roles they would each play. Kristal announced to Veronica, “You can be an assistant,” while Cherell volunteered, “I can model.” The conversation then continued with the students discussing who would be the manager and make-up designer. Subsequently, the girls decided to name their business “Threads Design and Modeling” and then began discussing the many products they would sell. The girls extensively debated about the types of shoes and dresses they would sell, including what would be their “signature shoes” and “signature dress.” Kristal even sketched out an example of what their signature dress might look like to show the group.

These discussions that went beyond the problem in non-mathematical ways sometimes led students to consider the reality of the situation. After discussing the cost of toppings for their cupcakes in GRP-PERS-B, Analise's group exchanged comments about whether it was necessary to get into that level of detail:

Analise: We're not doing this for real, you know, so we don't need to get into that.

Jared: Aw.

Arianna: We could do it at the school in the summer.

Reagan: We should, Arianna.

In this exchange, Analise first questioned the need to discuss specific details of their business since they're "not doing this for real." Arianna then challenged her comment, suggesting that they could "do it at the school in the summer" to which Reagan agreed. This excerpt highlights both tensions between students regarding what level of detail is productive to discuss and the effectiveness of GRP-PERS-B, in particular, at stimulating students to use mathematics in similar ways outside the classroom.

Engagement with mathematical practices. Finally, students' ways of engaging with mathematics while working on the four problems did not appear to differ in any systematic way. Table 5.10 highlights the occurrences of three particular mathematical practices – critiquing other's reasoning or thinking, inquiring about mathematics to another student, and justifying one's reasoning. Students' critiqued each other's reasoning most frequently in the decontextualized task and contextualized tasks, as well as GRP-PERS-B. In contrast, students justified reasoning most frequently while working on GRP-PERS-A, though justifying reasoning occurred nearly as frequently on GRP-PERS-B and GRP-CON. Mathematical inquiry also occurred a comparable amount during all tasks with the fewest inquiries occurring during GRP-CON (*n*=20) and the greatest number of inquiries occurring during GRP-PERS-B (*n*=27). The type of inquiries across tasks differed more significantly, however. While working on GRP-DECON, students were most likely to ask procedural questions (*n*=6) and to check their thinking (*n*=5). On the contextualized task, students were most likely to ask clarification questions (*n*=6) and substantive conceptual questions (*n*=6). On GRP-PERS-A, students were most likely to check their thinking (*n*=7) and ask substantive conceptual questions, while on GRP-PERS-B students were most likely to seek ideas from their groupmates (*n*=9) and ask calculation questions (*n*=9). Differences in frequencies of each type of action across problem-solving tasks were not statistically significant.

Table 5.10

Students' Mathematical Engagement with Group Problem-Solving Tasks

	GRP-DECON	GRP-CON	GRP-PERS-A	GRP-PERS-B
Critiquing other's thinking/reasoning	7	6	2	5
Mathematical inquiring to another student	24	20	24	27
Justifying reasoning	7	9	11	9

Discussion

In this section, I highlight some key themes that emerged from findings. First I discuss potential benefits of the personalization of tasks and use of collaborative participation structures, as well as consider the effectiveness of Personalized Group Task A as compared to Personalized Group Task B. Then I highlight some ways of thinking about usefulness that emerged as students discussed the usefulness of different tasks. In particular, I consider potential differences in the criteria used to judge usefulness at the task versus subject level. Finally, I discuss implications for classroom practice and future research, as well as reflect on lessons learned during task design and implementation. I conclude by highlighting questions to pursue and revisions to make in the next phase of design.

Benefits of Personalization and Participation Structure

Findings highlighted several benefits of both adding a personalized context to problem-solving tasks and using collaborative participation structures. In this section, I discuss three clear benefits that emerged from analyses: enhanced perceptions of usefulness, opportunities to draw on personal experiences, and interest in pursuing questions that go beyond the requirements of tasks. I conclude by speculating about the role of participation structure in students' success on problem-solving tasks.

Enhanced perceptions of usefulness. First, personalized contexts can, in fact, enhance students' perceptions of the usefulness of tasks, as illustrated by students' survey ratings and short answer responses. Personalized Task A was viewed as the most useful task overall, and students on average rated Personalized Task A as significantly more useful than all but one of the other tasks. As contextualized tasks were not viewed as significantly more useful than decontextualized tasks, these findings highlight that simply adding a context is not enough to increase students' perceptions of usefulness; that context needs to be personalized to have the desired effect. Responses to short answer prompts provide added support for these findings, as the majority of students viewed the personalized tasks as the most useful tasks in each problem set.

Working in groups, rather than individually also seemed to enhance perceptions of usefulness, particularly for contextualized tasks. For decontextualized tasks, differences in perceptions of usefulness for independent versus group tasks were not statistically significant; however, for contextualized tasks, students viewed the tasks as significantly more useful when they were working in groups. I offer two possible interpretations of these findings. First, altering the participation structure to encourage collaboration might not be enough to enhance perceptions of usefulness of decontextualized problems. Students might need additional framing from their teachers to be able to view decontextualized tasks as useful. However, ratings were slightly higher for GRP-DECON than IND-DECON, so this question is worth exploring again with a larger sample. Second, these findings suggest that students might find the support of others to be especially useful when working on tasks that involve contexts. While additional research should be done to determine exactly why students find groups to be helpful in these circumstances, one possibility is that collaborators are especially helpful for interpreting problem contexts. Peers might have familiarity with problem contexts and be able to offer expertise in interpreting what the problem is asking. Alternatively, simply having multiple people work together to decipher problem contexts might be especially beneficial. As such issues typically do not arise with decontextualized problems, working with others might offer a particular benefit when students are engaging with contextualized tasks.

Opportunities to draw on everyday experience. Next, personalized tasks provided the greatest opportunity for students to make connections to their own personal experiences. These tasks encouraged students to draw upon their funds of knowledge – in both mathematical and non-mathematical ways – more than typical contextualized tasks. One reason this ended up being especially important was for interpretation of the problem context. As discussed previously, students in the two focal groups incorrectly interpreted the problem context of the contextualized task. They did not appear to have prior experiences they could apply to help them understand that $95 was the cost for someone to create a design, not the cost of the bandanas. Certainly, issues of language and reading comprehension might have also played a role in students' misunderstanding. However, had students been able to draw on prior knowledge of purchasing designs to print on apparel, they might have been able to overcome any reading comprehension issues. As with the concentrate examples discussed in Chapter 4, I argue that students' lack of knowledge of contexts can impact their ability to understand problems, whereas applying prior funds of knowledge can improve students' ability to accurately decipher problem-solving tasks. In this case, both groups struggled to understand the bandana task and eventually reached incorrect answers, while they had very little trouble interpreting the contexts of personalized tasks, despite the much longer directions and thus greater amount of material to decipher.

Interest in going beyond the requirements of tasks. Third, students were most likely to go beyond the problem – again, both mathematically and non-mathematically – on the personalized tasks than on either other task type. Going beyond the problem in a non-mathematical way illustrated students' views of the problems as both interesting and authentic, as they treated the scenarios as realistic. Several students in the focal groups even mentioned wanting to carry out their plans for GRP-PERS-B with their friends outside of class. Meanwhile, going beyond the problem mathematically illustrated not only students' sense of the problem as authentic but also students' interest in the power of the mathematics. By pursuing additional mathematical questions, students illustrated their engagement with the material in a way that they did not exhibit on contextualized and decontextualized tasks. It is also worth noting that while students' engagement with mathematical practices – such as inquiring, justifying reasoning, and critiquing others' thinking – did not occur significantly more frequently during the personalized tasks than other tasks, they also did not occur less frequently. Thus, going beyond the problem and making connections to personal experiences did not prevent students from engaging with the mathematics content of the tasks in rich ways.

Success on problem-solving tasks. In terms of student performance, it is difficult to say what impact, if any, personalization had on performance, as most groups did not have enough time to complete the personalized problems. Similarly, it is somewhat challenging to determine the impact of working with groups on student performance, as Justine provided some level of support to many of the groups while they were working. However, it is noteworthy that the class as a whole performed quite poorly on the Do Now tasks, despite the fact that their perceptions of the difficulty of the tasks were comparable. In particular, the decontextualized tasks posed in both contexts were nearly identical; thus, no differences in performance should be attributed to differences in the features of the tasks. Rather, it seems likely that working independently was especially difficult for students and that they were in need of additional support, be it in the form of peers or their teacher.

On contextualized tasks, it is possible that features of the tasks also played a role in students' performance. The contextualized tasks students completed when working independently required students to write and solve equations in which the variables had coefficients. In contrast, the equation students wrote to solve the group contextualized task included a variable that was already isolated. Thus, it is possible that despite students' perceptions of the tasks as equally difficult, this mathematical difference worked together with the participation structure to influence students' success on independent versus group tasks.

A Comparison of Personalized Group Tasks

While students went beyond the problem and made connections to their own personal experience in both personalized problem, students' ratings of usefulness were significantly higher only for Personalized Group Task A. One question this raises is why students viewed GRP-PERS-A as more useful than GRP-PERS-B. I see at least two possibilities for this difference. First, it might be that the role of family in GRP-PERS-B was too far removed from the focus of the problem. If being able to use mathematics to help one's family is a key motivator in seeing a problem as useful, GRP-PERS-A included a direct application of mathematics to help one's family, whereas GRP-PERS-B involved students first taking action to make money and then using that money to then help their families (a step which was not discussed in any real detail in the task). This degree of separation might have caused students to view GRP-PERS-B as less useful than GRP-PERS-A.

A second possibility is that GRP-PERS-A involved students using mathematics in ways that they have seen others using mathematics and that they expect to have to do themselves one day. In contrast, GRP-PERS-B involved students using mathematics in ways they might be able to imagine using it and might have seen others use it, but also might not. While nearly all students are guaranteed to play some role in purchasing groceries and/or budgeting for their families in the future, some students might not have an interest in or a need to start a business to earn money for their families – at least not in early adolescence. Such a perspective might differ when students get older, highlighting a potential developmental dimension to the selection of task contexts. In this study, differences in the relevance of the context to students' own lives might have influenced students' perceptions of the usefulness of the tasks. However, given that six students reported in one of the prompts that they can imagine encountering a similar problem to GRP-PERS-B in life, this explanation is perhaps less likely than the first. Finally, it is also worth noting that students did not mention the current versus future focus of the problems, so it seems unlikely that differences in perceptions of usefulness are related to the time scale.

Expanding on Students' Conceptions of Usefulness

In addition to offering insight into the benefits of personalization and participation structure, study findings also highlight some new criteria that students used to make judgments about the usefulness of tasks. More than 20% of the students discussed the usefulness of the personalized tasks in terms of the challenge of those tasks. In particular, students tended to view tasks that challenged them as useful and tasks that did not challenge them as not useful. Since challenge rarely surfaced during students' discussions of the usefulness of mathematics in earlier surveys and interviews, it might be that perceptions of challenge are especially relevant when students make judgments about usefulness at the task level, rather than the subject level. In Chapter 3, I briefly discussed this tension between different levels at which perceptions of usefulness are measured. While few studies acknowledge the potential significance of measuring perceived utility of a subject overall versus a particular topic or task, this study illustrates some important differences that might surface.

When students are thinking about mathematics at the subject level, they might be less likely to consider the level of challenge, as challenge likely fluctuates over the course of a semester or year. However, when students are making judgments about the usefulness of a task in the moment, their experiences of the level of challenge of the task are likely to play a much greater role, as illustrated by this chapter's findings. For example, if a student feels that a task is easy and doesn't further her understanding in any way, yet she values challenge, then she might be more likely to view the task as less useful than a comparable challenging task.

Additionally, several students seemed to judge the usefulness of tasks based on their authenticity, another factor that is more relevant for considering the usefulness of tasks than subjects. As addressed in Chapter 4, students frequently attended to the authenticity of tasks during classroom observations. If these judgments of authenticity are tied to students' perceptions of the usefulness of tasks, as this chapter's findings suggest they might be, it will be important to consider the authenticity of tasks when designing for usefulness in the future.

Also related to perceptions of usefulness at the task level are perceptions of the usefulness of task contexts. While some students connected the usefulness of task contexts to applications of mathematics content, others focused solely on the usefulness of the contexts. In such cases, students highlighted the fact that tasks focused on family or related to their lives. These responses suggest that the selection of task contexts can influence students' perceptions of the usefulness of tasks in the moment, yet they are unlikely to surface when students discuss the usefulness of the subject of mathematics overall. One particular example involves the connection to one's family. While students reported goals of wanting to help their families in the future and give back to their families, and students commented on the usefulness of problem-solving tasks that involved thinking about family, very few mentions of the usefulness of mathematics related to family surfaced in Chapter 3 analyses. Thus, building bridges to help students see how mathematics can be useful for helping their families might serve to enhance students' perceptions of usefulness overall.

Finally, the two categories of conceptions of usefulness discussed in Chapter 3 both emerged as ways in which students thought about the usefulness of problem-solving tasks. First, some students discussed the applicability of the mathematics content to their everyday lives (including family- and money-related activities) and their future careers. Thus, despite the relatively young age of these students, they still saw usefulness in applying mathematics to future endeavors. Second, features of the learning experience again emerged as influencing students' perceptions of usefulness. Several students wrote about the usefulness of the strategies involved in problem-solving (an aspect of the structure of the activity), the collaborative participation structure, and the type of task. Thus, while some new factors emerged that influenced students' perceived usefulness of the tasks, these two primary conceptions of usefulness surfaced at both the subject and task level.

Implications and Future Directions

In this final section, I consider implications of these findings, as well as future directions for research. I begin by highlighting potential implications for classroom practice. Subsequently, I propose new directions for the expectancy-value model based on findings of this work. Finally, I reflect on lessons learned regarding task design and implementation, including directions for future design research.

Implications for Classroom Practice

Several implications for classroom practice arise from this work. First, at the broadest level, one way for teachers to help students see tasks as useful is to personalize tasks for students.

Certainly, this requires additional work on the part of teachers, who are often overextended to begin with. However, many teachers such as Ms. Sanchez already spend time altering curriculum problems to make them relevant to their students. For those teachers who already spend time designing or altering tasks for their students, the aspects of personalization considered in this study might help to provide direction regarding ways of altering tasks that are likely to enhance students' perceptions of usefulness.

Related, providing students with problems with which they can connect encouraged students to go beyond the requirements of problem-solving tasks in both mathematical and non-mathematical ways. One consideration for teachers relates to the affordances of students going beyond the task in non-mathematical ways. Although we might be tempted to put a stop to such explorations, as they do not appear to be mathematically pertinent, I argue that these non-mathematical explorations might open up possibilities for new, meaningful mathematical explorations. Teachers might benefit from using in situ observations to document the ways in which students expand problems in non-mathematical ways. These explorations might then create openings to introduce new and even more complex content that is driven by student interest and curiosity. Related, as with the discussion of students' out-of-school practices, teachers might help students to become more metacognitive about the strategies and processes they use in problem-solving. When students go beyond tasks in mathematical ways, teachers might help them to make connections between their own explorations and school mathematics content.

Finally, designing contextualized and personalized tasks for collaborative participation structures might also allow students to engage with the mathematics in more complex ways.

While working with a group might have benefitted students across all problems, personalized tasks explicitly encouraged students to use and record multiple problem-solving strategies. At times, students were also asked to compare those strategies, providing them with an opportunity to consider the effectiveness of various problem-solving solutions. Such comparisons could also be done in a whole class participation structure after students have worked on problems in small groups. Either way, teachers might incorporate into tasks opportunities to express multiple solution pathways in order to provide a helpful foundation for students to consider more complex mathematical questions. Additionally, in collaborative participation structures groups are able to draw on the funds of knowledge of many students, rather than just one's own self. When completing the personalized tasks in this study, students brought to bear – and shared with their groupmates – different kinds of knowledge related to purchasing groceries and materials, budgeting, baking, and designing clothes. Thus, continuing to develop personally relevant tasks for groups, in particular, can provide students with access to a larger source of prior knowledge – the knowledge of the collective – than they would have had if they were working alone.

Considerations for the Expectancy-Value Model

Next, findings from this research offer potential implications for the expectancy-value model. One question this work raises is whether there is a place in the model for students' conceptions of usefulness related to features of the learning experience. While these features appeared to directly influence students' perceived utility value, they do not fit clearly within either of the direct influences on utility value that are illustrated in the expectancy-value model – namely, one's affective reactions and memories of previous achievement-related experiences, or one's goals and self-schemata (Eccles & Wigfield, 2002). Additionally, while the model considers expectancies and orientations within a given domain, this research suggests that some conceptions based on features of the learning experience might not be domain-specific. In other words, students' experiences engaging with everyday activities outside the domain of mathematics might influence their perceptions of the usefulness of mathematics via features of the learning experience.

Second, and related, this research raises the question of whether there is a place in the model for the influence of students' experiences with everyday activities. While student characteristics and achievement-related experiences are included in the model, students' everyday experiences appear to be absent. However, we know from this study and prior research that students frequently make connections between their mathematics learning in the classroom and their everyday experiences. In order to consider these types of connections, it would be important for the model to account for the ways in which classrooms are organized – an aspect that is certainly domain-specific and thus perhaps not supported by the model. In the final chapter, I will propose a new model that takes up these various influences on students' perceptions of utility and highlights the many pathways through which students might come to view mathematics as useful.

Reflections on Task Design and Implementation

In this final section, I reflect on lessons learned through task design and implementation. First I highlight logistical considerations of implementation that will be important to attend to during future iterations. Next I reflect on the affordances and constraints of this study's design principles, as well as the effectiveness of various prompts used in the study. Finally, I conclude with implications and questions to pursue for future research.

Logistical considerations. During this study, many lessons were learned regarding the design and implementation of tasks and surveys that will be used to inform the next cycle of design. First, the time at which tasks were administered impacted students' focus and engagement with the tasks. Students were distracted and also likely realized that their performance on these tasks was inconsequential, as they knew that Justine's other classes were doing fun activities on those same days. As a result, it was difficult to get students to focus their attention fully on the tasks and do the best they could (especially the students not in the focal groups). In the future, tasks should be piloted earlier in the year if possible and not immediately before or after any breaks. Additionally, it would likely be helpful for the tasks to be better incorporated into students' regular classwork schedule and for students to have a sense that their teacher would be checking the tasks. In terms of better incorporating the tasks, it would likely be productive to engage students in some true groupworthy tasks that would require a similar type of interaction and participation prior to administration. Then students would have familiarity with the mode of engagement and also be less likely to see these tasks as different and thus not worth their full effort. Additionally, it would be preferable to have the teacher's other classes working on similar tasks on days of administration so that students are not expecting fun activities, only to find out that they have serious work to do.

Affordances and constraints of design principles. Next, it is important to consider the affordances and constraints of the four design principles used in this study. The first design principle was to select contexts that students perceive as useful and with which students have prior experience. Applying this principle to task design resulted in many affordances for student engagement. As previously discussed, students were able to apply their own prior experiences to the personalized tasks in ways that they weren't able to when working on contextualized curriculum tasks. Additionally, students did in fact perceive the personalized tasks – and especially Personalized Task A – as more useful than the other tasks. They also appeared to view the tasks as authentic and were motivated to go beyond task requirements in both mathematical and non-mathematical ways. One constraint did surface, however, related to students' desire to go beyond requirements of the task: Students spent a significant amount of time considering non-essential aspects of the problems at the expense of being able to complete the key components of the tasks – especially in GRP-PERS-B.

While I see great benefit in students being so excited about the problems that they wanted to discuss many details of the scenarios, time spent on these aspects did detract from the amount of time students were able to spend on mathematical components of the tasks. This finding highlights one tension that might arise when we design tasks that students perceive as useful and with which students have prior experience. There are several possible ways to overcome this constraint, however. First, as previously discussed, teachers might observe students in situ and capitalize on students' mathematical and non-mathematical pursuits to open up new questions to students. Rather than being viewed as extraneous, these pursuits might actually provide a foundation for exploring more complex mathematical ideas. Another option is to create more extended personalized tasks that are designed for students to work on over the course of several days or weeks. In that case, additional time spent by students to make realistic estimates or add details to problem contexts would be more worthwhile, as those moves would be in service of work students would be doing over a longer period of time. Alternatively, explicitly offering a five-minute period at the start of a task for students to discuss such details would allow students to still express their creativity to some degree without taking up too much time.

The second design principle was to align problem contexts with mathematics content to ensure that the mathematics content in tasks is authentic to those contexts. On the personalized tasks, students in the focal groups made far fewer comments questioning the authenticity of task contexts than they did when working on the contextualized curriculum task. Thus, aligning mathematics content with contexts seemed to help students view the tasks as authentic. Several challenges did arise in pursuing this alignment, however. First, due to the breadth of students' prior knowledge and experiences, authentic practices vary across individuals. Thus, while features of the task were authentic to some students, other students engaged in practices that were not reflected in the tasks. For example, in Personalized Group Task A, attempts were made to align mathematics content with authentic practices by having students estimate their weekly cost of groceries and then use that amount to determine monthly and yearly costs (a practice likely used by many adults). However, students referenced other approaches that did not begin with determining a weekly cost, such as buying a large amount of groceries at the start of the month and then making smaller supplementary trips throughout the month. Students who had been exposed to such purchasing practices from their parents/guardians were unable to fully apply their prior knowledge to this problem, as the setup did not support that approach. Thus, despite my best efforts to design tasks in which mathematics content and contexts were aligned in authentic ways, the resulting tasks were not authentic to all students' practices.

Additionally, engaging in authentic practices sometimes resulted in students achieving different levels of accuracy in their solutions. While real world contexts often support the use of different strategies that vary in their level of precision, such variation might not always be desirable in the mathematics classroom. As an example, on GRP-PERS-A, students used two different authentic strategies to calculate the monthly cost of groceries, which resulted in two different answers. While some students multiplied the weekly cost of groceries by 52, others multiplied the monthly cost by 12. Although multiplying one's monthly cost by 12 to determine the yearly cost might achieve a sufficient level of precision in the real world, that solution pathway results in an estimate that is considerably less accurate than the estimates calculated by students who multiplied the weekly cost by 52. This tension does certainly raise some challenges; however, it also provides an opportunity to help students engage with the mathematics more deeply. Teachers might highlight the different methods used by students to discuss the advantages and drawbacks of each method, as well as the level of precision that would be desirable given the context.

The third design principle was to select numbers that both facilitate exploration of the focal mathematics topic and are authentic to the chosen contexts. In this research, students were able to select their own numbers, which revealed both benefits and challenges of that practice. As desired, students appeared to view the tasks as authentic, perhaps in part because of the opportunity to come up with their own estimates. Students drew on their own personal experiences to justify their choices, bringing to bear knowledge they had gained in their everyday practices. Additionally, students were able to use their own expertise to provide their peers with feedback on the authenticity of their estimates. Thus, allowing students to determine their own estimates had many affordances.

This practice also has several constraints, however. One main challenge that surfaced in this study was the amount of time it took for students to determine estimates, as some students were much more thorough than expected in selecting accurate estimates. For example, in order to estimate the amount she would spend on groceries for her family for one week, Julie wrote out an entire shopping list of the ingredients she would need and then estimated the cost of each ingredient. Similarly, one of the non-focal groups spent nearly the entire time on task GRP-PERS-B figuring out what ingredients they would need to buy and how much each would cost to make the lemonade they would sell. These practices highlight a tension in the ways that different students think about authentic mathematical practices. While many students drew on their prior experiences or others' expertise to relatively quickly develop estimates, others seemed to strive for a greater degree of accuracy in their estimates, taking the time to do additional calculations to inform their decisions. Working with students to model appropriate levels of precision and sharing different students' estimation strategies with the class might help students to converge around more efficient ways of forming estimates. Thus, in future work it might be beneficial to give students experience with forming estimates and determining appropriate levels of precision prior to the administration of tasks.

Additionally, though it did not surface as an issue in this study, allowing students to select their own estimates can constrain the mathematical possibilities of tasks. As discussed earlier, unless guidelines are provided for the types of numbers to select (which might detract from students' perceptions of the authenticity of tasks), teachers will be limited by the numbers that students choose. Thus, this principle should not be applied in all design situations. If a teacher wants students to work with fractions, for example, it would not be appropriate to allow students to choose any number to work with, as they might not select a fraction (depending on the context).

Several other strategies can be applied in such cases to balance the tension between striving for authentic numbers and opening doors for mathematical exploration. First, as other researchers have suggested, teachers might provide students with a range of numbers from which to choose (Aguirre et al., 2012; Drake et al., 2015; Tyminski et al., 2014). As these researchers have discussed, this practice can help provide students ownership over their own learning by allowing students to select appropriate levels of challenge for themselves. Additionally, I suggest this practice might also improve students' perceptions of the authenticity of tasks, while still allowing teachers to provide direction regarding the types of numbers with which students will be working. Alternatively, teachers might select numbers for students but take extra care to a) select numbers that are likely to be viewed as authentic by students and b) explain to students the conditions under which such numbers would be authentic to individuals' practices. Although this option would not allow for self-expression and application of prior knowledge on the part of students, it would enable teachers to direct students' mathematical explorations while still striving to present tasks that students view as authentic.

The fourth and final design principle stated that tasks should incorporate features of engagement that students perceive as useful. In particular, these tasks were designed to be worked on in groups and to encourage students to show their thinking and express multiple problem-solving strategies. Working in groups did indeed appear to be advantageous to students, as they were able to draw on multiple groupmates' funds of knowledge and see many ways in which the tasks applied not only to their own lives but also to the lives of group members. It is unclear what impact showing their thinking had on students' perceptions of usefulness; thus, that outcome will need to be explored in future research.

A few challenges did arise in applying this principle to design useful problem-solving tasks. Creating tasks that aligned with the other design criteria but were also truly groupworthy was challenging. For example, allowing students to select their own estimates in the context of working with others raised questions about how to effectively navigate collaborative work that involves self-expression and personal connection. At times, students were asked to consider a question independently and then report back to their group and compare strategies. However, in practice, students often developed those estimates together and were then confused when they read the direction to share their estimates with their groups. Additionally, prompts that asked students to compare problem-solving strategies appeared to vary in their effectiveness. While some groups did attempt to make comparisons, in other groups only select students expressed their strategies, or students sometimes all used the same strategy, which did not facilitate comparison. Thus, figuring out how to effectively incorporate useful features of the learning experience into task design while also meeting the other design criteria remains a challenge.

Effectiveness of prompts. Finally, before considering implications and questions for future design research, I would like to reflect on the effectiveness of the four survey prompts used in this study. Several students appeared to misinterpret the question focused on engagement, so that prompt needs revision if used in future iterations of this work. The two prompts focused on usefulness (one comparative, one not) both appeared to be understood by the students and provided insight into their ways of thinking about usefulness at the task level. Such prompts might be beneficial to use in the future to better understand what makes students judge specific tasks, as opposed to entire academic subjects, as useful. Finally, the prompt that focused on whether students expected to deal with similar situations in their lives also seemed to be difficult for some students to interpret. Overall, student responses to that prompt were informative, though they did not provide the richness of responses that resulted from the usefulness questions. Again, alternative versions of the prompt might be considered for future iterations of this study.

Questions and implications for future design research. These findings raise many additional questions to pursue in future research. First, it would be worthwhile to explore the relationship between different types of personalization and the forms of mathematical engagement that they support. As personalization can take many forms, examining benefits and drawbacks of different types of personalization would be helpful for guiding the design of useful tasks moving forward. Additionally, in work with a larger sample, it would be beneficial to explore the association between different characteristics of students and their perceptions of the usefulness of tasks. For example, are students' perceptions of their own competence related to their likelihood of viewing challenging tasks as useful? Future research will also be needed to further explore effective ways of capitalizing on students' views of useful features of learning environments. Since some students highlighted useful features of the tasks – such as the problem-solving strategies involved – it would be worthwhile to explore which features most influence students' perceptions of the usefulness of tasks. Finally, related to features of the learning experience, it is important to examine whether students do, in fact, connect features of their everyday interactions with mathematics to features of engagement with mathematics in school. The student who commented that "life isn't independent" when asked to work independently certainly seems to be making such connections. However, a systematic exploration of whether students typically acknowledge differences in the features of learning environments would shed light on the role of students' everyday engagement with mathematics in their perceptions of engagement with classroom mathematics.

Overall, design and implementation of the new tasks illustrated the potential for well-designed personalized tasks to be viewed as especially useful by students. Such tasks allowed students to make connections to their personal experiences and explore authentic scenarios that were of interest to them. Additional studies with a greater number of groups will be needed to examine the generalizability of these findings and to explore whether any differences emerge in mathematical engagement. Also, differences in instantiations of the design principles should be explored, such as providing students with a range of numbers from which to choose rather than having students select their own estimates from scratch. In future iterations of this work, tasks will also be more naturally incorporated into school coursework, and attempts will be made to either limit the amount of time students spend discussing details of the task contexts, or draw on those discussions to open doors to more complex mathematics. New tasks will be designed to examine whether the same themes emerge in students' discussions of the usefulness of mathematics at the task level, and new prompts will also be tested. Even without these changes, however, this study's findings show promise for drawing on students' everyday experiences and conceptions of usefulness to design tasks that students view as interesting and useful. This research also highlights the diversity of pathways through which students can come to see the subject of mathematics or mathematics tasks as useful, highlighting the need to attend to these various pathways in research and classroom practice moving forward.

6. Conclusion: Lessons Learned and Future Directions

In this final chapter, I reflect on some of the lessons learned throughout this research. Drawing on findings presented in Chapters 3-5, I first discuss what this research contributes to our understanding of utility value and students' conceptions and perceptions of usefulness. Then I consider applications of these findings to the field of mathematics in particular. Throughout both sections, I also discuss implications of the findings for the design of learning environments and classroom practice. Subsequently, I describe methodological contributions of this work and conclude by highlighting limitations of and future directions for this research.

Enhancing Our Understanding of Utility Value

Findings from this research make several contributions to our understanding of utility value. First, this work identifies different ways in which middle school students think about the usefulness of mathematics. Next, through exploring these conceptions of usefulness, two key questions emerge regarding factors that might influence students' perceptions of usefulness. One question relates to the impact of studying usefulness at the task, topic, or subject level, while the second question focuses on the impact of perceived usefulness being driven by the claims of significant others rather than students' own goals. In order to examine these questions, I will discuss a new construct referred to as "subject level zoom" and then consider the role of both significant others' claims and one's own goals and values in perceptions of usefulness. Drawing on these discussions, I will then present a model that highlights the multiple pathways through which students might come to develop different conceptions of usefulness. Finally, I will close with a consideration of issues that have the potential to affect the measurement of utility value.

New Conceptions of Usefulness

By exploring the ways students think about the usefulness of mathematics, this research reveals two main categories of conceptions of utility: applicability of content and features of the learning experience. Conceptions related to the applicability of content focus on how particular content can be applied in the world and reflect the way we tend to conceptualize utility in research (e.g. Fennema & Sherman, 1976; Harackiewicz, Rozek, Hulleman, & Hyde, 2012; Hulleman & Harackiewicz, 2009). In contrast, conceptions related to features of the learning experience focus on the usefulness of particular learning processes and ways of engaging with subject matter. This way of thinking about usefulness has not been considered in the literature thus far and so represents a new contribution to work on utility value (that may or may not be specific to mathematics).

Additionally, when students were asked about the usefulness of particular tasks, a new conception of utility emerged related to the challenge of tasks. Students tended to see usefulness in tasks that challenged them and to view tasks that were simple as less useful. This finding aligns with prior research that has illustrated the relationship between task challenge and student engagement (Newmann, Marks, & Gamoran, 1996; Shernoff et al., 2003) and identifies yet another criterion that students sometimes use to assess usefulness. However, students assessed usefulness in terms of challenge only when they considered the usefulness of particular tasks but not when they discussed the usefulness of the discipline of mathematics as a whole. This finding raises a question about the level at which we study utility value. While students are often asked about their perceptions of usefulness at the subject level, those perceptions might not align with their perceptions of the usefulness of particular tasks or topics within that academic subject. For example, while students might say that mathematics as a field is useful, they might not view particular problem-solving tasks or topics within mathematics as useful. Such differences, and their potential implications, are explored in the section below.

Subject Level Zoom: Examining the Usefulness of a Task, Topic, or Subject

Throughout this research, students' conceptions of usefulness emerged as connected to the levels at which we ask students about their perceived utility. I refer to this changing level of focus as *subject level zoom*. The lens of subject level zoom allows us to both zoom out to examine perceptions of usefulness at the level of an entire academic subject and also zoom in to examine perceptions of usefulness of particular tasks or topics within that subject. Although researchers have previously questioned individuals about their perceptions of utility at multiple levels, the level of zoom has not been acknowledged as a factor that likely influences individuals' reported utility value.

In existing research, the construct of utility value emerges as a component of one's *task* value (Eccles & Wigfield, 2002), yet many studies measure students' perceptions of usefulness by inquiring about the usefulness of academic subjects (e.g. mathematics, science) rather than tasks (Anderman et al., 2001; Battle & Wigfield, 2003; Fennema & Sherman, 1976; George, 2006; Hulleman et al., 2008; Parsons, 1980; Xiang et al., 2005). Though some studies do zoom in on the usefulness of particular techniques or topics within a content area or academic subject (Canning & Harackiewicz, 2015; Hulleman et al., 2010), potential differences in the criteria students use to make judgments about usefulness depending on the level of zoom have not typically been considered. This research highlights several possible reasons why attending to these differences might be important.

First, throughout this research, students were asked about a) their perceptions of usefulness of mathematics overall (using the modified Fennema-Sherman scale discussed in Chapter 3; Doepken et al., 2004), b) their perceptions of the usefulness of particular mathematics topics (discussed in the Chapter 3 case study of Katie), and c) their perceptions of the usefulness of specific problem-solving tasks (discussed in Chapter 5). When comparing students' ratings of usefulness at the task and subject level, reported perceptions of usefulness were significantly higher at the subject level than at the task level. In other words, while students viewed the subject of mathematics overall as either useful or very useful, students tended to view particular tasks as ranging from only slightly to somewhat useful. These two sets of responses paint very different pictures of students' perceptions of the usefulness of mathematics and raise the question of why this difference in reported responses emerged.

One possible reason for this discrepancy is that students apply different criteria to consider the usefulness of specific tasks or topics than to assess the usefulness of an entire academic subject. While at the subject level students seemed to generally consider the applicability of content or features of the learning experience when making judgments about usefulness, at the task level students also considered factors such as the level of challenge and degree of authenticity. Additionally, the context of problems seemed to play an important role in students' judgements regarding the applicability of content at the task level, whereas problem contexts were not discussed when students considered usefulness at the subject level.

If students use different criteria to assess the usefulness of an academic subject than to assess the usefulness of specific tasks or topics within that subject, then measures of perceived usefulness at the subject level (e.g. Fennema & Sherman, 1976) might not capture the entirety of students' perceptions of usefulness. For example, in the case study presented in Chapter 3, Katie reported that mathematics is useful for many things; however, when she was asked about how she might use particular mathematics topics, she was unable to report any ways in which she herself expected to use the topics. Reflecting back on Eccles and Wigfield's (2002) expectancy-value model, this discrepancy raises the question of how – or even whether – Katie's perceived usefulness of the subject of mathematics will drive her academic-related choices and outcomes if she is unable to view individual mathematics topics or tasks as useful. While future research should certainly examine this tension, we might question in the meantime how students come to view the subject of mathematics as useful if they tend to not view tasks or topics as very useful on a day-to-day basis. One possible answer centers on students relying on the claims of significant others, which is considered in the section below.

Role of Significant Others

While the expectancy-value model highlights only two direct influences on students' perceived utility value, I suggest that significant others' claims regarding usefulness might also directly influence perceptions of utility, particularly at the subject level. Since many students reported strong perceptions of usefulness at the subject level but not at the task level, it is possible that students have come to believe mathematics is useful through their parents and teachers, who are increasingly exposed to rhetoric focused on the usefulness of STEM fields – in the Common Core State Standards and initiatives from the U.S. government's Committee on STEM Education, for example. In fact, several students explicitly reported that their teachers and parents told them that mathematics is useful, and Ms. Sanchez was observed talking with students about the usefulness of mathematics. While some students did not find her claims motivating, Liliana had a particularly close relationship with Ms. Sanchez and periodically referenced Ms. Sanchez's claims about usefulness as trusted claims (even though she found herself sometimes questioning the usefulness of particular mathematics topics). Such examples illustrate the indirect influence that rhetoric surrounding STEM education can have on students via the adults who come into direct contact with that rhetoric, a perspective that aligns with Bronfenbrenner's (1992) ecological systems theory.

Given this shift towards emphasizing the usefulness of mathematics in the United States, I propose that socializers' beliefs might no longer only indirectly affect task values, as highlighted in the expectancy-value model, but also directly affect task values. In other words, teachers and parents might be starting to effectively convince students of the usefulness of mathematics without students' goals or self-concept playing a mediating role. If this is the case, students might report perceiving mathematics as useful at large even if they do not experience mathematics as useful at particular moments in the classroom. Stated differently, students' broad sense of usefulness regarding the subject of mathematics might not always trickle down to influence their in-the-moment feelings of usefulness as they engage with mathematics in the classroom.

As with cognition, we can conceive of utility value as situated and thus as influenced by the many contextual factors present in a given setting (Goodwin, 1997; Lave & Wenger, 1991). Each of those factors – such as the form of learning, the participation structure, the problem context, and the mathematics content – might be considered by students as they work with mathematics in the classroom and decide whether the learning in which they are engaged is useful. Thus, students' performance, engagement, and persistence in the moment might be differentially affected depending on whether students see the work they are doing as useful (defined in terms of one or more of the dimensions previously described) – regardless of whether they view the subject of mathematics as useful.

If this is the case, how will students' performance-related choices and outcomes be affected by their utility value when their perceptions of utility are driven by significant others' claims rather than their own goals and values? Will perceptions of utility have the same effect as they would if students' perceptions derived specifically from their own personal and social identities, for example? Although such relationships would be difficult to explore given the many influences on students' performance-related choices and outcomes, we might begin by examining in a range of students the strength of the link between a) one's own goals and personal/social identity and b) one's task values, as presented in the expectancy-value model (Eccles & Wigfield, 2002). More deeply exploring the connection between goals and values might shed light on the various pathways through which students come to believe that mathematics is or is not useful. Additionally, we might consider providing students with more direct experiences that illustrate the usefulness of mathematics. Subsequently, we can examine the benefits of experiencing firsthand the ways in which mathematics can be useful to one's own life and goals, as compared to relying on words of wisdom from teachers and parents to trickle down to students.

Role of Student Goals and Values

In addition to significant others' claims playing a potentially important role in students' reported utility value, Eccles and colleagues' expectancy-value model (Eccles & Wigfield, 2002) highlights the important influence of goals on one's utility value. One particular facet of goals that has not typically been considered in prior utility value research emerged in this study as especially important for students: a focus on interdependence. Thus far, most items measuring utility value have been guided by independent conceptions of usefulness. However, in this research, students spoke about the importance of working with others and helping out others in their everyday activities, and they highlighted family-related reasons as the top three reasons for wanting to do well in school. This focus on interdependence aligns with the results of prior research on interdependent values in working class (Grossmann & Varnum, 2011; Stephens et al., 2012; Stephens et al., 2007) and Latin@ communities (Esparza & Sanchez, 2008; Sabogal et al., 1987; Valdés, 1996) and suggests that interdependent values might be especially important to consider in utility value research moving forward. Additionally, students' emphasis on working together in the classroom aligns with prior work that has emphasized adolescents' focus not only on academic goals but also social goals in the classroom, such as cooperation, dependability and responsibility (Wentzel, 1989, 1993). These connections illustrate the importance of understanding the findings of this research from a sociocultural and developmental perspective, as students' goals, values, and even conceptions of usefulness are strongly linked to their backgrounds and experiences as adolescents.

Given the role of goals in influencing task values, we might question whether students' value of interdependence influences their perceptions of utility. In discussions of the usefulness of mathematics, few students specifically mentioned the usefulness of mathematics for achieving interdependent goals. Thus, this might be an area where we can help students to create bridges between their own values and their perceptions of the usefulness of mathematics. Personalized Task A, for example, encouraged students to consider the usefulness of mathematics for providing for their families in the future. Students in turn viewed that task as significantly more useful than all other tasks they completed in their problem sets. In addition to strengthening the connection between students' values and perceptions of usefulness, it is also important to consider the ways in which we measure utility value. If most items focus on independent uses of mathematics, perceptions of usefulness for interdependent purposes will not be captured. Thus, moving forward it will be important to find ways for students to express their perceptions of the usefulness of subjects or tasks for achieving both independent and interdependent goals.

Emerging Issues Related to Reported Utility Value

Related to the ways in which we measure perceptions of usefulness, the three aforementioned factors (subject level zoom, the role of significant others' claims, and the role of values and goals) raise a potential issue regarding the measurement of utility value. In particular, existing items used to measure utility value might not capture the feelings regarding usefulness that students experience on a day-to-day basis when engaging with classroom material. Such a difference might emerge for a number of reasons, related to the topics discussed above. (Note: Although I do not claim that these reasons are specific to mathematics, I will discuss them in terms of the subject of mathematics, as that focus aligns with the findings from this study.) First, students might have been told by valued others that mathematics is useful, yet they don't see that usefulness in their own classroom mathematics. Alternatively, students might have a sense that the subject of mathematics is useful – either in general or for particular types of people – though they don't see concrete ways in which they personally will use mathematics. Another possibility is that students see mathematics as useful at the broadest level, but when they begin to engage with particular mathematics topics and tasks, they do not see the usefulness of those tasks and topics. Fourth, as described above, features of the learning experience might influence students' perceptions of utility as they engage with school mathematics. While students tend to report viewing the subject of mathematics as useful, they might see varying levels of usefulness in the ways in which they are asked to engage with mathematics in the classroom.

Two additional possibilities relate to connections between classroom mathematics and students' goals, values, and everyday experiences. It might be that students do believe that mathematics is useful, yet they feel that the goals of their classroom mathematics learning are not useful. As discussed in earlier chapters, students had goals related to interdependence and self-expression in their everyday activities, yet goals in the mathematics classroom tended to be more artificial and focused on completion. Thus, students might believe that the subject of mathematics can be useful, but the purposes they are given for completing particular tasks in the classroom are not useful. It also might be that students see a lack of authenticity in the mathematics they learn in the classroom, despite the fact that they believe there is useful mathematics to learn. This discrepancy in experiences could result in students reporting that mathematics is useful, yet questioning the usefulness of classroom mathematics when they are in the midst of engaging with problem-solving tasks that seem inauthentic.

These varied reasons for discrepancies between a student's reported utility value of an entire academic subject and one's experience of the usefulness of particular tasks point to numerous pathways through which conceptions of usefulness play out. In other words, a variety of different influences can alter the ways students think about usefulness in different situations, which in turn impact students' perceived utility value. In the section below, I present a model designed to highlight the multiple pathways through which different conceptions of usefulness might be triggered to influence one's perceived utility value.

Conceptions of Usefulness Model

In Figure 3, I present the Conceptions of Usefulness model, which illustrates some of the many pathways through which a) students might come to develop various conceptions of usefulness and b) conceptions of usefulness might influence perceived utility value. Several aspects of this model build on prior work that was discussed in Chapter 2. In particular, Eccles and colleagues' expectancy-value model seeks to specify a variety of indirect and direct influences on one's perceived utility value, including one's short- and long-term goals, as well as perceptions of socializers' beliefs and attitudes (e.g. Eccles & Wigfield, 2002). In this model, I include additional influences and also specify the mediating role that conceptions of usefulness plays in students' reported utility value. Within conceptions of usefulness, I also highlight two main categories – applicability of content, which reflects current ways of thinking about usefulness that exist in the literature (and were discussed in the framework presented in Chapter 2), and features of the learning experience, which is a new category. The way in which one conceptualizes usefulness in a given situation mediates his/her perceived utility value in that moment. Below I elaborate on the various components of the model.

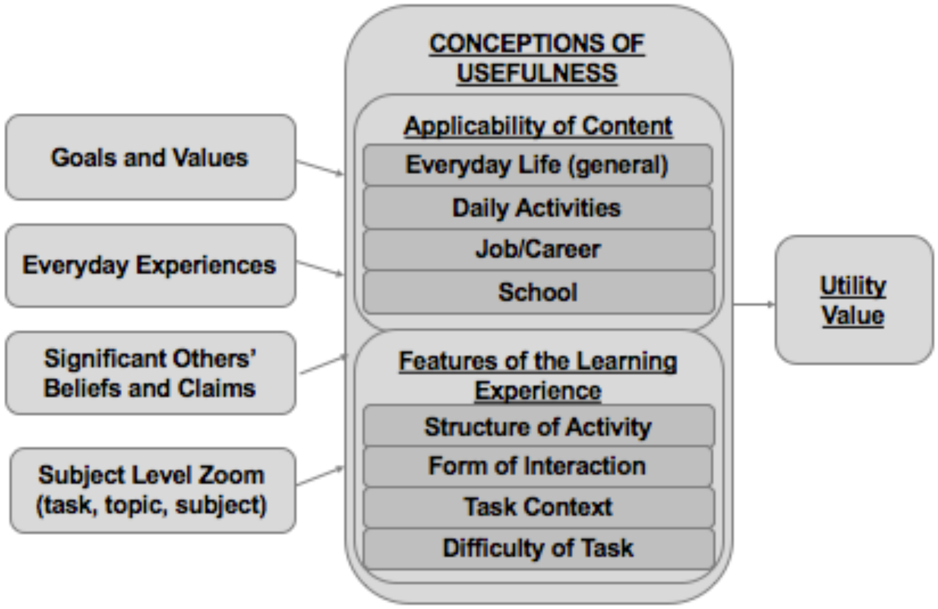

Figure 3. Conceptions of Usefulness Model

Four main types of influences on conceptions of usefulness have emerged thus far – personal goals and values, everyday experiences, significant others' beliefs and claims, and subject level zoom. Considering the first two categories, both individual goals/values and one's own everyday experiences might influence the way one thinks about what it means for mathematics to be useful. For example, a student like Arianna who regularly engages in building activities with her parents might come to see mathematics as useful for accomplishing particular everyday activities. Alternatively, because Victoria has a strong value of career, she might be more likely to conceive of usefulness in terms of whether a particular subject or task is useful for helping her to get or eventually perform a job. Thus, students' values and everyday experiences influence the conceptions of usefulness they apply to assess the utility of various tasks and subjects.

In the third category, significant others' beliefs and claims can also influence students' conceptions of usefulness. For example, Liliana believed mathematics was very useful for everyday life and for jobs/careers, at least in part because she was told so by a trusted source – Ms. Sanchez. However, this belief might not align with her own personal values or experiences as described above, though it does likely influence the way she thinks about the usefulness of mathematics. This category is similar to Eccles and colleagues' discussion of the influence of socializers' beliefs and attitudes. However, while the expectancy value-model shows students' perceptions of socializers' beliefs affecting utility value indirectly, I propose that it might also directly affect utility value (being moderated only by the conceptions of usefulness that those beliefs incite).

Fourth, the subject level zoom can influence the conceptions of usefulness that students apply to different situations. As we saw in this research, students conceived of usefulness in terms of the level of challenge only when they zoomed in to examine the usefulness of particular tasks. In contrast, when zooming out to consider the usefulness of the subject of mathematics, students never discussed the level of challenge and instead conceived of usefulness mainly in terms of the applicability of content and sometimes in terms of the form of interaction or structure of activity used in the classroom.

All four of these categories of influences directly affect students' perceived utility value via the conceptions of usefulness that they trigger. Currently, two primary categories of conceptions of usefulness have been identified: applicability of content and features of the learning experience. While some specific links have been identified – such as that applying a subject level zoom at the task level triggers conceptions of usefulness related to both categories, while zooming out to the subject level triggers conceptions of usefulness primarily related to the applicability of content – other links must be explored in future research. Thus, in this version of the model I solely highlight the various influences and conceptions of usefulness that emerged in this research and that appear to affect one's perceived utility value. Below I discuss some additional links that might be explored in future research to inform subsequent versions of the model.

Implications for research. This model highlights new areas to explore as we move forward in research on utility value. First, it will be important to consider the subject level zoom when measuring perceived utility value to further examine what conceptions of usefulness emerge when different levels of zoom are applied (task versus subject, for example). Related, analyses might be conducted to examine the conceptions of usefulness that are most frequently encouraged by the claims of significant others, as well as the conceptions that tend to emerge as a result of students' everyday experiences with domain content. Additionally, new methods will be needed to explore students' perceptions of usefulness not only in terms of applicability of content but also in terms of features of the learning experience. Open-ended items and interview tasks, such as the card-sorting task described earlier, can be used to better understand what features of learning experiences students view as useful and what factors might influence those conceptions of usefulness. Finally, the possibility of additional influences on conceptions of usefulness, as well as new categories of conceptions of usefulness, should be explored in future research.

Implications for instruction. There are also several important implications of this new model for classroom instruction. A first question that might emerge in examining the model is how teachers can accommodate this diversity in student perspectives and conceptions of usefulness. Rather than viewing the multiple conceptions of usefulness as an unfortunate complexity, I propose that this range of perspectives allows for a breadth of access points through which teachers might connect with students' conceptions of usefulness. Currently, many teachers tend to highlight the applicability of particular mathematics content, which is likely to be especially helpful if those areas of applicability align with students' own goals and values. However, focusing on the applicability of content is not the only way to emphasize the usefulness of mathematics. This model suggests that the way in which the classroom is organized, including the particular pedagogical practices that are used, are equally important. Since students saw usefulness in particular activity structures and forms of interaction in the classroom, teachers might work to engage students in those particular ways – or, to illustrate to students why other activity structures and forms of interaction are useful. This focus also allows teachers to connect with CCSSM's standards for mathematical practices, as highlighted in previous chapters. Thus, while socializing students into specific key practices of the discipline of mathematics, teachers also have the opportunity to engage students in mathematics that they view as useful.

Another point of access for teachers to align instruction with students' conceptions of usefulness – or socialize students to conceive of usefulness in new ways – is through task design. Since this model highlights the importance of considering students' beliefs about usefulness not only at the subject level but also at the task level, teachers can use task design to draw on students' different conceptions of usefulness. In particular, teachers might work to align both content and contexts with students' own prior experiences and values to increase the likelihood of students viewing tasks as useful. Furthermore, when engaging in task design, teachers might also consider the pedagogical practices that students will engage with as they work on tasks. As described above, these pedagogical practices and particular ways of interacting and organizing activity in the classroom can also be leveraged to either align with students' existing conceptions of usefulness or expose students to new conceptions of usefulness.

Additionally, it will be important for teachers to consider the multitude of influences on students' own conceptions of usefulness. In line with prior research on funds of knowledge and everyday practices involving mathematics (e.g. Brenner, 1998; Civil, 2002, 2007; Saxe, 1988; Taylor, 2012a), this work highlights the importance of drawing on students' everyday experiences when attempting to teach mathematics that students perceive as useful. Learning about students' everyday experiences with mathematics will allow teachers to both design tasks that draw on those experiences and highlight areas of applicability that are relevant to students' own lives. Furthermore, this research explores the possibility of drawing on forms of interaction and features of engagement that students experience when using mathematics in their everyday lives. I suggest that it might be fruitful to understand how students engage with useful mathematics in the world as we think about how best to engage students in mathematics they view as useful in the classroom.

Finally, it is worth noting that while I have discussed this model in terms of its application to the field of mathematics, it can be applied to explore conceptions and perceptions of usefulness in other domains, as well. Teachers in other subject areas might consider similar influences on students' perceptions of usefulness and might also examine everyday experiences and values that align with the practices of other disciplines. In fact, such research on students' conceptions of usefulness in different domains would be fruitful. In the next section, however, I focus in on what this model and findings from this research tell us about supporting student learning and task design in mathematics in particular.

Applications to Mathematics Education

While this research has implications for our understanding of utility value at large, findings can also be used to inform mathematics instruction in particular. In the previous sections, some examples of applications to the field of mathematics were already discussed. However, in this section I dive more deeply into the applications of this research to middle school mathematics education. I begin by discussing applications of students' conceptions of the usefulness of mathematics, and then I explicitly draw attention to the connection between particular mathematics practices and conceptions of usefulness. Finally, I describe implications of these findings for the design of mathematics problem-solving tasks.

Students' Conceptions of the Usefulness of Mathematics

One key way in which this research can inform classroom practice centers on students' conceptions of usefulness related to features of the learning experience. This conception of usefulness focuses on the processes and practices of mathematics, including showing one's thinking, working with others, and selecting appropriate representations. Such processes are consistent with two main areas of research and practice in the fields of mathematics and science. First, this new conception of usefulness aligns with current standards that focus on valued practices used by mathematicians and scientists in the field. For example, the Common Core State Standards in Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) specify eight standards for mathematical practice that teachers should strive to help their students develop. These standards focus on important mathematical practices such as “critique[ing] the reasoning of others,” “model[ing] with mathematics,” and “us[ing] appropriate tools strategically.” Similarly, the Next Generation Science Standards (NGSS Lead States, 2013) detail eight practices of science and engineering that are crucial for success in conducting scientific inquiry, such as “asking questions,” “constructing explanations,” and “engaging in argument from evidence” (Appendix F, p. 1). Second, conceiving of usefulness in terms of features of the learning experience is also consistent with models of teaching and learning that promote deep understanding and equitable instruction. For example, many of the practices specified above are also central in math-talk learning communities (Hufferd-Ackles et al., 2004) and in ambitious mathematics instruction (Franke, Kazemi, & Battey, 2007).

The connection between effective teaching practices and students’ conceptions of usefulness regarding particular features of learning highlights at least two possibilities for leveraging the alignment between practices that students view as useful and practices encouraged by the standards. First, for students who view these features of learning mathematics as useful, shifting mathematics classroom practices to be more aligned with the mathematical practices stated in CCSSM might also serve to enhance students’ perceptions of usefulness. Second, for students who tend to think about usefulness in terms of applicability of content and struggle to see the usefulness of mathematics, this new conception might provide a helpful lens. Teachers can highlight the usefulness of particular mathematical practices and processes, including the application of various modes of interaction to other settings (such as advantages of collaboration for the workplace or for navigating relationships). In particular, students might benefit from gaining direct exposure to individuals who use such mathematical practices in their everyday lives and careers to see the authenticity of those practices, as well as potential areas of application. As discussed above, it will likely be especially useful if those areas of application align with students' own goals and values.

Two additional conceptions of usefulness that emerged frequently among students might provide points of leverage for helping students to view mathematics as useful. In particular, students conceived of usefulness in terms of its application to a) future jobs and careers and b) everyday activities involving money. One question we might pose is whether one of these conceptions is more productive to leverage when working with middle school students. Here, students' prior experiences with careers versus money-related activities might come into play. Regarding money, many researchers have documented the rich ways in which adolescents engage with money outside the classroom (Brenner, 1998; Guberman, 1996; Nunes, Schliemann, & Carraher, 1993; Saxe, 1988; Taylor, 2009; Taylor, 2012b). Similarly, many students in this study reported either observing or helping their parents handle money. Thus, drawing on students' experiences with money might be helpful for developing rational number understanding in the mathematics classroom while also helping students to see mathematics as useful.

In contrast, when considering the usefulness of mathematics for jobs or careers, middle school students likely have fewer experiences on which to draw. While some students in this study reported seeing or hearing about family members using mathematics in their careers, others simply had a sense of mathematics as useful for various careers – perhaps because others told them it was. If students do not have firsthand experience with the careers that they believe involve mathematics, they are less likely to know what kinds of mathematics those careers involve. As a result, their ideas about the usefulness of mathematics for careers might sit more at the global level discussed earlier, yet not serve to influence students' in-the-moment experiences of mathematics. In order to capitalize on students' ideas about the usefulness of mathematics for careers, teachers might help students to better understand the specific ways in which different people use mathematics in their careers and connect particular mathematics topics with the careers in which they are used. Building such bridges and helping students to see more concrete ways in which mathematics is useful for careers might help those perceptions of usefulness to further increase student engagement in the mathematics classroom.

Related, it is important to note that we must not only draw on the careers with which students have had experience, as students' exposure might be limited by their socioeconomic status or geographical location, among other factors. Rather, we want to engage in equitable instruction by encouraging students to imagine a range of future possible selves (Oyserman, Bybee, & Terry, 2006; Oyserman & Fryberg, 2006). Thus, some elements of directly communicated utility value might be helpful to expose students to new careers in which mathematics is useful (Durik & Harackiewicz, 2007; Durik et al., 2014), seeking to make connections whenever possible between those careers and students' own values and lived experiences.

Supporting Connections Between Mathematics Practices, Content, and Conceptions of Usefulness

Before moving on to discuss implications for the design of mathematics problem-solving tasks, I would like to consider more deeply how we might connect specific mathematics content with students' conceptions of usefulness. One question that often arises when discussing the usefulness of mathematics is whether all mathematics content should be seen as useful and what connections can be made to mathematics in the world. As stated in the introduction, I do not believe that we should attempt to prove to students that all mathematics topics are useful. Rather, we should begin from students' own conceptions of usefulness and see where connections can be made. However, having said that, this new conception of usefulness related to features of the learning experience might be helpful for some students who strongly value usefulness but cannot see the applicability of many mathematics topics. If students value certain modes of interaction or processes of learning, we can help them see usefulness in those processes even if they do not view the mathematics content as applicable. For example, consider a high school student who is learning about logarithms. Hearing that some engineers and computer scientists use logarithms is unlikely to be motivating if that student has no interest in pursuing engineering or computer science as a career. However, highlighting the usefulness of the mathematical practices students engage with while working on logarithms – such as identifying patterns and critiquing reasoning – might provide a valuable lens for such students to see usefulness in their mathematics learning and experience success in the process.

Related to this example, I propose three ways in which we can approach making such connections between the content of middle school mathematics and students' conceptions of usefulness. One approach is to first identify the ways in which students actually use mathematics in their lives and then draw on those uses to make connections to classroom mathematics. In this study, students reported using only very basic types of mathematics – in particular, measurement and counting. While some types of measurement did occur with fractions, which aligns with middle school mathematics content, much of the mathematics students used involved whole numbers, which are primarily explored in the elementary grades. In such cases, we might be able to help students see additional mathematics in the activities with which they are engaging. While we must be careful to not be inauthentic in matching content with contexts (Taylor, 2012), it might be that students missed some connections between classroom mathematics and their everyday practices. For example, many students in this study reported cooking with their families, and some even mentioned changing the sizes of recipes for larger groups. However, only one student mentioned the connection to proportions, which the students had just begun to learn in school. Thus, we might draw on students' experiences with cooking for family parties to help them understand ratios and proportions. Overall, the goal with this approach is to identify connections that already exist, even if students do not see them on their own.

Second, we might begin by identifying students' goals, values, and prior experiences, and then finding ways in which we can draw on those values and experiences to connect with new mathematical ideas. For example, many students have experiences utilizing the various resources in their communities, such as grocery stores and banks, either on their own or with parents. In the adolescent years, students also often have a strong emphasis on fairness and equity (Alm\a as, Cappelen, Sørensen, & Tungodden, 2010; Lerner & Steinberg, 2009). We might draw on these experiences and values when teaching proportionality to examine the amount of resources per person in different neighborhoods, as prior researchers have done (Gutstein, 2005; Rubel & Lim, n.d.). This approach would allow students to engage with contexts that are both familiar and important to them as they examine new mathematical ideas, thus increasing the likelihood that they will view their learning as useful.

A third approach is to begin by identifying the ways in which students think about the usefulness of mathematics and then help them to see more concretely how mathematics is used in those ways. For example, many students think about the usefulness of mathematics in terms of its applicability for everyday activities – especially everyday activities involving money. To connect this conception of usefulness with a classroom mathematics topic, a teacher might teach equations using contexts such as loans and interest rates when purchasing cars and property taxes when purchasing homes in different neighborhoods. Since many students assess usefulness in terms of whether the mathematics content is applicable to various everyday activities, they are more likely to view equations as a useful topic if teachers can illustrate how equations can be applied to accomplish everyday activities that are important to them. Similarly, students' emphasis on the applicability of mathematics for money-related activities – as well as their focus on the use of mathematics in careers – can be used to teach rational number understanding, ratios and proportionality, and statistical variability in the classroom. Regardless of which approach is taken to make such connections, students' experiences with mathematics, values, and conceptions of usefulness can be drawn upon to inform task design in ways that increase students' perceptions of the usefulness of mathematics.

Design of Problem-Solving Tasks

Related to the prior discussion, a primary way in which this research can be applied to the field of mathematics is in the design of problem-solving tasks. In Chapter 5, I proposed several principles for engaging in the equitable design of high utility problem-solving tasks. I argued that it is important to begin with an understanding of students' own conceptions of utility, as well as to strive to develop tasks that are as authentic as possible – in terms of both the context/content match and the particular numbers involved. Here, it is worth noting that authenticity can be conceptualized in two different ways. First, in line with Taylor's (2012) usage, authenticity can refer to the degree to which students actually engage with such practices in their own lives. In other words, a task is authentic if students engage with that task outside the classroom. Second, I propose that we can also conceive of authenticity in terms of the degree of alignment between the features of a problem-solving task and the features of a comparable problem in life. For example, I strived to make Personalized Group Task A authentic in this way by following a process that mirrors the process many adults actually use to calculate budgets in their own lives. This latter way of conceptualizing authenticity is consistent with many of the questions students posed about the authenticity of mathematics in the classroom.

Given these two ways of conceptualizing authenticity, I propose several pathways for developing tasks that students view as authentic. First, and as discussed in Taylor's (2012) research, we might draw on authentic ways in which students currently use mathematics. However, while such connections are certainly fruitful, students' existing uses of mathematics are sometimes limited, as they were with the students in this study. Thus, another possibility is to begin with contexts that students have experience with in the world and then expand the mathematics in those contexts in authentic ways. Through this process, we can open students' eyes to new ways of engaging with mathematics in familiar settings, while still conveying a sense of the authenticity of task contexts. Finally, we might also consider drawing on students' familiarity with the ways in which their parents and other family members use mathematics. Although students themselves do not use mathematics in these ways, this familiarity might be enough for students to see problems as authentic, even if they themselves have not engaged with such problems in the world. Furthermore, if students are familiar with the contexts, they might still have everyday knowledge that they can apply to such problems, as students did when completing the personalized tasks for this study. Thus, students do not necessarily need to have engaged with similar problems in the world to find them authentic. It might be that students' ability to identify the features of tasks as comparable to features of real world problems provides another pathway towards viewing tasks as authentic.

Related, this work highlights several affordances of personalized tasks. Students were able to make the greatest number of connections to their own lives and to their personal experiences when working on personalized tasks. Additionally, students were most likely to go beyond the requirements of the tasks and to consider situations as authentic when they were engaged with personalized tasks. Students spent a considerable amount of time working on the personalized tasks and even added elements that were not initially requested in the tasks. Thus, developing personalized tasks offers several points of leverage for helping students to engage with classroom mathematics.

Methodological Contributions

In addition to having implications for our understanding of utility value and its application to mathematics classroom practice, this research also makes several methodological contributions. First, this study offers several new ways of eliciting students' ideas about usefulness. By including a card sorting task and a video response task, in addition to more traditional survey measures, new conceptions of usefulness emerged. Using such techniques in future research will be important for eliciting ideas about usefulness from new communities of students. Related, by examining students' perceptions of usefulness of both the subject of mathematics and particular mathematics tasks, this research highlights the importance of acknowledging the levels at which we ask students about usefulness. The types of prompts used in interventions will also have implications for the conclusions we can draw regarding students' perceptions of usefulness. For example, if students are asked about the usefulness of the subject of mathematics, claims cannot be made about students' likely perceptions regarding the usefulness of particular curriculum tasks or various features of learning mathematics.

More broadly, findings from this research highlight the importance of considering the methodological approach used to study utility value. Examining students' own perspectives is crucial for developing an equitable conception of utility that encompasses the views of our diverse population of students. Prior survey items and interventions have focused on very independent uses of mathematics and have primarily considered the applicability of content (e.g. Anderman et al., 2001; Durik, Vida, & Eccles, 2005; Fennema & Sherman, 1976; George, 2006; Harackiewicz et al., 2012; Hulleman et al., 2010). However, in this study, students reported strong interdependent values and emphases on collaborative modes of learning, which align with some of the themes of research on independent versus interdependent motives (Stephens, Fryberg, & Markus, 2012; Stephens, Fryberg, Markus, Johnson, & Covarrubias, 2012; Stephens, Markus, & Townsend, 2007). Additionally, these perspectives echo work in the area of social justice mathematics that explores ways of using mathematics to work towards liberation and achieve social change (Gutstein, 2006). Despite these connections to interdependent values in other bodies of research, such themes are generally not explored in utility value research. By including analytical tools that allow us to tap into students' own perspectives and by working with populations of students whose voices are underrepresented in this research, we can use our methodological approach to make utility value research more equitable moving forward.

On a related note, while the particular findings of this research regarding students' conceptions of usefulness are not generalizable, the processes used to learn about students' conceptions of usefulness are generalizable. Moving forward, other researchers can use similar card-sorting tasks and video response tasks to examine conceptions of usefulness of middle school students in different communities. Additionally, research is needed with students across a range of ages to examine how conceptions of usefulness change with age. This study highlighted one way that existing techniques can be modified depending on students' age, and in Chapter 3 I offered some additional possibilities regarding survey items and interventions that can be used in similar research with elementary or middle school students.

Limitations

This study has several limitations that are important to mention. First, the same students were not able to participate in each form of data collection. Instead, seventh-grade students participated in earlier phases of the study, while sixth-grade students participated in later phases. Although age-related differences might have influenced students' responses, efforts were made to ensure that the populations of students were similar on key features. Second, due to imposed restrictions and logistical issues, I was unable to capture audio and video recordings of classroom observations. While jottings were vigorously recorded throughout observations, comments and interactions were clearly missed, as audio or video is needed to more fully capture classroom events. Next, in both rounds of interviews, participants were disproportionately female. The lack of male students in the study raises questions of whether findings are representative of male students in addition to female students (in particular, findings related to students' emphases on helping family, working with others, and expressing their own creativity in their everyday activities). Fourth, this study was limited by logistical issues that surfaced during the design phase. Due to changes in the administration of problem-solving tasks, all students did not provide ratings for each of the six tasks, so regression analyses were unable to be performed. In future work, it will be important to conduct such analyses to examine the relationship between participation structure, task type, and students' perceptions of utility. Finally, due to issues of consent and scheduling, I was unable to include observations of students' out-of-school practices in this work. While some features of engagement are missed by not observing students in everyday settings, new insights can also be gained from examining students' perspectives on their own practices through interviews.

Future Directions

This research has opened the door to many new questions and pathways that are worthy of exploration. Regarding students' conceptions of usefulness, additional research exploring the new conception related to features of the learning experience is needed. Questions remain regarding the prevalence of this conception, when it is most likely to surface, and which students are most likely to consider usefulness through that lens. Additionally, this work raises the question of whether there are additional ways of thinking about usefulness that this study did not capture. Future research exploring innovative methodologies that we might use to access conceptions of usefulness would further contribute to this body of literature. Similar research should also be conducted in communities with adolescents of different ethnicities, classes, and ages to examine the generalizability of results and identify shared experiences across communities and age cohorts. Everyday practices across various communities and within age cohorts should also be examined to identify similarities and differences in features of engagement. Though I do not claim that the findings of this research will apply to other populations of middle school students, it is worth exploring where similarities and differences emerge and whether new considerations related to usefulness arise when additional student voices are heard.

Future work is also needed to explore the correlates and outcomes of particular ways of thinking about usefulness. Studies should examine whether students who think about usefulness in different ways have different values or beliefs about mathematics. For example, it might be the case that students who conceive of usefulness primarily in terms of applicability to everyday life view mathematics as more useful and more interesting than students who conceive of usefulness in terms of applicability to careers. Related, do the ways in which one conceives of usefulness play a role in the impact that utility value has on achievement-related choices (as specified in the expectancy-value model; Eccles and Wigfield, 2002)? Although a causal study examining this question might not be feasible, future research might examine the conceptions of usefulness of students who pursue advanced mathematics classes, as well as low- versus high-performing students within individual classes. Alternatively, within the context of a self-generated utility value intervention, students might be asked to write about how a particular technique, practice, or topic is useful to their lives. Conceptions of usefulness implicit in those responses might then be coded and connected with students' subsequent desire to pursue additional problem-solving tasks.

Another possibility for research to pursue centers on the levels at which we study utility value. As noted in this work, some differences in perceived utility value emerged when students were asked about the usefulness of the subject of mathematics versus the usefulness of particular tasks. The levels at which we question students about their perceptions of usefulness must be attended to in future work, and researchers should continue to examine differences in the ways students make judgments about usefulness at the task versus subject level. On a related note, future research should examine the relationship between students' perceptions of the usefulness of mathematics as compared to their perceptions of the usefulness of particular mathematics tasks. As noted above, there are many possible reasons why students might view mathematics as useful, yet not perceive usefulness in the moment when engaging with classroom mathematics. This theory should be explored, and implications for the design of future research should be considered.

Finally, while this research illustrated that it is possible to design personalized tasks that students view as more useful than traditional curriculum tasks, there is certainly complexity involved in this process. In this research, only one of the two personalized tasks was viewed as significantly more useful by students. Thus, future work should carefully examine the features of tasks that students view as useful versus not useful in order to identify which features provide the most leverage for design. In particular, exploring participation structures, types of tasks, authenticity, and connections to students' goals and values will help us to better understand how to design problem-solving tasks that students view as useful.

Conclusion

This research highlighted the importance of considering students' perspectives and experiences when studying utility in mathematics. By taking a sociocultural approach and introducing new methodologies to the study of utility value, different conceptions emerged that have not typically been considered in utility value research. This work also highlighted important differences in features of students' engagement with mathematics in the classroom and in out-of-school settings. Design principles were then proposed to consider how we might draw on students' conceptions of usefulness and interactions with mathematics in and out of the classroom to design problem-solving tasks that students view as useful. Moving forward, as we advance this work to explore additional ways to help students see the usefulness of mathematics, it is crucial that we continue to allow students' own voices to emerge and consider the prior experiences and everyday knowledge of the students with whom we work. By building on students' own experiences and conceptions of usefulness, we can more effectively design learning environments that maximize our potential to improve students' engagement with and achievement in mathematics.

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Appendix A – Survey Items and Modifications

Construct	Description of Construct/Connection to Utility Value	Instrument Used/Modified	Modifications
Perceptions of usefulness (independent)	Perceptions of usefulness are measured to assess utility value. In line with existing research, this construct measures perceptions of usefulness for independent purposes (e.g. Fennema & Sherman, 1976).	Fennema-Sherman Usefulness of Mathematics scale (Fennema & Sherman, 1976); Modified Fennema-Sherman Usefulness of Mathematics scale (Doepken, Lawsky, & Padwa, 2004)	All 12 scale items were included. Nine of the items reflected the changes made in the modified scale, while one item was kept in its original form (to ask about whether mathematics is "related" to students' lives). The language in two items was changed slightly during and after pilot testing to be more understandable to and appropriate for middle school students.
Perceptions of usefulness (interdependent)	Perceptions of usefulness for interdependent purposes have not been measured in the literature thus far. Such perceptions focus on usefulness for family and community as opposed to oneself.	Fennema-Sherman Usefulness of Mathematics scale (Fennema & Sherman, 1976)	Ten items were developed to focus on the importance of mathematics for helping one's family or community; pilot testing was used to identify items that fit together. Five items were selected for inclusion.
Perceptions of usefulness (now/future)	This construct focuses specifically on perceptions of usefulness for different points in time – now versus the future. While some existing measures have inquired about perceptions of usefulness for specific points in time, these items were designed specifically to examine this distinction.	Designed based on the Fennema-Sherman Usefulness of Mathematics scale (Fennema & Sherman, 1976)	Six items were developed to examine potential differences in perceived utility of mathematics for now versus the future. The reliability of the scale was mediocre, and no significant differences emerged during pilot testing; thus, the scale was dropped for the official administration to decrease survey length.
Perceived	Perceived competence	MSALT Math	All four of the items that were

competence	measures a student's views of his/her own competence. Emphasizing the utility of a subject or task can promote interest in individuals with high perceptions of competence, yet undermine interest for individuals with low perceived competence (e.g. Godes et al., 2007).	Efficacy and Math Self-Concept of Ability scales (www.rcgd.isr. umich.edu/msalt)	considered to be part of both the math efficacy and math self-concept scales were included. One additional item from the math efficacy scale was included, as it focused on careers, which is a common theme in utility value research.
Personal and situational interest	Personal interest measures the interest one brings to various contexts, while situational interest assesses the interest one acquires in a given setting ((Hidi & Baird, 1988; Mitchell, 1993) Utility value and interest are correlated (Hulleman et al., 2008), and studies have shown that perceptions of utility value can promote interest (Godes et al., 2007; Hulleman et al., 2010).	Personal Interest and Situational Interest scales (Mitchell, 1993)	All 10 items were used for pilot testing. One item was removed from the Situational Interest scale, and two items were removed from Personal Interest scale to reduce survey length. Reliability analyses still produced high alphas without the items, and a combination of positive and negative items remained.
Relative interdependence	Relative interdependence measures the degree to which one views oneself in relation to the surrounding context (including other individuals) versus as an independent, unique entity (Markus & Kitayama, 1991). The relationship between	Self-Construal scale items (Hardin, Leong, & Bhagwat, 2004; Singelis, 1994)	Items from this scale were added after pilot testing to more directly target independent versus interdependent motives. Due to time constraints, 11 of the 30 items in the scale were selected for inclusion in this survey. Items were selected based on their relevance and understandability to middle school students, and

	relative interdependence and utility value has not yet been examined. This construct is included due to existing research on the importance of interdependent values in working class and Latin@ communities.		additional changes in language were made as needed with students' developmental stage in mind. In total, six interdependent and five independent items were selected for inclusion.
Friends' and parents'/ caregivers' views of mathematics	This construct measures students' perceptions of their friends' and parents'/caregivers' attitudes towards mathematics. The expectancy-value model highlights the indirect influence of children's perceptions of socializers' attitudes and beliefs on students' perceived utility value (Eccles & Wigfield, 2002).	Subjective Norms in Mathematics items (OECD, 2013)	The word "caregivers" was added to any questions about parents in order to be more inclusive. One item was removed from the friend scale ("My friends enjoy taking math tests"), as pilot testing revealed that the reliability of the scale was low with it included. (While more than half of students reported that their friends work hard in math and do well in math, less than 25% reported that their friends enjoy math tests.) The following new item was added as a replacement: "Most of my friends like mathematics."
Important life values	This construct measures students' beliefs regarding the importance of various values in their lives. Life values are measured given the direct influence of one's goals and identity on his or her perceived utility value (Eccles & Wigfield, 2002).	Selection of items in "Your Plans for the Future" section from ICPSR's Sloan Study of Youth and Social Development, 1992-1997 (Schneider, 2013)	Ten of the 13 items in this scale were used with only minor language changes based on the age of the participants (e.g. changing "my line of work" to "my future job or career" and "leisure time" to "free time"). As the items were not intended to fit together as a scale, items were selected to include a mix of independent and interdependent values. After pilot testing, four additional items were

			removed to decrease the length of the survey (and because there was little variation in pilot survey responses).
Reasons for wanting to do well in school	This construct measures the reasons that students report for wanting to succeed in school and is included because one’s goals and identity directly influence perceived utility value (Eccles & Wigfield, 2002). Capturing students’ reasons for wanting to do well in school provides insight into their goals and values.	Motives for Attending College scale items (Stephens, Fryberg, Markus, Johnson, & Covarrubias, 2012)	Ten of the twelve scale items were used, and one new item was added (“Gain respect from my friends”). The two items selected for removal focused on motives that would be less relevant to middle school students. Again, language was changed to make items more understandable to and relevant for middle school students.

Appendix B – Conceptions of Usefulness Coding Scheme

Top Level Code	Second Level Code
Applicability of Content	Everyday life (general)
	Specific daily life activities
	Everyday life + daily activities
	Job/career
	Current or future schooling
Features of the Learning Experience	Method of interaction
	Structure of the activity
	Representation being utilized
Other	Affective
	Value of learning new/important things
	Contribution to one's personal skills/development
	"It's math"
	Miscellaneous
Not able to be coded	N/A

Appendix C – Classroom Observations Coding Scheme

Top Level Code	Second Level Code	Third Level Code	Fourth Level Code
Features of the Classroom Context	Class session features	Mathematics topic	Ratios, Linear relationships, Commission and mark-up, Unclear
		Month of the year	January – May
		Class period	5, 7, 8
	Features of the Activity	Participation structure (stated)	Independent work, Partner work, Small group work, Stations, Teacher-led group work, Whole class activity, Whole class discussion
		Observed participation structure	Same categories as above
		Task	Administrative, Do Now, Entry flashcards, Gallery walk, Graphing activity, Investigation, IXL, Note-taking, Studying, SumDog, Survey, Teacher-created real world problem, Unclear, Video, Worksheet (not otherwise specified)
Student actions/ interactions	Discussion	Contributing an idea	
		Critiquing other’s reasoning or work	Critiquing other student
			Critiquing teacher
		Debating points of view	
		Inquiring	To student (calculation, checking one’s thinking or answer, clarification, conceptual, definition, general, other, process-procedure)
			To teacher (calculation, checking one’s thinking or answer, clarification, conceptual, definition, general, other, process-procedure)

		Justifying reasoning	Inability to justify reasoning Justifying another's reasoning (voluntary, prescribed) Justifying one's own reasoning (voluntary, prescribed) Refusal to justify reasoning
		Lack of response	
		Requesting more time	
		Showing one's thinking	
	Helping/Help-seeking	Denying help to another student	
		Helping another student	Prescribed helping Voluntary helping
		Looking at another student's work	
		Offering to help another student	
		Requesting help or feedback from student	
		Requesting help or feedback from teacher	Announcing completion of work Approaching teacher Calling out to teacher Raising hand Stating need for help
	Off-task behavior		
	On-task behavior (general)		
Student Perceptions/ Orientations	Affective Response	Gestures/sounds	Blank stare, bouncing in chair, breathing heavily, cheering/applauding, crossed arms, frowning, furrowed brow, gasping, hands in air (cheering), hands in air (disbelief), head on desk or computer, head on hand, hitting table or notebook, huffing, laughing, moaning-groaning, nodding, opening eyes wide, opening mouth wide, pacing, playing with

		objects, poking, rolling eyes, rubbing eyes, scrunching face, shaking head, shrugging, sighing, slouching, smiling, staring off in a daze, tapping pen or fingers, whining, yawning/ stretching
	Negative comment	Distraction, frustration with other student, lack of enjoyment
	Positive comment	Interest/enjoyment, Pride, Realization
Comparison to others	Asking about another student's progress	
	Comparing self to another	
	Group comparison	
Connection to everyday life	Authenticity	Assuming lack of authenticity
		Questioning authenticity
		Stating or assuming authenticity
	Making connections to personal experience	Family connection
		Job/career connection
		Money connection
		Other connection
	Problem Context	Applying everyday knowledge to problem context
		Asking a question about problem context
		Challenging problem context
		Lack of understanding of problem context
	Utility	Documenting Utility
		Questioning Utility
Orientation towards challenge/ difficulty	Desire/request for challenge	
	Perception of difficulty	Subject/topic as difficult
		Task as difficult
	Perception of simplicity	Subject/topic as simple

			Task as simple
	Orientation towards collaboration	Challenging/opposing collaboration	
		Noting an imbalance in effort during collaboration	
		Opposing independent work	
		Requesting collaboration	
		Stating the importance of collaboration	
		Other	
	Orientation toward mistakes	Criticizing/highlighting student mistake	
		Mistake as learning opportunity	
		Mistake as thing to avoid	
	Orientation towards rules/norms	Enforcing rules/norms	
		Questioning rules/norms	
	Perceptions of competence	Lack of competence	Group as lacking competence
			Other as lacking competence
			Self as lacking competence (Subject/topic, task)
		Perception of competence	Group as competent
			Other as competent
			Self as competent (Subject/topic, task)

Appendix D – Personalized Problem-Solving Task #1

Flash forward 20 years. You're settled into a job and have started your family. You own a house and a car, and you're ready to do some budgeting to figure out how much you'll be spending on groceries for your family each year. Before you head to the grocery store for the week, you sit down at the kitchen table to think about your budget.

STEP 1: How many people do you imagine you'll be cooking for? On your group recording sheet, write down the number of people that each group member expects to be cooking for.

STEP 2: Individually, jot down an estimate (to the nearest dollar) for the amount you think it will cost to buy groceries for your family for a week. Be prepared to explain to your group how you came up with that estimate.

Once everyone has written an estimate, come back together as a group. Have one group member share his/her estimate, as well as how s/he came up with that estimate. Together discuss whether the estimate seems reasonable, depending on the size of the group member's future family. Once everyone has agreed on a reasonable estimate, record the number on your group recording sheet. Then move on to your next group member and repeat the process until you have recorded an estimate for every group member.

Below the table on your recording sheet, work together to write a couple sentences explaining explain how you came up with your estimates.

STEP 3: As a group, write equations for each group member to help you answer the following question: How much will it cost for you to feed your family for one month (4 weeks)?

Again, start by looking at one group member's estimate. Work together to write an equation that would help you figure out how much it would cost for that group member to feed his/her family for one month. Record the equation on your group recording sheet, and then move on to the next group member. Make sure all group members agree on the recorded equations.

STEP 4: Compare all of the equations. Write a sentence or two on your group recording sheet answering the following questions: What is different about the equations? What is the same?

STEP 5: Individually, solve your equation to answer the following question: How much will it cost you to feed your family for one month (4 weeks)? Record your answers on the group recording sheet.

CHALLENGE (if your group has time):

a) Write a new equation to solve the following question: How much will it cost to feed your family for a year (52 weeks)?
b) Solve the equation. How much will it cost to feed your family for a year (52 weeks)?

Appendix E – Personalized Problem-Solving Task #2

Outside of school, many students like to make or build things with family and friends to have fun and be creative. Think about some of the things you like to make or build. Do you make jewelry? Bake muffins or cupcakes? Grow vegetables in a garden? Build models? Since your family means a lot to you, you recently decided that you want to start selling the things you make so that you can earn money to help out your family. You decide to start a business with some of your math classmates.

STEP 1: As a group, decide on the product you want to sell. Depending on your interests/skills, you might consider making jewelry, building something, or growing fruits/vegetables in a garden. Write down your product below.

STEP 2: Together decide how much money seems reasonable to charge per item. (You might want to account for the cost of making the item, as well as how much people would be willing to pay for it. Be sure to talk with your group members who have made your product before to find out how much they have paid for ingredients/materials in the past.)

STEP 3: Individually, on a separate piece of paper, use any strategy you want to answer the following question: How many items would you need to sell to earn $100? Be prepared to share your strategy with your group members in the next step.

♦ *STOP! Work out your solution on a separate piece of paper. Then be prepared to share your strategy with your group*

STEP 4: Come back together as a group. Compare your answers, and have each group member explain his/her strategy. In the space below, record the different strategies used by members of your group. Once you have agreed on an answer, record it below

STEP 5: After reviewing everyone's strategies, work together to write an equation that you could use to figure out how many items you would have to sell to make $100. Use n to represent the number of items. (Keep in mind that this does not include the cost of the materials needed to make your product.) Record your equation below.

STEP 6: Using the equation you just wrote as a model, write a new equation to answer the following question: How many items will you need to sell to earn $1,000? Then solve your equation.

CHALLENGE: Your group will need to spend money to buy the materials to make your product. Right now your equations only consider the money you're making but not the money you spent. How would you change your equations to include the money spent on materials? Use the space below to brainstorm strategies and show your work.

Now write a few sentences explaining how you might change your equations to include the money spent on materials.