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Essays on Horizontal Mergers and Dynamic Contract Breach

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ABSTRACT

Essays on Horizontal Mergers and Dynamic Contract Breach

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This collection of essays on horizontal merger enforcement and dynamic contract breach addresses the question of what role, if any, should government play in enforcing the contracts of private parties? The government's appropriate role in these various settings depends, among other things, on the nature of the competition between private parties, the nature of their contracts, and the potential effects that their contracts have on third parties who are not involved in contract negotiations.

In Chapter 1, I begin my study of horizontal merger enforcement in a static model of Cournot competition. I show that asymmetric information about efficiency gains from proposed horizontal mergers can lead to the optimality of approval policies that are non-monotonic in industry concentration and entry costs. The intuition is that changes in these parameters induce a selection effect in the set of mergers that are proposed, which informs an antitrust authority's posterior beliefs about the size of a proposed merger's efficiency gain.

Next, Chapter 2 studies the design of optimal liquidated damages when breach of contract is possible at more than one point in time. It offers an intuitive explanation for why cancellation fees for some services (e.g., hotel reservations) are observed to increase as the time for performance approaches. I also show that even when renegotiation is possible, the efficient breach and investment decisions can be implemented with the same efficient expectation damages that implement the efficient outcomes absent renegotiation. Hence, to the extent that courts are able to calculate efficient expectation damages, or to the extent that contract parties are rational in drafting liquidated damage clauses, a court of law should do nothing more than simply enforce the contracts of private parties (assuming they impose no externalities)

Finally, Chapter 3 considers horizontal merger enforcement in a dynamic environment where merger, exit, entry, and investment decisions constitute a Markov perfect equilibrium. Every period a single pair-wise merger is possible, and all active firms are assumed to engage in capacity-constrained Cournot competition. I examine the structural and welfare differences between the dynamic merger enforcement policy and the myopic merger enforcement policy.

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CHAPTER 1

Inference about Proposed Horizontal Mergers with Efficiency Gains

1.1. Introduction

Horizontal mergers, or mergers between firms competing in the same market, are a common industrial phenomenon.[1] In the United States, firms exceeding certain size thresholds that wish to merge are required by law (the Hart-Scott-Rodino Act of 1976) to notify antitrust authorities such as the Federal Trade Commission (FTC) and the Department of Justice (DOJ) of their intention to merge. One of these agencies then evaluates the benefits and costs of the proposed merger and decides whether to approve or reject it.[2] Since the antitrust authority usually performs this analysis with less information than is available to the firms, the enforcement decision is based on the proposed merger's *expected* welfare effects. In carrying out this analysis, the agency usually views the presence of many non-merging firms and evidence of easy entry as factors mitigating any potential harm that the merger might cause. In other words, the FTC and DOJ are more likely to

[1] Between 1981 and 1998, Gugler et al. (2003) find that 69,605 mergers were announced worldwide, with 44,600 mergers completed that involved transactions valuing at least $1 million (1995 U.S. dollars). Among completed deals, 41.7% were horizontal mergers (deals involvings companies with sales in the same primary four-digit SIC industry), 4.0% were vertical mergers, and 54.3% were conglomerate mergers (defined as mergers which were neither horizontal nor vertical in nature). The average transaction (among all types of mergers) was valued at $220 million.

[2] I assume antitrust authority can always prevent a merger from occurring when it wishes to do so. In practice, there is a lengthy review process and possibly negotiations between the antitrust authority and the merging parties (e.g., an agreement to sell off assets in return for regulatory approval). I abstract away from these details, as does much of the literature on mergers; an exception is Farrell (2003).

approve a merger if, ceteris paribus, industry concentration and entry costs are relatively low. Are these views sensible for an antitrust agency concerned with maximizing either consumer or aggregate surplus? This paper presents a model of mergers with efficiency gains and quantity competition that addresses this question.

The profit and welfare effects of horizontal mergers have been widely studied in the economics literature. Williamson (1968) was the first to frame the welfare effects of a horizontal merger as a trade-off between reduced competition and increased productive efficiency. Efficiency gains are frequently claimed by the merging parties and are central to evaluations of the likely effects of proposed mergers.[3] However, they are difficult to observe or verify by antitrust authorities in practice. Hence, I focus on how more readily observable industry characteristics, such as the number of active firms and entry conditions, should affect the analysis of mergers when efficiency gains are not directly observable.

Farrell and Shapiro (1990) address the issue of efficiency gains by assuming only privately profitable mergers are proposed. They derive a sufficient condition – one requiring mergers to have a "positive net external effect"[4] – under which proposed mergers would

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[3]In the U.S., for example, the Horizontal Merger Guidelines (§4, revised in 1997) states, "Competition usually spurs firms to achieve efficiencies internally. Nevertheless, mergers have the potential to generate significant efficiencies by permitting a better utilization of existing assets, enabling the combined firm to achieve lower costs in producing a given quantity and quality than either firm could have achieved without the proposed transaction. *Indeed, the primary benefit of mergers to the economy is their potential to generate such efficiencies.*" (emphasis added) A copy of the Guidelines can be found at http://www.usdoj.gov/atr/public/guidelines/horiz_book/hmg1.html and http://www.ftc.gov/bc/docs/horizmer.htm.

[4]A merger is said to have a positive net external effect if it benefits non-merging firms more than it hurts consumers. Farrell and Shapiro derive restrictions on pre-merger market shares that imply this condition. However, they do not consider the possibility of entry.

increase aggregate welfare. For the purposes of this paper, however, I also need to consider the likelihood that a profitable merger which does not satisfy Farrell and Shapiro's condition nonetheless increases consumer surplus or aggregate surplus. Notably, this involves Bayesian updating on the likely distribution of efficiency gains after a merger is proposed, which does not arise when considering only mergers satisfying Farrell and Shapiro's sufficient condition. Therefore, this paper contributes to the literature on horizontal mergers by analyzing the profit and welfare effects of mergers when the regulator is asymmetrically informed about the merger's cost savings. The subsection below ("Related Literature") discusses the few studies in the previous literature that have addressed this issue and contrasts them with the present paper.

I begin, in Section 1.2, by studying the effects of merger-induced cost synergies[5] in a simple Cournot model with linear (ex-ante symmetric) costs and without the possibility of entry or exit.[6] Because a larger synergy implies higher total output and total profit, only those mergers that generate sufficiently large synergies should be approved. However, as the merger synergy is unobservable to the antitrust authority, a merger is approved if and only if the expected welfare with the merger, conditional on it being proposed or privately profitable, exceeds the welfare without it.

[5]These merger-specific synergies, which uniformly reduce the merged firm's marginal cost, may arise, for example, from shared patents or integration of other unique assets. Coordinating production might also lead to rationalization, or production re-shuffling, whereby output decisions are more efficiently allocated among different plants (e.g., in order to avoid diseconomies of scale). But if the joint production possiblity frontier of the merging firms does not shift outwards, or if the cost reduction is *feasible* for one firm to achieve unilaterally without the merger, then the merger is said to generate no synergies. This is the same sense in which Farrell and Shapiro (1990) use the word synergy. For a more detailed discussion of the distinction between synergies and other types of merger-related efficiency gains, see Farrell and Shapiro (2000). For evidence of real mergers with various efficiency gains, see Fisher and Lande (1983).

[6]I assume fixed costs are sunk and therefore do not affect the merger decision.

First, I show that if the number of firms initially in the industry, n, is sufficiently large, any merger inducing a cost synergy is privately profitable, increases consumer surplus, and increases total surplus. Furthermore, not only is the minimum synergy required for a merger to be profitable non-monotonic in n, expected welfare *conditional on proposed mergers* may be non-monotonic in n as well. In particular, I show that a proposed merger which would raise expected consumer surplus may instead lower it if there is a moderate *increase* in n. This observation highlights a selection effect from changes in industry concentration that does not arise when a merger's cost synergy is observable. To the extent that antitrust authorities always view mergers more favorably when industry concentration is lower, this selection effect is often overlooked in practice.

Closely linked to considerations of industry concentration, or market power, is the issue of ease of entry. It is crucial to account for entry conditions when evaluating the social desirability of proposed horizontal mergers. The Horizontal Merger Guidelines (§3), for example, argue that: "A merger is not likely to create or enhance market power or to facilitate its exercise, if entry into the market is so easy that market participants, after the merger, either collectively or unilaterally could not profitably maintain a price increase above premerger levels."

In Section 1.3, the main model of this paper, I consider the profitability and social desirability of mergers while allowing for the possibility of entry. The setting is still quantity competition, but now I allow for convex cost functions. By studying the possibility of merger-induced entry, this paper differs from most of the existing literature on horizontal mergers. (For a discussion of some notable exceptions, see the subsection below on the related literature.) In particular, I examine how the evaluation of merger proposals is affected by the interaction between the entry-inducing effect of mergers and the unobservability of cost synergies.

I assume that a potential entrant has the same variable cost function as the non-merging incumbent firms, but it incurs a fixed entry cost if it enters. Following a merger, entry is profitable if the gross profit upon entry is larger than the cost of entry. Thus, by holding fixed the merger's synergy, the direct effect of an increase in the entry cost is to make entry less likely. This effect is bad for consumer surplus because total output increases with the number of firms. Since post-merger entry affects the ultimate profitability of the merger, changes in the entry cost will also affect the set of mergers that are proposed in the first place. I show that this indirect or selection effect from an increase in the entry cost also hurts consumers. The reason is that a larger entry cost implies that proposed mergers are on average (weakly) less efficient and therefore result in lower total output and higher price.[7] Hence, I find that the expected consumer surplus from approving a proposed merger is (weakly) decreasing in the entry cost. This means that an antitrust authority using consumer surplus as the welfare standard should approve a proposed merger if and only if the entry cost is sufficiently low.

However, if the antitrust authority takes firms' profits into account and evaluates mergers using a total surplus standard, easier entry may make a merger socially undesirable. The intuition is that private entry incentives tend to be socially excessive in homogeneous goods models (see Mankiw and Whinston (1986)). This inefficiency is exacerbated in the

[7]The intution is as follows. The entrant's gross profit from entry is decreasing in the merger's cost synergy. So the maximum synergy of a merger that induces entry is decreasing in the entry cost. Therefore, the minimum synergy needed for a merger to be profitable also (weakly) decreases with the entry cost, since for any synergy, a merger is more profitable without entry than with entry.

present model when a less efficient entrant enters to steal business away from the more efficient merged firm.

If the entry cost is low enough that entry occurs with positive probability after a profitable merger, an increase in the entry cost decreases the probability of post-merger entry. This consequence has a positive direct effect on total surplus because any entry is socially excessive. In addition, it turns out that when a profitable merger induces entry with positive probability, there is no selection effect with respect to the set of mergers that are proposed. Therefore, I show that an *increase* in the entry cost, when it is sufficiently low, has an overall *positive* effect on expected total surplus. On the other hand, if the entry cost is high enough that entry never occurs after a profitable merger, small changes in the entry cost would have no direct effect. But as the selection effect is (weakly) negative in this case, an increase in the entry cost when it is sufficiently high has a (weakly) negative overall effect on expected total surplus.

Provided that it is neither optimal to approve, nor reject, all proposed mergers, I find that the optimal policy when using a total surplus standard is to approve a proposed merger if and only if the entry cost is neither too high nor too low. Given that in practice most antitrust authorities focus on consumer surplus, as opposed to total surplus, their concern with whether entry is sufficiently easy, and not whether entry is sufficiently difficult, seems justified in light of the predictions of this model.

In Section 1.4, I return to the question of how the initial number of firms affect the welfare consequences of proposed mergers with unknown efficiency gains, but continue to allow for the possibility of post-merger entry. I assume in this section that instead of observing entry costs, the antitrust authority forms beliefs about the distribution of entry costs that are consistent with the observed number of firms being in a pre-merger free-entry equilibrium. When the antitrust authority holds the most pessimistic belief possible about the likelihood of entry (that is consistent with the observed number of firms), I argue that the non-monotone consumer welfare effect identified in the model without entry (Section 1.2) continues to be possible when the possibility of entry is allowed. Finally, I conclude in Section 1.5 and leave for the Appendix all proofs omitted from the main text.

1.1.1. Related Literature

There is a large literature on horizontal mergers. Most early works, such as Salant, Switzer, and Reynolds (1983), Davidson and Deneckere (1985), and Perry and Porter (1985), do not allow for cost improvements. (As mentioned above, Williamson (1968) and Farrell and Shapiro (1990) are two exceptions.) Instead, they mainly focus on the profitability of mergers under different assumptions about firms' cost functions and whether competition is in quantities or prices. More recently, Bian and McFetridge (2000) derive the minimum cost synergies required for a merger to be profitable and to meet various welfare criteria. However, they assume that the antitrust authority can perfectly observe these synergies, and do not allow for entry.

Most relevant to this paper are studies that focus on how the welfare evaluation of mergers is affected by imperfectly observed cost synergies or the possibility of post-merger entry.

Besanko and Spulber (1993) consider a situation in which the antitrust authority is unable to observe the efficiency gain of the proposed merger. By focusing on a merger from duopoly to monopoly, they do not address how the proposed merger affects non-merging firms' profits, nor do they consider how the number of firms and the possibility of entry affect the evaluation of the merger. Instead, Besanko and Spulber show that when it is costly to propose a merger, an antitrust authority may wish to adopt a welfare standard that is biased in favor of consumer surplus, even if total surplus is of ultimate concern.[8] My paper is similar to their work in that it, too, incorporates an informational asymmetry between the antitrust authority and the merging firms, and it is also concerned with the formulation of optimal merger enforcement policies. However, by assuming that merger proposals are costless to make, my model abstracts away from the possible feedback effect that enforcement policies have on which mergers are proposed in the first place (an effect that lies at the heart of Besanko and Spulber's analysis).

Amir, Diamantoudi, and Xue (2005) also assume a merger's cost synergies are imperfectly observed. Unlike this paper, however, they consider a post-merger Baynesian Cournot equilibrium in which non-merging firms are unable to observe the merged firm's costs.[9] But if non-merging firms have better information about demand conditions or each other's costs, they would presumably learn the value of the merger synergy before the antitrust authority. While some of Amir, Diamantoudi, and Xue's results are similar to mine, they do not address the issues surrounding entry. Nevertheless, this paper and theirs can be viewed as providing complementary analyses which are appropriate for

[8] In particular, because proposing a merger is costly in their model, only "better-than-average" mergers are proposed. So, the antitrust authority will tend to approve too many proposed mergers if it uses a total surplus standard. It is then optimal to offset this tendency of approving too many mergers by placing a greater emphasis on consumer surplus than producer surplus.

[9] Amir et al. assume that the post-merger constant marginal cost is either the same as before, or some lower value. Thus, their results are characterized in terms of the size of the cost-reduction for the merged firms, the likelihood of the merger realizing it, and the types of beliefs held by the non-merging parties about the merger's cost reduction. The three cases considered are where outsiders believe that the merger is certain to reduce costs, certain to not reduce costs, or will result in the expected costs (Amir et al. assume linear demand and costs).

merger evaluations conducted under differing temporal considerations and informational assumptions.

The horizontal mergers literature has also mostly abstracted away from the issue of how mergers affect (ex-post) entry incentives. Exceptions, however, include Werden and Froeb (1998), Spector (2003), and Davidson and Mukherjee (2007).[10] Werden and Froeb argue that if a profitable merger does not generate synergies, it is unlikely to induce entry. They attribute this finding, in part, to the following observation: If the industry is in a free-entry equilibrium before the merger, then the lowest possible value of the entry cost must exceed the gross profits of any potential entrant who is less efficient than the incumbents (otherwise, the potential entrant would have already entered). Spector (2003) writes: "Since ease of entry and the presence or absence of merger-specific synergies are among the most important criteria used by antitrust authorities when assessing whether an attempted merger should be challenged, an improved understanding of the relationship between these two elements is of paramount importance for policy." However, like Werden and Froeb, Spector also does not allow for synergies in his analysis. Instead, he shows that a profitable merger generating no synergies necessarily increases price, regardless of entry conditions.

Davidson and Mukherjee (2007) do allow for cost synergies, but they do not address the issue of optimal merger enforcement by an asymmetrically informed antitrust authority. On the other hand, they allow for entry by more than one firm after a merger, and

[10]Cabral (2003) also analyzes how entry affects the consequences of a merger, but focuses on spatial competition and mergers from duopoly to monopoly. He finds that entry is less likely when the merged firm is more efficient, which accords with the prediction of this model.

furthermore, show that with price competition, mergers that induce relatively small cost synergies are not privately profitable when there is free entry.

1.2. Mergers without Entry

1.2.1. The Model

I begin by considering how the profit and welfare effects of proposed horizontal mergers are affected by the number of firms initially in an industry. (The possibility of entry is ruled out for now but examined in Section 1.3.) Initially, there are $n \geq 3$ identical firms in the industry, numbered $i = 1, 2, ..., n$, competiting in quantities. Each firm has constant marginal cost $c > 0$ and no fixed costs. (Equivalently, fixed costs can be thought of as being sunk, so that they do not affect the merger decision.) Aggregate demand is characterized by the downward-sloping inverse demand function $P(X)$, where X is total industry output and $P'(\cdot) < 0$.

Firms 1 and 2 have the opportunity to (propose to) merge, where a merger causes their marginal cost to become $c - \delta$, while non-merging firms' marginal costs remain at c. The parameter δ is realized prior to the merger decision and is observed by all the firms. Firms 1 and 2 propose to merge if and only if the merger would be privately profitable for them, i.e., if and only if joint profit maximization (with marginal cost $c - \delta$) yields a higher payoff than their combined pre-merger profits.[11] After the merger decision, each firm chooses the quantity it wishes to produce, payoffs are realized, and the game ends.

[11] Farrell and Shapiro (1990) implicitly make the same assumption. See Budzinski and Kretschmer (2007) for an analysis of the possibility that unprofitable mergers may be proposed.

By restricting attention to non-negative values of δ, this parameter can be interpreted as a merger-induced "synergy" or efficiency gain.[12] Such (marginal-)cost reductions might arise because the merger allows firms 1 and 2 to share patents or combine non-contractible assets to improve their joint production capabilities. However, if the merger only enables merging firms to reallocate their production decisions, then $\delta = 0$, since firms initially have symmetric costs.

In charge of approving or rejecting merger proposals is an antitrust authority who only knows the cumulative distribution function $G(\cdot)$ from which δ is drawn, but does not observe the actual realization of δ. Since a merger is privately profitable only when the synergy is sufficiently large (as will be shown below), the antitrust authority updates its belief about the likely values of δ when it is confronted with a proposed merger. It then evaluates the welfare (consumer surplus or total surplus) that can be expected to result from approving the proposed merger, where the expectation is over synergy values δ and is conditional on the merger being privately profitable. The proposed merger is approved if and only if the expected welfare with the merger is higher than the pre-merger welfare. The goal of this section of the paper is to study how the initial number of firms present in the industry affects the antitrust authority's approval/rejection decision.

Drastic Synergies. If the cost-reduction associated with the merger synergy is sufficiently large, or drastic, the non-merging firms may wish to produce nothing in equilibrium. More precisely, a synergy is said to be *drastic* if a monopolist (in this case, the merged firm) with cost $c - \delta$ would find it optimal to set a price below c, or equivalently,

[12] In reality, various factors such as incompatible corporate cultures might cause some mergers to lead to efficiency losses, or negative values of δ. However, by assuming that δ is perfectly observed prior to the merger decision, a merger will never be profitable in this model (and hence will never be proposed) if it involves a negative δ (and at least one non-merging firm is present).

if it would produce more than the perfectly competitive total output in an industry with common marginal cost c.[13] Given a merger with a drastic synergy, no non-merging firm would want to produce a positive quantity (otherwise it would make a negative profit margin on each unit of output sold), and therefore the merged firm would indeed be a monopolist.

Consumer surplus is always higher after a merger with a drastic synergy than without any merger, since the merged firm produces a larger output than the competitive output, and the total output from no merger (i.e., in a symmetric n-firm Cournot equilibrium) is smaller than the competitive output.[14] Furthermore, the merged firm's (monopoly) profit with a drastic synergy is larger than the profit of a monopolist with cost c, which is in turn larger than the aggregate industry profit without any merger.[15] Thus, producer surplus, and hence total surplus, is also higher after a drastic synergy merger than without the merger. Note that this also implies that any drastic synergy merger is privately profitable.

[13] The merger synergy is drastic if $P(X^m(c-\delta)) < c$, where $X^m(c-\delta)$ denotes the output of a monopolist with marginal cost $c-\delta$. In other words, the synergy is drastic if $\delta > \overline{\delta}$, where $\overline{\delta}$ is implicitly defined by $P(X^m(c-\overline{\delta})) = c$. (Recall that demand is downward sloping and a monopolist's output is decreasing in its marginal cost.) Note that what constitutes a drastic synergy is independent of how many firms are initially in the industry.

[14] The claim is that $X^m(c-\delta) > X^c > X^{\overline{M}}(n)$ for all drastic δ and all $n \geq 3$, where $X^m(c-\delta)$ denotes the merged firm's (i.e., total industry, or monopoly) output after a merger occurs with synergy δ, X^c is the competitive total output in an industry with common marginal cost c (it satisfies the market demand at price c), and $X^{\overline{M}}(n)$ denotes total output after no merger (i.e., the total output in a symmetric n-firm Cournot equilibrium). Since $P'(\cdot) < 0$, this is equivalent to $P(X^m(c-\delta)) < c < P(X^{\overline{M}}(n))$. The first inequality is the definition of δ being drastic, while the second inequality follows from a firm's first order condition in any Cournot equilibrium.

[15] Let $\pi^m(c-\delta)$ denote the merged firm's (monopoly) profit with a drastic synergy, let $\pi^{\overline{M}}(n)$ denote the per-firm profit in a symmetric n-firm Cournot equilibrium (the outcome with no merger), and let $X^m(c)$ denote the output of a monopolist with marginal cost c. Then, $\pi^m(c-\delta) \equiv [P(X^m(c-\delta)) - (c-\delta)]X^m(c-\delta) \geq [P(X^m(c)) - (c-\delta)]X^m(c) \geq [P(X^m(c)) - c]X^m(c) \geq [P(X^{\overline{M}}(n)) - c]X^{\overline{M}}(n) = n[P(X^{\overline{M}}(n)) - c]X^{\overline{M}}(n)/n = n\pi^{\overline{M}}(n)$. (The first and third inequalities are due to revealed preferences.)

Therefore, a merger that would generate a drastic synergy (with certainty) will be proposed and should be approved under either a consumer surplus standard or total surplus standard, regardless of n. However, merger opportunities that lead to drastic synergies are arguably less likely to arise than those that generate non-drastic synergies. Furthermore, none of the qualitative features of the results I derive below would be changed if merger synergies are potentially drastic but with sufficiently low probability. Hence, for simplicity I restrict attention to non-drastic synergies throughout the rest of this paper.

1.2.2. A Competitive Limit Result

First, I introduce some notation and study the profit and welfare effects of horizontal mergers when the initial industry structure approaches perfect competition, i.e., as $n \to \infty$.

If firms 1 and 2 do not merge, the outcome is the standard symmetric Cournot equilibrium with n firms: each firm produces $x^{\overline{M}}(n)$, gets a profit of $\pi^{\overline{M}}(n)$, and total output is $X^{\overline{M}}(n) = nx^{\overline{M}}(n)$, where superscript $\overline{M}$ denotes the case of no merger occurring. If firms 1 and 2 do merge, the result is an asymmetric Cournot equilibrium with $n-1$ firms,[16] where the merged firm M has marginal cost $c-\delta$, and the $n-2$ non-merging firms still have marginal cost c. Let superscript M denote the case of a merger occurring, and let subscripts M and i index the merged firm and an arbitrary non-merging firm, respectively. Total output is $X^M(\delta, n) = x_M^M(\delta, n) + (n-2)x_i^M(\delta, n)$, where the merged firm produces $x_M^M(\delta, n)$ and each of the non-merging firms $i \in \{3, ..., n\}$ produces $x_i^M(\delta, n)$. The merged

[16]The assumption of non-drastic synergies implies that all $n-2$ non-merging firms remain active after a merger.

firm's profit is written as $\pi_M^M(\delta, n)$, and the profit of each of the $n-2$ non-merging firms is $\pi_i^M(\delta, n)$.[17]

I make the following assumption in order to derive the competitive limit result concerning the profit and welfare effects of mergers.[18]

Assumption 1. *For any δ, total output after a merger, $X^M(\delta, n)$, remains bounded as $n \to \infty$.*

After observing the synergy δ, firms 1 and 2 find it privately profitable to merge if and only if their profit from merging exceeds their combined profits if they do not merge, or $\pi_M^M(\delta, n) > 2\pi^{\overline{M}}(n)$. It can be shown that firms 1 and 2's profit from merging is increasing in the synergy. Therefore, they benefit from the merger if and only if $\delta > \delta^*(n)$, where $\delta^*(n)$ is implictly defined by[19]

$$\pi_M^M(\delta^*(n), n) = 2\pi^{\overline{M}}(n).$$

On the other hand, the merger increases consumer surplus (relative to no merger at all) if and only if $\delta > \delta^{CS}(n)$, and increases total surplus if and only if $\delta > \delta^{TS}(n)$, where

$$\begin{aligned} CS^M(\delta^{CS}(n), n) &= CS^{\overline{M}}(n), \\ TS^M(\delta^{TS}(n), n) &= TS^{\overline{M}}(n). \end{aligned}$$

[17] Even though there are only $n-1$ firms in the industry after a merger, n appears in the arguments of π_M^M and π_i^M because it is an exogenous parameter that represents the *initial* (not current) number of firms in the industry.

[18] Assumption 1 is satisfied in standard Cournot oligopoly models (for example, with linear demand and linear or convex costs).

[19] Since $\pi^M(0, n) = \pi^{\overline{M}}(n-1)$, $\delta^*(n) > 0$ if we assume $\pi^{\overline{M}}(n-1) < 2\pi^{\overline{M}}(n)$. Kamien and Zang (1990) show that this is satisfied if n is sufficiently large (their Lemma A.2) or if demand is weakly concave (Lemma A.4).

Proposition 1. *Any merger inducing a positive synergy is privately profitable, increases consumer surplus, and increases total surplus for n sufficiently large.*[20]

Proof. See Appendix 1.6.1 for all omitted proofs. □

The intuition for this result is as follows. First note that the equilibrium price after a merger must approach c as $n \to \infty$. This is because a limiting price above c would imply an infinite supply by the non-merging firms, contradicting Assumption 1. On the other hand, a limiting price *below* c would imply that all non-merging firms eventually produce nothing (their profit margin would be negative). In this case, however, the merged firm's best response is the monopoly quantity, which implies a price *above* c (recall synergies are non-drastic). Given that the equilibrium price after a merger approaches c as $n \to \infty$, any merger generating a positive synergy must then be profitable if n is sufficiently large. To see this, note that without the merger, the profit of each firm (in a symmetric n-firm Cournot equilibrium) approaches zero as $n \to \infty$. However, if the merger leads to a marginal cost below c, the merged firm eventually makes a positive profit margin, produces a positive quantity, and hence earns a positive profit.

If n is sufficiently large, any merger generating a positive synergy must also increase consumer surplus, or equivalently, lower the equilibrium price. Intuitively, this is due to the fact that the post-merger price approaches c faster than the no-merger price, since the merged firm has a lower cost and hence expands output.[21] Because any merger generating a positive synergy eventually increases consumer surplus when n becomes large enough,

[20]That is, for any $\overline{\delta} > 0$, there exists $\overline{n}_{CS}$ such that $\delta^{CS}(n) < \overline{\delta}$ for all $n > \overline{n}_{CS}$, and similarly for $\delta^{TS}(n)$.

[21]Note that this argument applies to the case of n being sufficiently large but finite. Since total output with and without the merger both approach the same competitive limit as $n \to \infty$, *in the limit* consumer-surplus is unaffected by a merger with a positive but non-drastic synergy.

it must also increase total surplus. This follows from the observation that the aggregate profit of non-merging firms approaches zero, with or without the merger. Thus, because the merger eventually raises the profits of the merging firms, it must increase aggregate profit and hence total surplus for n large enough.

Proposition 1 concerns the effects of mergers when the number of firms initially in the industry is large. It implies that the minimum synergies needed for a merger to increase consumer surplus and total surplus both approach zero as $n \to \infty$. Therefore, regardless of the welfare standard, all proposed mergers should be approved if there are sufficiently many firms. This justifies the leniency with which the FTC and DOJ view horizontal mergers when industry concentration is sufficiently low. However, as the following discussion shows, private profitability and expected welfare may be non-monotone with respect to n when n is smaller.

1.2.3. Linear Demand

To illustrate that the expected change in welfare from approving a merger (with a given δ) is non-monotonic in the number of firms initially in the industry, it suffices to consider the case of linear demand, say $P(X) = a - X$, with $c < a \leq 2c$. As before, firms initially have the same constant marginal cost c, and firms 1 and 2's marginal cost after merging is $c - \delta$. To focus on non-drastic synergies, suppose that $\delta \in [0, \overline{\delta}]$, where $\overline{\delta} \equiv a - c > 0$.

It is straightforward to show that the equilibrium outputs and profits after a merger and after no merger are given by the expressions in Table 1.1.

Firms 1 and 2's profit if they merge is $\pi_M^M(\delta, n)$, while they make $2\pi^{\overline{M}}(n)$ in combined profits if they do not merge. Therefore, they find it profitable to merge if and only if

Table 1.1. Outputs and profits assuming linear demand ($P(X) = a - X$), constant marginal cost (c), and non-drastic synergies

	No merger ($\overline{M}$)	Merger (M)
Output of the merged firm M	–	$x_M^M(\delta, n) = \frac{a-c+(n-1)\delta}{n}$
Output of a non-merging firm $i \in \{3, ..., n\}$	$x^{\overline{M}}(n) = \frac{a-c}{n+1}$	$x_i^M(\delta, n) = \frac{a-c-\delta}{n}$
Total output	$X^{\overline{M}}(n)$ $= nx^{\overline{M}}(n)$ $= \frac{n(a-c)}{n+1}$	$X^M(\delta, n)$ $= x_M^M(\delta, n) + (n-2)x_i^M(\delta, n)$ $= \frac{(n-1)(a-c)+\delta}{n}$
Profit of the merged firm M	–	$\pi_M^M(\delta, n) = \left[\frac{a-c+(n-1)\delta}{n}\right]^2$
Profit of a non-merging firm $i \in \{3, ..., n\}$	$\pi^{\overline{M}}(n) = \left(\frac{a-c}{n+1}\right)^2$	$\pi_i^M(\delta, n) = \left[\frac{a-c-\delta}{n}\right]^2$

$\pi_M^M(\delta, n) > 2\pi^{\overline{M}}(n)$, or equivalently, if and only if

$$\delta > \delta^*(n) \equiv \frac{[(\sqrt{2}-1)n-1]}{n^2-1}(a-c).$$

The graph of $\delta^*(n)$ is shown in Figure 1.1. Note that a merger from duopoly to monopoly is profitable even if there are small cost *increases* associated with the merger, since $\delta^*(2) = \frac{1}{3}(2\sqrt{2}-3)(a-c) < 0$. Salant, Switzer, Reynolds (1983) (henceforth SSR) show that with quantity competition, linear demand and costs, and $n > n^{SSR} \equiv \frac{1}{\sqrt{2}-1} \cong 2.4$ firms, a pair-wise merger is unprofitable if it generates no cost savings. Consistent with this, I find that when merger-induced synergies are possible, $\delta^*(n^{SSR}) = 0$ and $\delta^*(n) > 0$ for all $n > n^{SSR}$. That is, when there are more than n^{SSR} firms in the industry, a pair-wise merger that is profitable must generate synergies.

Furthermore, observe that $\delta^*(n)$ is non-monotonic in n: it is increasing in n for all $n \in (1, n_{\delta^*}^{\max})$ and decreasing in n for all $n \in (n_{\delta^*}^{\max}, \infty)$, where $n_{\delta^*}^{\max} \equiv \frac{1+\sqrt{2(\sqrt{2}-1)}}{\sqrt{2}-1} \cong 4.6$ is the root of the equation $\frac{d}{dn}\delta^*(n) = 0$ that is greater than 1. (In particular, note that $n_{\delta^*}^{\max}$ is independent of (a, c).) This implies that as the initial number of firms in the industry *increases* beyond five, the minimum cost synergy needed for a merger to be profitable decreases, but as n *decreases* below five, the minimum cost synergy needed for a profitable merger also decreases.

The intuition for the non-monotonicity of $\delta^*(n)$ comes from the following observations. When n is small, the increase in market power from a merger is large, and so not as large a synergy is needed in order for the merger to be profitable. A small increase in n causes a relatively large loss in market power, implying that a larger cost reduction is needed for a profitable merger. When n is large, the market power effect of a merger is small, but so are profits without the merger. Therefore, not as large a synergy is needed for the merger to be profitable when n is large.

1.2.3.1. Non-monotone Welfare Effects. When evaluating horizontal merger proposals, the antitrust authority may adopt either a consumer surplus (equivalently, price) standard or a total surplus standard. Let $\delta^{CS}(n)$ denote the minimum cost synergy needed for a merger to increase consumer surplus (or decrease price), and let $\delta^{TS}(n)$ denote the minimum cost synergy needed for a merger to increase total surplus. Assuming demand is linear, it can be shown that[22]

$$CS^M(\delta, n) > CS^{\overline{M}}(n) \iff \delta > \delta^{CS}(n) \equiv \frac{a-c}{n+1},$$

[22]Note that a merger increases consumer surplus, or equivalently increases total output, if and only if the merging firms' combined output expands (and the non-merging firms' combined output contract) after the merger (see Table 1). This also implies that non-merging firms gain from a merger if and only if the merger lowers consumer surplus (i.e., raises price).

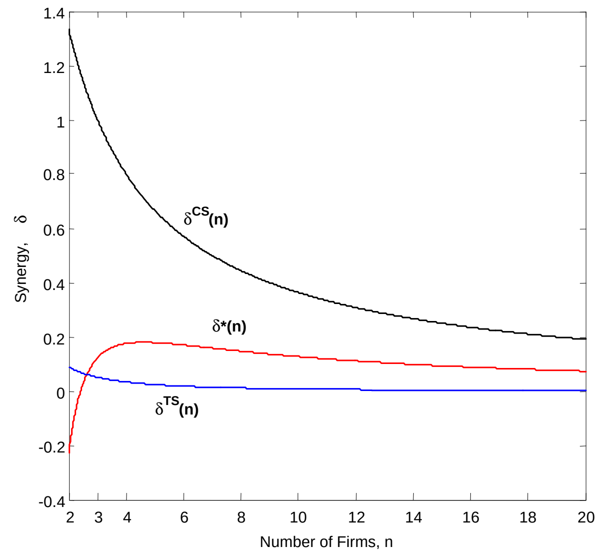

Figure 1.1. Minimum synergies for a merger to be profitable ($\delta^*(n)$), reduce price ($\delta^{CS}(n)$), and increase total surplus ($\delta^{TS}(n)$), when $(a, c) = (10, 6)$.

and[23]

$$\begin{aligned} TS^M(\delta, n) &> TS^{\overline{M}}(n) \iff \delta > \delta^{TS}(n), \text{ where} \\ \delta^{TS}(n) &\equiv \tfrac{-(n+1)^2+\sqrt{(n+1)^4+(2n^2-2n-1)(2n+1)}}{(2n^2-2n-1)(n+1)}(a-c). \end{aligned}$$

Figure 1.1 shows how $\delta^*(n), \delta^{CS}(n)$, and $\delta^{TS}(n)$ vary with the initial number of firms in the industry for the parameters values $(a, c) = (10, 6)$. Observe that while a marginally profitable merger reduces consumer surplus ($\delta^*(n) < \delta^{CS}(n)$), any profitable merger increases total surplus ($\delta^{TS}(n) < \delta^*(n)$) for all $n \geq 3$. Therefore, any merger that increases consumer surplus must also increase total surplus (since $\delta^{CS}(n) > \delta^{TS}(n)$ for all n).

[23] $\delta^{TS}(n)$ is defined as the *positive* value of δ such that $TS^M(\delta, n) = TS^{\overline{M}}(n)$; there is also a negative root which is ignored because no cost-increasing merger is profitable when $n \geq 3$.

It can also been seen from Figure 1.1 that as $n \to \infty$, not only do profitable mergers necessarily increase total surplus, but there are also fewer profitable mergers that would increase price (mergers that are proposed but should not be allowed under a consumer surplus standard). These observations illustrate Proposition 1 and explain why antitrust authorities tend to view proposed horizontal mergers with increased leniency when industry concentration is relatively low.

When there are relatively few firms initially, the following proposition shows that the analysis is more involved. Let $\Delta CS^M(\delta, n) \equiv CS^M(\delta, n) - CS^{\overline{M}}(n)$ denote the change in consumer surplus from approving a merger.

Proposition 2. *The expected change in consumer surplus from approving a proposed merger,*
$E_\delta \left[\Delta CS^M(\delta, n) | \delta > \delta^*(n)\right]$ *, is non-monotonic in* n *for some distributions of* δ*.*

In particular, it is possible that a proposed merger which would raise expected consumer surplus may instead lower it if there is a moderate *increase* in n, i.e., a moderate *decrease* in the pre-merger industry concentration.

To prove this non-monotonicity result, it sufficies to consider the following example. (Appendix 1.6.1 fills in the details of the proof. Also, see Figure 1.2.) Suppose there are two types of mergers. "Good" mergers generate a synergy sufficiently high ($\delta_H > \delta^{CS}(2)$) as to increase consumer surplus, even if they involve a duopoly merging to monopoly. "Bad" mergers, which generate a low synergy ($\delta_L < \delta^*(n_{\delta^*}^{\max})$), would increase consumer surplus only if there are enough firms in the industry ($n > n_3$, so that the negative effect

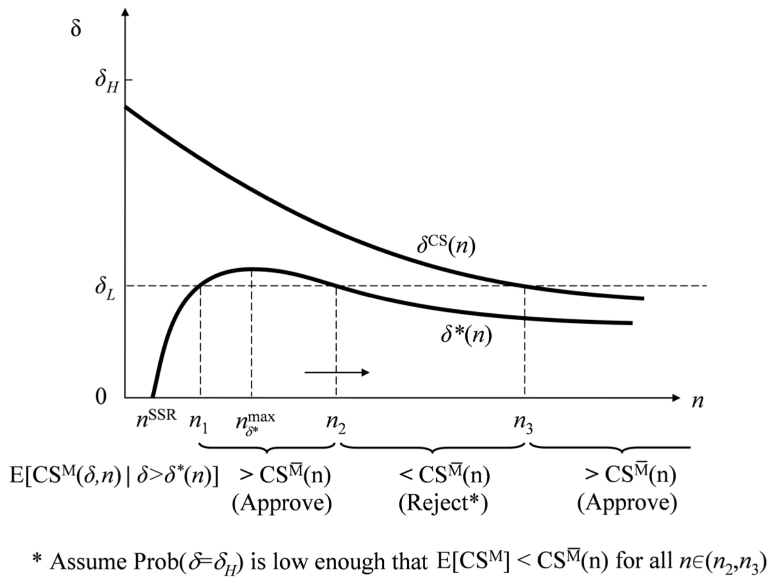

Figure 1.2. A proposed merger would raise expected consumer surplus if there are initially $n \in (n_1, n_2)$ firms in the industry, but may instead lower it if n increases to $n \in (n_2, n_3)$ and the probability of δ_H is sufficiently low. If $n > n_3$, then proposed mergers once again raise expected consumer surplus.

on consumer surplus from increased market power is outweighted by the positive effect of efficiency gains).

First, consider a situation in which there are sufficiently few firms in the industry as to make only good mergers profitable ($n \in (n_1, n_2)$). In this case, expected consumer surplus from approving a *proposed* merger is higher than from rejecting the merger because only good mergers are proposed. All proposed mergers would therefore be approved. Now, consider a moderate increase in the number firms initially in the industry ($n > n_2$), so that bad mergers are also profitable and hence are proposed as well. However, bad mergers would lower consumer surplus if there are not too many more firms than before ($n < n_3$). Hence, if the antitrust authority cannot observe the merger's type and is sufficiently pessimistic about the (unconditional) probability of a merger being of the good type, it will conclude that expected consumer surplus is lower with a proposed merger than without it (when $n \in (n_2, n_3)$). In this case, absent additional observable structural parameters, such as entry costs, the antitrust authority would reject all proposed mergers. Of course, if there are sufficiently many more firms than before ($n > n_3$), bad mergers also increase consumer surplus. When this is true, all proposed mergers should be approved once again.

This example shows that as n increases, a proposed merger's expected impact on consumer surplus, $E_\delta\left[\Delta CS^M(\delta, n) | \delta > \delta^*(n)\right]$, can change from being positive to negative to positive again. The non-monotone sign of this change in expected consumer surplus stems from the fact that when δ is unobservable to the antitrust authority, a change in n induces not only a direct effect on welfare (holding δ fixed), but also an indirect selection effect (via the effect of n on $\delta^*(n)$). The direct effect of an increase in n is always positive because $\delta^{CS}(n)$ is monotonically decreasing in n. However, the selection effect is positive (negative) if and only if $\delta^*(n)$ is increasing (decreasing), or equivalently, if and only if n is sufficiently small (large). As a result, depending on n and the distribution of δ, the overall effect may change signs more than once.

This would not be true if δ were observable to the antitrust authority, for then there would be no selection effect. In this case, the optimal approval policy with a consumer surplus standard is a one-tailed test in n: a proposed merger should be approved if and only if n is sufficiently large (so that $\delta > \delta^{CS}(n)$). Therefore, the non-monotone sign of the expected consumer surplus effect can be attributed to the selection effect, which arises only when δ is unobservable.

With respect to total surplus, $E_\delta\left[\Delta TS^M(\delta,n)|\delta>\delta^*(n)\right]$ is always positive if $n\geq 3$, since any profitable merger increases total surplus for all $n\geq 3$ (see Figure 1.1). Nevertheless, it can be shown with a similar example that the expected *gain* in total surplus from approving a proposed merger may be non-monotonic in n as well. In contrast, if δ were known, the optimal approval policy with a total surplus standard is again a one-tailed test: a proposed merger should be approved if and only if $n\geq 3$ (since $\delta^*(n)>\delta^{TS}(n)$ for all $n\geq 3$).

Finally, it is informative to compare this simple model to that of Farrell and Shapiro (1990). According to their Proposition 5, a profitable merger that raises price would necessarily raise welfare, or total surplus (in their words, have a positive net external effect) if pre-merger market shares satisfy $\sum_{i\in I}s_i<\sum_{i\in O}\lambda_i s_i$, where I and O are, respectively, the set of merging firms ("insiders") and non-merging firms ("outsiders"), $\lambda_i=-\frac{p'(X)+x_ip''(X)}{c^i_{xx}(x_i)-p'(X)}$, and all quantities are evaluated at the pre-merger levels.[24] In my model, an environment with linear demand and linear cost, this condition reduces to $\frac{2}{n}<\sum_{i=3}^n 1\cdot\frac{1}{n}=\frac{n-2}{n}$, or $4<n$. Thus, Farrell and Shapiro's model tells us that $\delta^{CS}(n)>\delta>\delta^*(n)$ implies $\delta>\delta^{TS}(n)$ for $n>4$. In contrast, the above model shows that this is actually true for $n\in(n^o,4)\cong(2.6,4)$ as well. In particular, if the merger is really occurring in order to realize cost savings, and there are initially three firms in the industry, then this model predicts that any profitable merger raises total surplus, whereas Farrell and Shapiro's model would be silent on the welfare effect.

[24]This result assumes $p'',p''',c^i_{xx}\geq 0$ and $c^i_{xxx}\leq 0$, which are all satisfied when demand and costs are linear.

1.3. Mergers with Entry

When an antitrust authority evaluates the costs and benefits of a proposed horizontal merger, it often considers the industry's entry conditions and views ease of entry as a factor mitigating any potential anti-competitive harm that might be caused by the merger. In the U.S., for example, the Horizontal Merger Guidelines (§3) considers the timeliness, likelihood, and sufficiency of entry when evaluating merger proposals. To capture an antitrust authority's reasoning about the effects of post-merger entry on the incentives to merge, and on the merger's likely welfare effects, I now modify the previous model to allow for the possibility of entry after the merger decision is made. The focus will be on the the second of the three entry-related issues identified in the Horizontal Merger Guidelines, namely the likelihood of entry, which will be measured by the size of the sunk cost of entry.

The structure of the game is the same as before, except now a potential entrant, E, decides whether to enter the industry. The potential entrant is assumed to observe firms 1 and 2's synergy draw δ as well as their merger decision and outcome (i.e., whether a proposed merger is approved) before making its entry decision. If it chooses to enter, E must pay an entry cost of $K \in [\pi^{\overline{M}}(n+1), \pi^{\overline{M}}(n)]$ and will have the same cost function as the non-merging incumbents. (Recall that $\pi^{\overline{M}}(n)$ is the per-firm Cournot profit when there are n ex-ante symmetric firms.) Because the n incumbent firms are assumed to have already sunk the entry cost K, the bounds on the possible values of K reflect an implicit assumption that it is an equilibrium for there to be exactly n firms in the industry initially. In other words, $K \geq \pi^{\overline{M}}(n+1)$ because otherwise an $(n+1)$-st firm would have entered the industry, and $K \leq \pi^{\overline{M}}(n)$, because otherwise there would have been fewer than n firms initially in the industry.[25,26] One implication is that because the potential entrant has the same cost structure as the non-merging incumbents, it will not enter if there is no merger, as $K \geq \pi^{\overline{M}}(n+1)$.

The entry cost K is observed by all firms, so when firms 1 and 2 decide whether to merge they know precisely how the entry decision will depend on their merger outcome. The entry cost is also observed by the antitrust authority, but it is unable to predict with certainty whether entry will occur if a merger is permitted, because it does not know the size of the merger synergy δ.[27] (In other words, firms view the entry decision as deterministic, but the antitrust authority views it as random.)

I also modify the previous model to allow for more general cost functions. Specifically, suppose that the $n \geq 3$ firms initially in the industry each has the (variable) cost function $c(x)$, with $c'(x)$ and $c''(x) \geq 0$ for all $x \geq 0$.[28] If firms 1 and 2 merge, their new cost function is $c^M(x, \delta)$, where

$$c^M(x, \delta) \equiv \min_{0 \leq x_1, x_2 \leq x} \left\{ c(x_1) + c(x_2) - \delta x \mid x_1 + x_2 = x \right\}.$$

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[25]Note that none of the results are affected if we instead only assume the lower bound $\pi^{\overline{M}}(n+1)$ on K. This would allow for the possibility that entry costs have increased after they were sunk by the n incumbents, but the incumbents cannot exit in the short-run. As we show below, the potential entrant does not enter (even after a merger) whenever $K \geq \pi_i^{ME}(\delta^{M\overline{E}})$, where $\pi_i^{ME}(\delta^{M\overline{E}}) < \pi^{\overline{M}}(n)$. Therefore, all of the results stated below that apply to the case $K \in [\pi_i^{ME}(\delta^{M\overline{E}}), \pi^{\overline{M}}(n)]$ would also apply for all $K \in [\pi_i^{ME}(\delta^{M\overline{E}}), \infty)$ in the short-run, when exit is not possible.

[26]An implicit assumption with regard to the bounds on the entry cost is that the merger opportunity is unanticipated by the incumbents when they sunk their entry cost.

[27]However, as will be made clear in the following analysis, once the Antitrust Authority is confronted with a proposed merger, it knows that the realization of δ is sufficiently large for the merger to be profitable. In this case, the Antitrust Authority *is* able to perfectly predict that no entry will occur when K is sufficiently large.

[28]Appendix 1.6.3 discusses how the results of this section can be extended to the case of ex-ante asymmetric costs.

In contrast to the previous section, which assumed constant returns to scale technologies, now a merger generating no synergies still involves a (marginal) cost reduction if the production technology satisfies decreasing returns to scale. (It can be verified that $c^M(x,0) = c(x/2)+c(x/2) = 2c(x/2)$, which implies $c^M_x(x,0) = c'(x/2) < c'(x)$.) Furthermore, $c^M_\delta(x,\delta) < 0$ and $c^M_{x\delta}(x,\delta) < 0$ for all $x > 0$ and all $\delta \in (0, \overline{\delta}]$,[29] i.e., a larger synergy δ reduces both cost and marginal cost. Finally, we have $c^M_{xx}(x,\delta) \geq 0$, and assuming the marginal cost of the first unit of output is sufficiently high ($c'(0) > \overline{\delta}$), $c^M_x(x,\delta) \geq 0$ for all $x \geq 0$ and all $\delta \in [0, \overline{\delta}]$ as well.

As before, the inverse market demand function is given by $P(X)$, where X is total output and $P'(X) < 0$ for all X. Following Farrell and Shapiro (1990), I make the following assumption in order to guarantee that the number of firms, n, and merger synergy δ have reasonable effects on firms' output choices and profits.

$$P''(X)x + P'(X) < 0 \text{ for all } x < X, \tag{1.1}$$

Inequality (1.1) implies that each firm's marginal revenue decreases in the output of others, or that reaction curves are downward sloping. It is satisfied, for example, if market demand satisfies $P''(X)X + P'(X) < 0$ for all X. ((1.1) is satisfied if $P(\cdot)$ is concave.) Together, (1.1) and $c''(x) > 0 > P'(X)$ imply that the equilibrium total output increases if and only if, given the pre-merger outputs of the non-merging firms, the merging firms expand output after a merger when there is no entry.[30]

[29] As before, I assume non-drastics synergies, or that δ is sufficiently small so that each non-merging incumbent firm still produces a positive amount of output in equilibrium after a merger. Analogous to footnote 13, $\overline{\delta}$ is implicitly defined by $P(X^m(\overline{\delta})) = c^M_x(X^m(\overline{\delta}), \overline{\delta})$, where $X^m(\delta)$ is the output of a monopolist with cost function $c^M(x,\delta)$.

[30] See Farrell and Shapiro (1990) or Whinston (2006).

Table 1.2. Notation for outputs and profits with potential post-merger entry

	No Merger ($\overline{M}$)	Merger & No Entry ($M\overline{E}$)	Merger & Entry (ME)
Output of the merged firm M	–	$x_M^{M\overline{E}}(\delta)$	$x_M^{ME}(\delta)$
Output of a non-merging firm $i \in \{3, ..., n\}$	$x^{\overline{M}}(n)$	$x_i^{M\overline{E}}(\delta)$	$x_i^{ME}(\delta)$
Output of the entrant	–	–	$x_i^{ME}(\delta)$
Total output	$X^{\overline{M}}(n)$	$X^{M\overline{E}}(\delta)$	$X^{ME}(\delta)$
Profit of the merged firm M	-	$\pi_M^{M\overline{E}}(\delta)$	$\pi_M^{ME}(\delta)$
Profit of a non-merging firm $i \in \{3, ..., n\}$	$\pi^{\overline{M}}(n)$	$\pi_i^{M\overline{E}}(\delta)$	$\pi_i^{ME}(\delta)$
Profit of the entrant	–	–	$\pi_i^{ME}(\delta) - K$

I solve for the subgame perfect equilibrium. The analysis begins with the potential entrant's entry decision conditional on whether firms 1 and 2 have merged and then considers firms 1 and 2's merger decision.

1.3.1. The Entry Decision

First of all, observe that without a merger, the potential entrant would not find it profitable to enter because its entry cost K is least $\pi^{\overline{M}}(n+1)$, i.e., the gross profit from entry after no merger (since the potential entrant is identical to the non-merging incumbents).[31] However, the entrant may find it profitable to enter after a merger if δ and K are not too large. To fix the notation, Table 1.2 lists firms' outputs and profits in each possible outcome. (When there is a merger, the dependence on n is suppressed to simplify the notation.)

With no merger, the outcome is again a Cournot equilibrium with n symmetric firms. After a merger followed by entry, the entrant produces the same amount of output as a

[31] Assume the potential entrant does not enter when it is indifferent between entering and not entering.

non-merging incumbent (because it has the same variable costs) but must incur the entry cost K (which incumbents have already sunk). Therefore, its output and gross profit are indexed by subscript i as well. It can be shown that, *without or without entry, the merged firm's output and profit and total industry output are increasing in* δ, *while each non-merging firm's output and profit are decreasing in* δ. (Appendix 1.6.4 proves these properties in a more general environment where firms have asymmetric pre-merger costs.)

Conditional on a merger occurring, E will enter if and only if its entry cost is less than its gross profit, or equivalently, if the merged firm is not too much more efficient than E :

$$K < \pi_i^{ME}(\delta), \text{ i.e., } \delta < \delta^E(K),$$

where $\delta^E(K)$ is the inverse function of $\pi_i^{ME}(\cdot)$. Because $\pi_i^{ME}(\delta)$ is strictly decreasing, so is $\delta^E(K) \equiv \left(\pi_i^{ME}\right)^{-1}(K)$. If the merged firm is sufficiently efficient, i.e., $\delta > \delta^E(\pi^{\overline{M}}(n+1))$, entry would never be profitable.

Figure 1.3 shows the graph of $\delta^E(K)$, and the values of (δ, K) for which entry occurs or does not occur, conditional on a merger occurring. Notice that after a merger with no synergies ($\delta = 0$), the (gross) profit of a non-merging firm (and the entrant) after entry, $\pi_i^{ME}(0)$, is less than its profit when there is no merger, $\pi^{\overline{M}}(n)$. The reason is that while there are n firms in the industry in both cases, even a merger without synergies lowers the merged firm's marginal cost of producing any given amount of output.[32] The merged firm therefore expands its output, which causes the other $n-1$ non-merging firms to reduce their outputs and earn lower profits.

[32]This assumes pre-merger marginal costs are strictly increasing, or $c_{xx}(\cdot) > 0$. If pre-merger marginal costs are constant, then ex-ante symmetry would imply $\pi_i^{ME}(0) = \pi^{\overline{M}}(n)$. None of the results would be affected in this case.

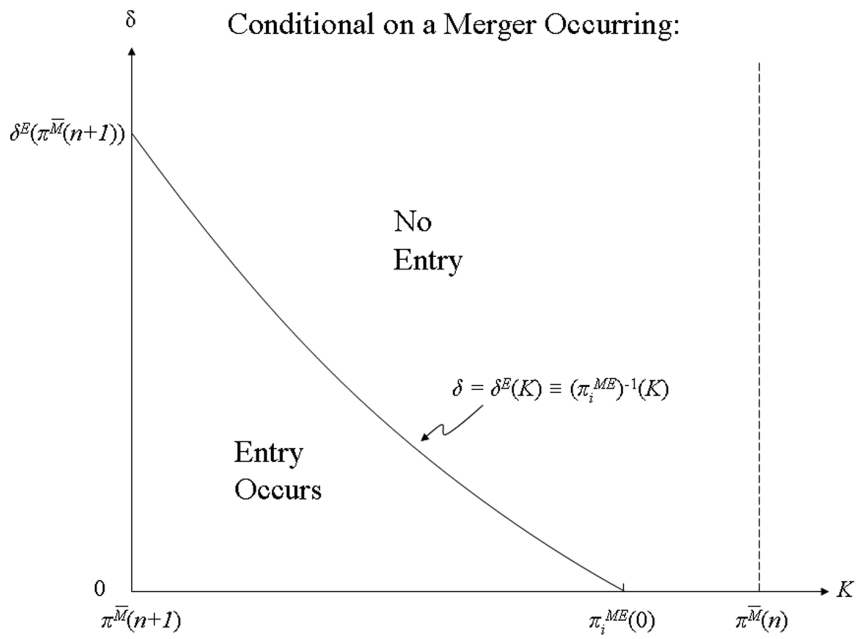

Figure 1.3. Conditional on a merger, entry occurs if $K < \pi_i^{ME}(\delta)$, or $\delta < \delta^E(K)$, and otherwise does not occur.

Finally, it will be important for the following analysis to recall that the potential entrant never enters if there is no merger, and that its (gross) profit from entry is given by $\pi_i^{ME}(\delta)$ only after a merger has occurred.

1.3.2. The Merger Decision

Given a merger synergy δ and entry cost K, define $\pi_M^M(\delta, K)$ to be firms 1 and 2's profit from merging, accounting for the entrant's equilibrium entry decision. Since entry after a merger occurs if and only if $\delta < \delta^E(K)$, we have[33]

$$\pi_M^M(\delta, K) = \begin{cases} \pi_M^{ME}(\delta), \text{ if } \delta < \delta^E(K) \\ \pi_M^{M\overline{E}}(\delta), \text{ if } \delta > \delta^E(K) \end{cases}$$

Let δ^{ME} and $\delta^{M\overline{E}}$ denote the minimum synergies that are needed for a merger to be profitable when entry will, and respectively will not, occur. They are defined by the equations[34]

$$\pi_M^{ME}(\delta^{ME}) = 2\pi^{\overline{M}}(n) \text{ and } \pi_M^{M\overline{E}}(\delta^{M\overline{E}}) = 2\pi^{\overline{M}}(n),$$

because firms 1 and 2's combined profit from not merging is $2\pi^{\overline{M}}(n)$.

In order to rank δ^{ME} and δ^{ME}, I make the following assumption.

Assumption 2. *After a merger, total output is higher, and individual output is lower, with entry than without entry:*

(1.2) $$X^{ME}(\delta) > X^{M\overline{E}}(\delta), \textit{ for all } \delta$$

(1.3) $$x_i^{ME}(\delta) < x_i^{M\overline{E}}(\delta) \textit{ for all } \delta \textit{ and all } i \textit{ (including } i = M).$$

Appendix 1.6.2 shows that with linear and symmetric pre-merger costs, (1.2) is satisfied, and (1.3) is satisfied if demand is weakly concave. This set of assumption implies that the non-merging firms' combined output after a merger is larger with entry than without entry, so that the merged firm's profit is lower with entry than without, for any

[33]Because δ is a continuous random variable, the zero-probability event $\delta = \delta^E(K)$ is ignored.

[34]Assume firms 1 and 2 merge when they are indifferent between merging and not merging.

synergy δ. Thus, because the merged firm's profit is increasing in δ regardless of whether entry occurs,[35]

$$\delta^{ME} > \delta^{M\overline{E}}.$$

In other words, because competition is less intense without entry, a merger does not require as large a synergy in order to be profitable when entry does not occur.

I also make the following assumption to ensure that entry sometimes occurs in equilibrium for sufficiently low entry costs.

Assumption 3. $\pi_i^{ME}(\delta^{ME}) > \pi^{\overline{M}}(n+1)$.

In other words, entry is profitable when it is least costly and the merger synergy makes firms 1 and 2 just indifferent about merging – given that there will be entry. This assumption is equivalent to $\delta^E(\pi^{\overline{M}}(n+1)) > \delta^{ME}$, and it is automatically satisfied, for example, when demand and costs are linear.

The following proposition, and Figure 1.4, illustrate how the equilibrium merger and entry incentives depend on (δ, K), or equivalently, the various possible equilibrium outcomes when all proposed mergers are approved.

Proposition 3. *Equilibrium merger and entry incentives are as follow.*

[35]By definition, $X^{ME}(\delta) = (n-1)x_i^{ME}(\delta) + x_M^{ME}(\delta)$ and $X^{M\overline{E}}(\delta) = (n-2)x_i^{M\overline{E}}(\delta) + x_M^{M\overline{E}}(\delta)$. Thus (1.2) and (1.3) imply $(n-1)x_i^{ME}(\delta) > (n-2)x_i^{M\overline{E}}(\delta)$ for all δ, and so it follows from downward sloping demand that $\pi_M^{M\overline{E}}(\delta) = P(X^{M\overline{E}})x_M^{M\overline{E}} - c^M(x_M^{M\overline{E}}, \delta) \geq P((n-2)x_i^{M\overline{E}} + x_M^{ME})x_M^{ME} - c^M(x_M^{ME}, \delta)$ $> P((n-1)x_i^{ME} + x_M^{ME})x_M^{ME} - c^M(x_M^{ME}, \delta) = \pi_M^{ME}(\delta)$. (The first inequality follows from the merged firm's optimal output choice being $x_M^{M\overline{E}}$, and not x_M^{ME}, when there is no entry.) Since this holds for all δ, we have $\pi_M^{ME}(\delta^{ME}) \equiv 2\pi^{\overline{M}}(n) \equiv \pi_M^{M\overline{E}}(\delta^{M\overline{E}}) > \pi_M^{ME}(\delta^{M\overline{E}})$, which implies $\delta^{ME} > \delta^{M\overline{E}}$ because $\pi_M^{ME}(\delta)$ is increasing in δ.

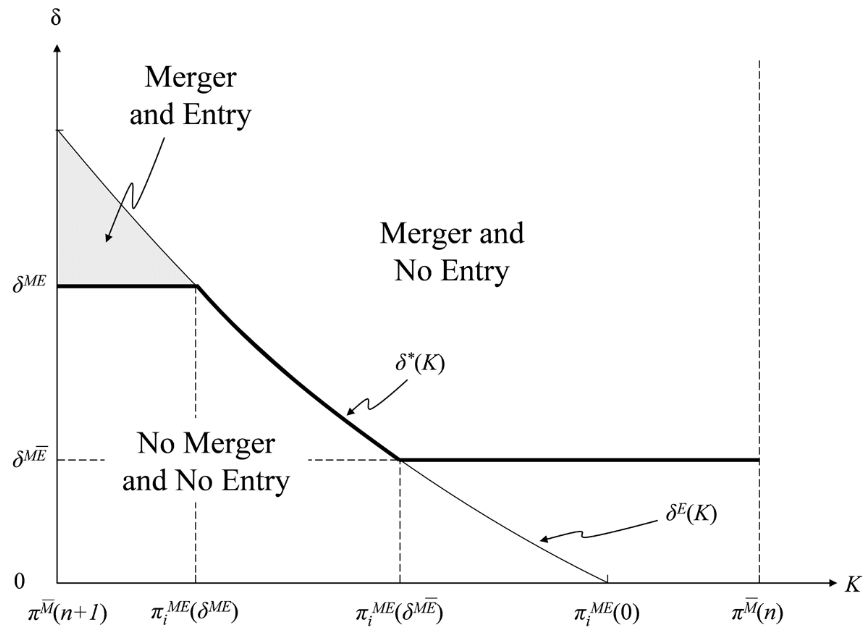

Figure 1.4. A merger is profitable if and only if $\delta > \delta^*(K)$.

(i) When $K \in [\pi^{\overline{M}}(n+1), \pi_i^{ME}(\delta^{ME})]$*, a merger is profitable if and only if* $\delta \geq \delta^{ME}$*, and (post-merger) entry is profitable if and only if* $\delta < \delta^E(K)$*.*

(ii) When $K \in (\pi_i^{ME}(\delta^{ME}), \pi_i^{ME}(\delta^{M\overline{E}}))$*, a merger is profitable if and only if* $\delta \geq \delta^E(K)$*, and entry is never profitable after a (profitable) merger.*

(iii) When $K \in [\pi_i^{ME}(\delta^{M\overline{E}}), \pi^{\overline{M}}(n)]$*, a merger is profitable if and only if* $\delta \geq \delta^{M\overline{E}}$*, and entry is never profitable after a (profitable) merger.*

Proof. (i) First suppose $K \in [\pi^{\overline{M}}(n+1), \pi_i^{ME}(\delta^{ME})]$. If $\delta \geq \delta^{ME}$, the merger is profitable even if it induces entry (by the definition of δ^{ME}). Conversely, if the merger is profitable, $\delta \geq \delta^{ME}$ must hold; otherwise, the merger would induce entry (as $\delta^{ME} \leq \delta^E(K)$ for these values of K) and hence would not be profitable. By definition, (post-merger) entry is profitable if and only if $\delta < \delta^E(K)$.

(ii) Next, consider $K \in (\pi_i^{ME}(\delta^{ME}), \pi_i^{ME}(\delta^{M\overline{E}}))$, where we have $\delta^{M\overline{E}} < \delta^E(K) < \delta^{ME}$. If $\delta \geq \delta^E(K)$, the merger does not induce entry and so it is profitable because $\delta \geq \delta^E(K) > \delta^{M\overline{E}}$ (by the definition of $\delta^{M\overline{E}}$). On the other hand, a profitable merger must involve $\delta \geq \delta^E(K)$; otherwise, $\delta < \delta^E(K) < \delta^{ME}$, which implies the merger would induce entry and hence be unprofitable. Thus, for any K in this range, a profitable merger implies $\delta \geq \delta^E(K)$ and so makes entry unprofitable.

(iii) Finally, suppose $K \in [\pi_i^{ME}(\delta^{M\overline{E}}), \pi^{\overline{M}}(n)]$, so that $\delta^{M\overline{E}} \geq \delta^E(K)$. If $\delta \geq \delta^{M\overline{E}}$, the merger does not induce entry and so is profitable. Conversely, if a merger is to be profitable, δ must be at least $\delta^{M\overline{E}}$ or else the merger would not be profitable even without entry. Since $\delta^{M\overline{E}} \geq \delta^E(K)$, entry is never occurs after a profitable merger. □

Let $\delta^*(K)$ denote *the minimum synergy required for a merger to be profitable when the entry cost is* K :

$$\pi_M^M(\delta^*(K), K) = 2\pi^{\overline{M}}(n).$$

Proposition 3 implies that

$$\delta^*(K) = \begin{cases} \delta^{ME}, & \text{if } K \in [\pi^{\overline{M}}(n+1), \pi_i^{ME}(\delta^{ME})] \\ \delta^E(K), & \text{if } K \in [\pi_i^{ME}(\delta^{ME}), \pi_i^{ME}(\delta^{M\overline{E}})] \\ \delta^{M\overline{E}}, & \text{if } K \in [\pi_i^{ME}(\delta^{M\overline{E}}), \pi^{\overline{M}}(n)] \end{cases}$$

The graph of $\delta^*(K)$ is shown in Figure 1.4. Recall that when there is no merger, entry is never profitable. Therefore, there is no entry in the region where $\delta < \delta^*(K)$, even if $\delta < \delta^E(K)$. According to Werden and Froeb (1998), a merger generating no synergies and inducing entry is not profitable, suggesting that firms merge only if they expect high synergies or high entry cost. The fact that the function $\delta^*(K)$ is (weakly) decreasing in K precisely captures this intuition: *the more costly entry is, the smaller the synergy required for a merger to be profitable.*[36] As observed earlier, this negative selection effect simply reflects the fact that competition is less intense without entry, so not as large a synergy is required for a profitable merger. In particular,

$$\frac{d\delta^*(K)}{dK} = \begin{cases} \frac{d\delta^E(K)}{dK} < 0, & \text{if } K \in (\pi_i^{ME}(\delta^{ME}), \pi_i^{ME}(\delta^{M\overline{E}})) \\ 0, & \text{otherwise.} \end{cases} \tag{1.4}$$

1.3.3. The Effect of Mergers on Consumer Surplus

Antitrust authorities in most countries adopt a welfare standard that is close to consumer surplus when evaluating horizontal mergers. For example, the U.S. Horizontal Merger Guidelines (§4) states: "... the Agency considers whether cognizable efficiencies likely would be sufficient to reverse the merger's potential to harm *consumers* in the relevant market ..." (emphasis added). As noted above, ease of entry is one factor that determines the extent of a merger's anticompetitive effects. Therefore, I begin the welfare analysis by examining how proposed mergers affect expected consumer surplus, and how this depends on post-merger entry conditions.

Assuming for the moment that all profitable mergers occur, Proposition 3 implies that equilibrium total output is given by

$$X^M(\delta, K) \equiv \begin{cases} X^{\overline{M}}, & \text{if } \delta < \delta^*(K) & \text{(no merger)} \\ X^{M\overline{E}}(\delta), & \text{if } \delta \geq \max\{\delta^E(K), \delta^*(K)\} & \text{(merger \& no entry)} \\ X^{ME}(\delta), & \text{if } \delta^*(K) \leq \delta < \delta^E(K) & \text{(merger \& entry)} \end{cases}$$

[36] Note that this result does not dependent on the specific functional forms of demand or cost.

Equilibrium consumer surplus can then be written as

$$CS^M(\delta, K) = \int_0^{X^M(\delta,K)} \left\{P(s) - P[X^M(\delta, K)]\right\} ds,$$

which is the area between the demand curve and equilibrium price, up to equilibrium total output. Define $CS^{ME}(\delta)$, $CS^{M\overline{E}}(\delta)$, and $CS^{\overline{M}}$ analogously. Notice that because demand is downward sloping, consumer surplus is increasing in total output: $\frac{\partial}{\partial X^M} CS^M(\delta, K) = -P'(X^M(\delta, K)) > 0$. Furthermore, since total output is increasing in δ for $\delta > \delta^*(K)$, so is consumer surplus: $\frac{\partial}{\partial \delta} CS^M(\delta, K) = -P'(X^M(\delta, K)) \frac{\partial}{\partial \delta} X^M(\delta, K) > 0, \forall \delta > \delta^*(K)$.

Suppose the antitrust authority adopts consumer surplus, as the welfare criterion by which it evaluates merger proposals. Then it will only approve a proposed merger if, for the given level of entry cost, consumer surplus is not expected to decrease as a result of the merger, or equivalently, if price is not expected to increase. Thus, for each value of K, the antitrust authority calculates the expected consumer surplus from approving the proposed merger:

$$E_\delta[CS^M(\delta, K)|\delta > \delta^*(K)] = \frac{1}{1 - G[\delta^*(K)]} \int_{\delta^*(K)}^{\overline{\delta}} CS^M(\delta, K) g(\delta) d\delta,$$

where $g(\cdot)$ is the probability density function associated with $G(\cdot)$ and has support $[0, \overline{\delta}]$.

Proposition 4. *All else equal, a reduction in the entry cost weakly increases the expected gain in consumer surplus from approving a proposed merger:*

$$\frac{d}{dK} E_\delta[CS^M(\delta, K)|\delta > \delta^*(K)] \leq 0 \quad \textit{for all } K \notin \{\pi_i^{ME}(\delta^{ME}), \pi_i^{ME}(\delta^{M\overline{E}})\},$$

with strict inequality for all $K \in (\pi^{\overline{M}}(n+1), \pi_i^{ME}(\delta^{M\overline{E}}))$.[37]

The intuition for this result can be obtained from Figure 1.4. When K decreases but remains above $\pi_i^{ME}(\delta^{M\overline{E}})$, a profitable merger never induces entry, and the set of mergers that are profitable or proposed does not change. Hence expected consumer surplus remains constant. When K falls below $\pi_i^{ME}(\delta^{M\overline{E}})$ but remains above $\pi_i^{ME}(\delta^{ME})$, a profitable merger still does not induce entry. However, as K decreases in this range, the set of profitable/proposed mergers now involves larger synergies on average. This positive indirect selection effect implies higher expected total output, and hence increases expected consumer surplus. Finally, when K falls below $\pi_i^{ME}(\delta^{ME})$, the set of profitable/proposed mergers remains constant, but the subset of these profitable mergers that induce entry becomes larger. This positive direct effect from the possibility of an additional firm also causes expected consumer surplus to increase.

In other words, a change in the entry cost potentially induces two effects: it could change the set of profitable mergers that induce entry – the direct effect – and it could change the set of mergers that are profitable – the indirect, or selection, effect. Because at most one of these two effects is present for any particular value of the entry cost, the analysis of the overall effect of a proposed merger on consumer surplus is simplified.

[37]The derivative $\frac{d}{dK} E_\delta[CS^M(\delta, K) | \delta > \delta^*(K)]$ is undefined at $K \in \{\pi_i^{ME}(\delta^{ME}), \pi_i^{ME}(\delta^{M\overline{E}})\}$ because the lower limit of integration $\delta^*(K)$ is not differentiable at $K \in \{\pi_i^{ME}(\delta^{ME}), \pi_i^{ME}(\delta^{M\overline{E}})\}$.

Let $\overline{K}$ be the entry cost that makes the antitrust authority indifferent between approving or rejecting a proposed merger:[38]

$$E_\delta[CS^M(\delta,\overline{K})|\delta > \delta^*(\overline{K})] = CS^{\overline{M}}.$$

Then Proposition 4 implies that an antitrust authority using a consumer surplus standard simply has to perform a one-tailed test (assuming $K \geq \pi^{\overline{M}}(n+1)$) to determine whether to approve a proposed horizontal merger: it should approve the merger if and only if the entry cost is less than $\overline{K}$ (but still above $\pi^{\overline{M}}(n+1)$). This provides one possible explanation for why antitrust authorities that are concerned with consumer surplus, e.g., the DOJ and FTC in the U.S., would view proposed horizontal mergers with greater leniency when they believe entry is inexpensive.[39]

1.3.4. The Effect of Mergers on Total Surplus

Although antitrust authorities usually favor a consumer surplus standard when evaluating horizontal mergers, it is nevertheless instructive to study the effect of mergers on aggregate welfare, or total surplus. For example, a total surplus standard is of interest if one wishes to focus exclusively on the issue of allocative efficiency, or if redistribution via means other than antitrust policy (e.g., taxes) is more effective.

[38] Observe that $\pi^{\overline{M}}(n+1) < \overline{K} < \pi^{\overline{M}}(n)$ is satisfied whenever $G(\cdot)$ does not put too much mass on large values of δ. To see this, note that (a) $CS^{ME}(\delta) > CS^{\overline{M}}$ for all δ, while (b) $CS^{M\overline{E}}(\delta) < CS^{\overline{M}}$ if and only if $\delta <\underline{\delta}$ where $\underline{\delta}> \delta^{M\overline{E}}$. Property (b) implies that for $K \geq \pi_i^{ME}(\delta^{M\overline{E}})$, there exists values of $\delta > \delta^{M\overline{E}} \equiv \delta^*(K)$ such that $CS^{M\overline{E}}(\delta) < CS^{\overline{M}}$. To see why Property (b) holds, note that $CS^{M\overline{E}}(\delta)$ increasing in δ, and $CS^{M\overline{E}}(\delta^{M\overline{E}}) < CS^{\overline{M}}$ (since $\delta^{M\overline{E}} \equiv \delta^*(n) < \delta^{CS}(n)$, using the notation of Section 1.2.)

[39] None of the above arguments requires that the antitrust authority be able to observe the precise value of K. To determine whether a proposed merger should be approved, the antitrust authority only needs to know whether K is above or below $\overline{K}$.

As with consumer surplus, it can be shown that total surplus is also increasing in the size of the merger's cost synergy. Therefore, the selection effect on total surplus from increases in the entry cost is (always weakly) negative because proposed mergers involve smaller synergies on average when the entry cost is higher. However, private entry incentives tend to be socially excessive in markets with homogenous products. Thus, in contrast to the finding for consumer surplus, I show that the direct effect on total surplus from a change in the entry cost is non-monotonic. This implies that if it is neither optimal to approve, nor reject, all proposed mergers, then the optimal policy is to approve a proposed merger if and only if the entry cost is neither too high nor to low.

To begin the analysis, observe that equilibrium gross producer surplus, excluding the entry cost, is given by

$$\begin{aligned} PS^{\overline{M}} &= n\pi^{\overline{M}}(n) && \text{(no merger)} \\ PS^{M\overline{E}}(\delta) &= (n-2)\pi_i^{M\overline{E}}(\delta) + \pi_M^{M\overline{E}}(\delta) && \text{(merger \& no entry)} \\ PS^{ME}(\delta) &= (n-1)\pi_i^{ME}(\delta) + \pi_M^{ME}(\delta) && \text{(merger \& entry)} \end{aligned}$$

(Recall that the potential entrant's cost function is identical to that of any non-merging incumbent, and therefore so is its gross profit, $\pi_i^{ME}(\delta)$.) Equilibrium total surplus can then be written as the sum of consumer and producer surpluses, minus the cost of entry when it occurs:

$$TS^M(\delta, K) \equiv \begin{cases} TS^{\overline{M}} = CS^{\overline{M}} + PS^{\overline{M}} & \text{(no merger)} \\ TS^{M\overline{E}}(\delta) = CS^{M\overline{E}}(\delta) + PS^{M\overline{E}}(\delta) & \text{(merger \& no entry)} \\ TS^{ME}(\delta) - K = CS^{ME}(\delta) + PS^{ME}(\delta) - K & \text{(merger \& entry)} \end{cases}$$

Note that since incumbent firms have already sunk their entry costs, net total surplus only accounts for the entrant's entry cost.

Consider first the issue of how private equilibrium entry incentives compare with the social desirability of entry. After a merger, entry is socially efficient if and only if the entry cost is outweighed by the increase in gross total surplus from entry:

$$K < TS^{ME}(\delta) - TS^{M\overline{E}}(\delta).$$

On the other hand, entry is privately profitable (after a merger) if and only if:

$$K < \pi_i^{ME}(\delta).$$

As previously shown by Mankiw and Whinston (1986), there is a tendency toward socially excessive entry in homogenous product markets if a business-stealing effect is present, i.e., if the equilibrium output per firm declines as the number of firms grow.[40] Therefore, it is natural to make the following assumption.

Assumption 4. *Entry is never socially efficient after a merger. That is,*

$$TS^{M\overline{E}}(\delta) > TS^{ME}(\delta) - \pi^{\overline{M}}(n+1) \text{ for all } \delta \in [0, \overline{\delta}]. \tag{1.5}$$

This assumption can be shown to be satisfied, for example, when demand and costs are linear (see Appendix 1.6.1, immediately before the proof of Proposition 5). Notice

[40]This result of Mankiw and Whinston (Proposition 1), which concerns homogeneous product markets, also assumes that (i) firms are symmetric (ii) equilibrium aggregate output is increasing in the number of firms, and (iii) equilibrium price is not below marginal cost for any (finite) number of firms. All of these conditions are satisfied in my model. (Mankiw and Whinston also study the way in which product differentiation can counteract the tendency toward excessive entry.)

that it implies $TS^{M\overline{E}}(\delta) > TS^{ME}(\delta) - K$ for all $K \in [\pi^{\overline{M}}(n+1), \pi^{\overline{M}}(n)]$. In other words, even if the entry cost equals the lowest possible value that is consistent with an n-firm free-entry equilibrium before the merger decision, it would still not be socially desirable for an identical firm to enter the industry after a merger.

Recall that there is never any private incentive for entry if no merger occurs. Nevertheless, would a social planner want the potential entrant to enter when there is no merger? The answer, at least in the case of linear demand and costs, turns out to be "no." (See Appendix 1.6.1. Given that the entrant has yet to sink its entry cost and would be no more efficient than the non-merging incumbents, this result is not surprising at all.) However, when entry is sufficiently inexpensive ($K \leq \pi_i^{ME}(\delta^{ME})$), entry occurs with positive probability in equilibrium after a merger, and so Assumption 4 implies that there will be socially excessive entry in equilibrium.[41]

When total surplus is the welfare standard for evaluating merger proposals, the antitrust authority updates its beliefs as before when confronted with a proposed merger. Namely, it calculates expected total surplus for any proposed merger conditional on the merger being privately profitable. That expected total surplus has the following properties.

Proposition 5. *All else equal, an increase in the entry cost, when it is sufficiently large, does not increase the expected total surplus from approving a proposed merger. That is,* $\frac{d}{dK} E_\delta[TS^M(\delta, K) | \delta > \delta^*(K)] \leq 0$ *for all* $K > \pi_i^{ME}(\delta^{ME}), K \neq \pi_i^{ME}(\delta^{M\overline{E}})$*, with strict inequality for all* $K \in (\pi_i^{ME}(\delta^{ME}), \pi_i^{ME}(\delta^{M\overline{E}}))$*. Furthermore, if Assumption 4 is satisfied,*

[41] The potential entrant's equilibrium decision to not enter is socially efficient (i) when following no merger and (ii) when following a merger if the entry cost is sufficiently large ($K > \pi_i^{ME}(\delta^{ME})$).

the expect total surplus from approving a proposed merger: (i) is increasing *in* K *for* K *sufficiently close to but less than* $\pi_i^{ME}(\delta^{ME})$*, and (ii) achieves a global maximum at* $K = \pi_i^{ME}(\delta^{ME})$.

The intuition when $K \in (\pi_i^{ME}(\delta^{ME}), \pi^{\overline{M}}(n)]$ is the same as for the case of consumer surplus: entry never occurs for these values of K, and the set of profitable/proposed mergers remains the same. Therefore expected total surplus remains constant with respect to K for $K \geq \pi_i^{ME}(\delta^{M\overline{E}})$. When $K \in (\pi_i^{ME}(\delta^{ME}), \pi_i^{ME}(\delta^{M\overline{E}}))$, changes in K continue to have no direct effect because a profitable merger still does not induce entry; but as K decreases, the set of profitable/proposed mergers involves larger synergies on average. I show in Lemma 6 (see Appendix 1.6.1) that total surplus is higher when the merger synergy is larger. (A larger synergy increases total output by causing production to shift from the less efficient non-merging firms to the more efficient merged firm). Thus, lowering the entry cost in the range $(\pi_i^{ME}(\delta^{ME}), \pi_i^{ME}(\delta^{M\overline{E}}))$ has the indirect, and hence overall, effect of increasing expected total surplus.

When K falls below $\pi_i^{ME}(\delta^{ME})$ and continues to decrease, the set of profitable/proposed mergers remains constant, but the subset of these profitable mergers that induce entry becomes larger. Hence, expected consumer surplus increases, as previously shown. Although decreasing K makes entry less costly, the savings in entry cost is proportional to the probability of entry, which, to a first-order approximation, is zero when $K \cong \pi_i^{ME}(\delta^{ME})$. However, because Assumption 4 implies that entry is always socially inefficient after a merger, the overall effect on expected total surplus from a decrease in K is negative for K sufficiently close to (but less than) $\pi_i^{ME}(\delta^{ME})$.

Provided that it is neither optimal to approve, nor reject, all proposed mergers,[42] Proposition 5 implies that the optimal policy with a total surplus standard is to approve a proposed merger if and only if the entry cost is neither too high nor too low.[43] In practice, antitrust authorities usually focus on whether entry is sufficiently easy, and not on whether it is sufficiently difficult. Comparing Propositions 4 and 5 suggests that this is consistent with the common concern about consumer surplus, as opposed to total surplus.

1.4. Connecting the Two Models: Unobservable Entry Costs

In this section, I connect the previous two models by examining the effect of the initial number of firms in a setting in which entry is possible. The previous section focused on how entry costs affect the evaluation of proposed mergers when the number of firms initially present – say, n – is held fixed. Having identified these effects, it is then natural to ask how changes in n further influence the social desirability of proposed mergers.

To address this question, imagine that the antitrust authority only observes n, the number of firms initially in the industry, but does not observe K, the cost of entry. Instead, given the observed n, it updates its beliefs about the likely distribution of K. In particular, the antitrust authority believes that K is drawn from a distribution with support $[\pi^{\overline{M}}(n+1), \pi^{\overline{M}}(n)]$. Therefore, as n increases, the implied entry cost K must approach zero because $\pi^{\overline{M}}(n)$ and $\pi^{\overline{M}}(n+1)$ both approach 0 as $n \to \infty$.

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[42]The assumptions are that $E_\delta[TS^M(\delta, K)|\delta > \delta^*(K)] > TS^{\overline{M}}$ for $K = \pi_i^{ME}(\delta^{ME})$ and $E_\delta[TS^M(\delta, K)|\delta > \delta^*(K)] < TS^{\overline{M}}$ for $K \in \{\pi^{\overline{M}}(n+1), \pi^{\overline{M}}(n)\}$. These assumptions can be shown to hold with linear demand, constant marginal cost, and a beta distribution for the synergy δ, with appropriate choices of the parameters.

[43]That is, approve a proposed merger if and only if K lies in the range $(\underline{K}, \overline{K})$, where $\underline{K}$ and $\overline{K}$ satisfy the equation $E_\delta[TS^M(\delta, K)|\delta > \delta^*(K)] = TS^{\overline{M}}$.

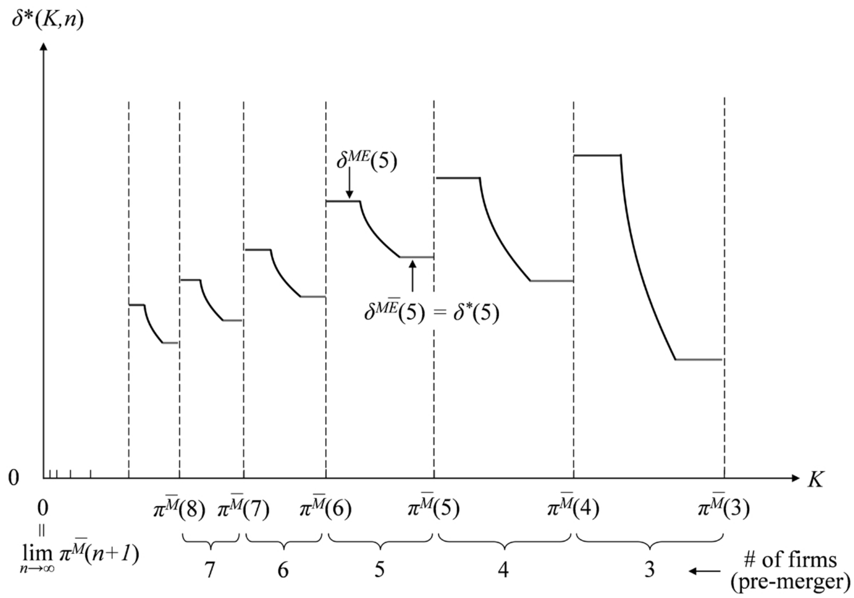

Figure 1.5. For any given n, a merger is profitable if and only if $\delta > \delta^*(K, n)$. The upper and lower bounds of $\delta^*(K, n)$ are given by $\delta^{ME}(n)$ and $\delta^{M\overline{E}}(n)$, respectively, where $\delta^{M\overline{E}}(n) \equiv \delta^*(n)$ by definition.

Recall from Section 1.3 that the minimum synergy needed for a merger to be profitable, $\delta^*(K, n)$ (with the dependence on n now made explicit), has upper and lower bounds given by $\delta^{ME}(n)$ and $\delta^{M\overline{E}}(n)$, respectively, and equals $\delta^E(K, n)$ for intermediate values of K in each interval $[\pi^{\overline{M}}(n+1), \pi^{\overline{M}}(n)]$. Assuming linearity of the demand and costs, it can be verified that:

$$\text{A merger is profitable absent entry} \iff \delta > \delta^{M\overline{E}}(n) \equiv \tfrac{[(\sqrt{2}-1)n-1](a-c)}{n^2-1} (= \delta^*(n))$$

$$\text{A merger is profitable with entry} \iff \delta > \delta^{ME}(n) \equiv \tfrac{(\sqrt{2}-1)(a-c)}{n}$$

$$\text{A merger induces entry} \iff \delta < \delta^{E}(K, n) \equiv a - c - (n+1)\sqrt{K}$$

Figure 1.5 illustrates how $\delta^*(K,n)$ changes as both K and n change. If there are sufficiently many firms initially in the industry, any merger involving a cost synergy ($\delta > 0$) will never induce entry and will be profitable. Entry will never be profitable (even with the lowest possible entry cost, $\pi^{\overline{M}}(n+1)$) because for n sufficiently large, $\delta^E(\pi^{\overline{M}}(n+1), n) = \frac{a-c}{n+2}$ is arbitrarily small. The merger's profitability is eventually guaranteed since the minimum synergy needed for a merger to be profitable, $\delta^*(K,n)$, is arbitrarily small for all $K \in [\pi^{\overline{M}}(n+1), \pi^{\overline{M}}(n)]$. This is because $\delta^{M\overline{E}}(n) \geq \delta^*(K,n) \geq \delta^{ME}(n)$ for all K, with the upper and lower bounds of $\delta^*(K,n)$ both approaching zero as n grows large.

Notice that $\delta^{M\overline{E}}(n) \equiv \delta^*(n)$ by the definition of $\delta^*(n)$ from the model without entry in Section 1.2. So while $\delta^{ME}(n)$, the upper bound of $\delta^*(K,n)$, is monotonically decreasing in n, the lower bound of $\delta^*(K,n)$, i.e. $\delta^{M\overline{E}}(n) \equiv \delta^*(n)$, is at first increasing and then decreasing in n (see Figure 1.5).

For each given n, one can imagine the antitrust authority holding one of the three following beliefs about the likelihood of entry, or equivalently, the distribution of entry costs. (i) The antitrust authority's may be pessimistic about the likelihood of entry and assign probability (approaching) one to $K = \pi^{\overline{M}}(n)$, implying that entry will not occur with certainty.[44] (ii) Alternatively, it may be optimistic about entry and assign probability (approaching) one to $K = \pi^{\overline{M}}(n+1)$, so that entry is believed to be relatively likely (given the distribution of δ, which I assume is independent of K prior to observing a merger proposal). (iii) Finally, the beliefs about the entry cost may be uniform on the interval $[\pi^{\overline{M}}(n+1), \pi^{\overline{M}}(n)]$, reflecting an unbiased prior.

[44]Recall $\pi_i^{ME}(0) < \pi^{\overline{M}}(n)$ so that entry with $K = \pi^{\overline{M}}(n)$ is unprofitable after a merger even if $\delta = 0$

When an antitrust authority holding pessimistic beliefs about entry is confronted with a proposed merger, it will infer that the minimum synergy induced by the merger is at least $\delta^*(\pi^{\overline{M}}(n), n) = \delta^{M\overline{E}}(n) = \delta^*(n)$. In this case, we are back to the first model without entry, and the non-monotone welfare effect identified in Section 1.2.3.1 is still possible for some prior beliefs about δ (that place sufficient weight on low values of δ). On the other hand, if beliefs about the likelihood of entry are always optimistic for any n, then the inference that is drawn from a proposed merger is that δ exceeds $\delta^*(\pi^{\overline{M}}(n+1), n) = \delta^{ME}(n)$, which is monotonically decreasing in n. In this case, the competitive limit result (Proposition 1) from the model without entry still applies.

Finally, if the antitrust authority believes that, for any given n, all possible values of the entry cost that are consistent with an n-firm free-entry equilibrium are equally likely, then $K \sim Unif[\pi^{\overline{M}}(n+1), \pi^{\overline{M}}(n)]$. In this case, when comparing the difference between (i) expected welfare conditional on approving the proposed merger and (ii) welfare without the merger, we must not only integrate over values of $\delta > \delta^*(K, n)$ but also integrate over all values of $K \in [\pi^{\overline{M}}(n+1), \pi^{\overline{M}}(n)]$. If K is uniformly distributed, then we simply have to compare the area of the region in which (i) exceeds (ii) with the area of the region in which (ii) exceeds (i), and approve (reject) the merger whenever the former area is larger (smaller) than the latter. Once again, the competitive limit result (Proposition 1) continues to hold, since $\lim_{n\to\infty} \delta^*(K, n) = 0$. However, the non-monotone welfare effect may re-emerge as well.

1.5. Conclusion

This paper contributes to the analysis of horizontal mergers when an antitrust authority is asymmetrically informed about mergers' efficiency gains, or cost synergies. A merger is privately profitable, and hence proposed, if and only if its synergy is large enough. Hence, upon being confronted with a proposed merger, an antitrust authority updates its prior beliefs about the size of the merger's synergy. Since the minimum synergy needed for merger profitability depends on industry concentration and entry costs, changes in these two parameters will induce changes in the set of mergers that are proposed.

First, I showed that if there are sufficiently many firms initially in the industry, any marginal-cost-reducing merger will be profitable, lower price, and raise total surplus. However, after a moderate increase in the initial number of firms, a merger which would have raised expected consumer surplus may now lower it. This non-monotone change in expected consumer surplus can be attributed to a selection effect that is present when the merger's cost synergy is unobservable to the antitrust authority.

Secondly, I studied how post-merger entry costs affect the set of proposed mergers, and how this selection effect influences entry incentives. Intuition suggests that a decrease in the entry cost has two opposing effects on entry. On the one hand, entry is more likely to occur after a decrease in the entry cost. This change implies that larger synergies are needed for mergers to be profitable and hence proposed. However, if the set of proposed mergers involves more efficient firms on average, then entry becomes *less* likely because competition is fiercer when the potential entrant faces a more efficient merged firm. I showed that for any given entry cost, at most one of these two effects is present in equilibrium.

As a result of this observation, the welfare analysis of horizontal mergers that potentially generate synergies and induce entry is greatly simplified. I showed that expected consumer surplus from approving proposed mergers is (weakly) decreasing in the entry cost. Hence, an antitrust authority using consumer surplus as the welfare standard should approve a proposed merger if and only if the entry cost is sufficiently low. However, if the antitrust authority also takes firms' profits into account and evaluates mergers using a total surplus standard, very easy entry may not be socially desirable either. The intuition is that private entry incentives tend to be socially excessive in homogeneous goods markets, an inefficiency which is exacerbated in this model when a less efficient entrant enters to steal business away from the more efficient merged firm. Thefore, under a total surplus standard, mergers should not be approved if the entry cost is sufficiently high *or* sufficiently low.

Given that the DOJ and FTC are primarily concerned with consumer surplus, as opposed to total surplus, their focus on whether entry is sufficiently ease, and not whether entry is sufficiently difficult, seems justified in light of the predictions of this model.

One limitation of the present analysis is that it abstracts away from considerations of industry dynamics. To deal with such issues, and endogenous mergers in particular, the model in this paper can be imbedded as the stage game of an infinitely-repeated game in which firms can exit as well as enter. Gowrisankaran (1999) considers Markov perfect equilibria in such an environment and fully endogenizes the merger formation process, but it does not deal with the optimal merger enforcement policy. Gowrisankaran

(1997) does address the issue of merger enforcement in such a dynamic framework, but restricts attention to concentration thresholds as the only policy instrument. Although concentration threshold policies are widely used in practice and relatively easy to model, it is not clear why they should fully characterize the optimal merger enforcement rule in a dynamic environment. Therefore, one goal for future research is to characterize the optimal enforcement policy in such a dynamic model.

1.6. Appendix

1.6.1. Omitted Proofs

PROOF OF PROPOSITION 1. First, note that $\lim_{n\to\infty} P(X^M(\delta,n)) = c$. To see this, consider the first order condition of an arbitrary non-merging firm $i \neq M$ after a merger,

$$P'(X^M(\delta,n))x_i^M(\delta,n) + P(X^M(\delta,n)) - c = 0.$$

Since $X^M(\delta,n) = x_M^M(\delta,n) + (n-2)x_i^M(\delta,n)$, this can be rewritten as

$$P'(X^M(\delta,n))\left[X^M(\delta,n) - x_M^M(\delta,n)\right]/(n-2) + \left[P(X^M(\delta,n)) - c\right] = 0.$$

Since $x_M^M(\delta,n) \leq X^M(\delta,n) < \infty$ for all n, the first term goes to zero as $n \to \infty$. Thus, the second term goes to zero as well. Note that $\lim_{n\to\infty} P(X^M(\delta,n)) = c$ is equivalent to $\lim_{n\to\infty} X^M(\delta,n) = X^c$, where X^c is the competitive total output without the merger (in an industry with common marginal cost c), i.e., $P(X^c) = c$.

Next, to show that any merger with a positive synergy is profitable for n sufficiently large, consider the first order condition of the merged firm,

$$P'(X^M(\delta,n))x_M^M(\delta,n) + P(X^M(\delta,n)) - (c-\delta) = 0.$$

Since $\lim_{n\to\infty} P(X^M(\delta,n)) - c = 0$, this implies that the merged firm's output approaches

$$\lim_{n\to\infty} x_M^M(\delta,n) = -\delta/P'(X^c).$$

Thus, the merged firm's profit approaches

$$\lim_{n\to\infty} \pi_M^M(\delta,n) = \lim_{n\to\infty}[P(X^M(\delta,n)) - (c-\delta)]x_M^M(\delta,n) = -\delta\cdot\delta/P'(X^c) > 0.$$

On the other hand, the combined profits of firms 1 and 2 from not merging is $2\pi^{\overline{M}}(n)$, which approaches zero as $n\to\infty$.[45] So for any $\delta > 0, \pi_M^M(\delta,n) > 2\pi^{\overline{M}}(n)$ for all n sufficiently large.

Furthermore, it can be shown that any positive-synergy merger increases consumer surplus, or equivalently, lowers equilibrium price, for n sufficiently large. To see this, note that the first order condition of an arbitrary firm when there is no merger (i.e., in an n-firm symmetric Cournot equilibrium) implies

$$P(X^{\overline{M}}(n)) = c - P'(X^{\overline{M}}(n))X^{\overline{M}}(n)/n.$$

[45] A simpler argument can be used to show $\lim_{n\to\infty}\pi^{\overline{M}}(n) = 0$, which follows from $\lim_{n\to\infty} P(X^{\overline{M}}(n)) = c$ and $\lim_{n\to\infty} x^{\overline{M}}(n) = \lim_{n\to\infty} X^{\overline{M}}(n)/n = 0$.

By summing the first order conditions of all $n-1$ firms after a merger and rearranging, we get

$$P(X^M(\delta, n)) = c - \delta/(n-1) - P'(X^M(\delta, n))X^M(\delta, n)/(n-1).$$

Hence, $P(X^M(\delta, n)) < P(X^{\overline{M}}(n))$ if and only if

$$\delta/(n-1) + P'(X^M(\delta, n))X^M(\delta, n)/(n-1) > P'(X^{\overline{M}}(n))X^{\overline{M}}(n)/n,$$

or

$$\delta + P'(X^M(\delta, n))X^M(\delta, n) > P'(X^{\overline{M}}(n))X^{\overline{M}}(n) \cdot \left(1 - \frac{1}{n}\right).$$

As $n \to \infty$, the second term on the left hand side and the term on the right hand side both approach $P'(X^c)X^c$. Thus, $P(X^M(\delta, n)) < P(X^{\overline{M}}(n))$ holds for n sufficiently large if and only if $\delta \geq 0$, which holds by assumption.

Finally, given that any positive-synergy merger eventually increases consumer surplus when n becomes large enough, it must also increase total surplus. This is because the aggregate profit of non-merging firms approaches zero, with or without the merger. Thus, as the merger eventually raises the profits of the merging firms, it must increase aggregate profit and hence total surplus for n large enough. □

Proof of Proposition 2. To see that the effect of n on expected consumer surplus from approving a proposed merger is not monotonic, it suffices to consider the example that accompanies Figure 1.2 in the main text. Suppose δ can take only two possible values δ_H and δ_L, with probabilities α and $1-\alpha$, respectively, where $\delta_H > \delta_L$. Assume $\delta_H > \delta^{CS}(2)$ so that $\delta_H > \delta^{CS}(n)$ for all $n > 2$, and assume $0 < \delta_L < \delta^*(n^{\max}_{\delta^*})$. Let n_1

and n_2 denote, respectively, the minimum and maximum number of firms (initially in the industry) for which a merger with synergy δ_L is just profitable. In other words, choose n_1 and n_2 so that $\delta_L = \delta^*(n)$ for $n \in \{n_1, n_2\}$; $n_1 < n_{\delta^*}^{\max} < n_2$; and $\delta_L > \delta^*(n)$ for all $n \in (n_1, n_2)$. Let n_3 be the minimum number of firms for which a merger with synergy δ_L will increase consumer surplus, i.e., $\delta_L = \delta^{CS}(n_3)$. Note that $n_3 > n_2$ (since $\delta^{CS}(n_2) > \delta^*(n_2) = \delta_L = \delta^{CS}(n_3)$ and $\delta^{CS}(n)$ is decreasing in n). Therefore, $E_\delta[CS^M(\delta, n)|\delta > \delta^*(n)] > CS^{\overline{M}}(n)$ for all $n \in (n_1, n_2)$ and for all $n \in (n_3, \infty)$.[46] However, if α is sufficiently close to 0, then the oppositive inequality holds when $n \in (n_2, n_3)$.[47] □

Proof of Proposition 4. Because the integrand $CS^M(\delta, K)$ is discontinuous in δ at $\delta = \delta^E(K)$ when $K \in [\pi^{\overline{M}}(n+1), \pi_i^{ME}(\delta^{ME}))$, we split up the region of integration at

[46]When $n \in (n_1, n_2)$, $\delta_H > \delta^*(n) > \delta_L$ and so $E_\delta[CS^M(\delta, n)|\delta > \delta^*(n)] = CS^M(\delta_H, n) > CS^{\overline{M}}(n)$ because $\delta_H > \delta^{CS}(n)$ for all $n > n_1$. When $n \in (n_3, \infty)$, $\delta_H > \delta_L > \delta^*(n)$ and so $E_\delta[CS^M(\delta, n)|\delta > \delta^*(n)] = E_\delta[CS^M(\delta, n)] > CS^{\overline{M}}(n)$ because $\delta_H > \delta_L > \delta^{CS}(n)$ when $n > n_3$.

[47]When $n \in (n_2, n_3)$, we have $\delta_H > \delta_L > \delta^*(n)$ and so $E_\delta[CS^M(\delta, n)|\delta > \delta^*(n)] = E_\delta[CS^M(\delta, n); \alpha] = \alpha CS^M(\delta_H, n) + (1-\alpha) CS^M(\delta_L, n)$, which is less than $CS^{\overline{M}}(n)$ for all $n \in (n_2, n_3)$ if $\alpha < \alpha(n_2)$, where $\alpha(n)$ is defined by $E_\delta[CS^M(\delta, n); \alpha(n)] = CS^{\overline{M}}(n)$. To see this, first note that $CS^M(\delta_L, n) < CS^{\overline{M}}(n) < CS^M(\delta_H, n)$ for all $n \in (n_2, n_3)$, so that $E_\delta[CS^M(\delta, n)]$ is increasing in α (as $CS^M(\delta_H, n) > CS^M(\delta_L, n)$). Furthermore, it can be shown that given linear demand and costs, we have (a) $\frac{\partial^2}{\partial\delta\partial n} CS^M(\delta, n) < 0$, which implies $\frac{\partial}{\partial n} CS^M(\delta_H, n) < \frac{\partial}{\partial n} CS^M(\delta_L, n)$, and (b) $\frac{d}{dn} CS^{\overline{M}}(n) > \frac{\partial}{\partial n} CS^M(\delta, n)$ for all $n \geq 3$ and all δ. Observations (a) and (b) imply $\alpha'(n) > 0$, so that $\alpha(n_2) < \alpha(n)$ for all $n \in (n_2, n_3)$. Hence, if $\alpha < \alpha(n_2)$, then $\alpha < \alpha(n)$ for all $n \in (n_2, n_3)$. Thus, if we assume $\alpha < \alpha(n_2)$, then $E_\delta[CS^M(\delta, n)] < CS^{\overline{M}}(n)$ for all $n \in (n_2, n_3)$. (Note that $\alpha(n_2) > 0$ because $CS^M(\delta_L, n_2) < CS^{\overline{M}}(n)$.)

$\delta^E(K)$ before differentiating. So if $K < \pi_i^{ME}(\delta^{ME})$),

$$\begin{aligned}
\frac{d}{dK}E_\delta[CS^M(\delta,K)|\delta &> \delta^*(K)] = \frac{1}{1-G[\delta^{ME}]}\frac{d}{dK}\left\{\begin{array}{c} \int_{\delta^{ME}}^{\delta^E(K)} CS^{ME}(\delta)g(\delta)d\delta \\ +\int_{\delta^E(K)}^{\overline{\delta}} CS^{M\overline{E}}(\delta)g(\delta)d\delta \end{array}\right\} \\
&= \frac{1}{1-G[\delta^{ME}]}\left\{\begin{array}{c} CS^{ME}(\delta^E(K)) \\ -CS^{M\overline{E}}(\delta^E(K)) \end{array}\right\} g(\delta^E(K))\frac{d\delta^E(K)}{dK},
\end{aligned}$$

which is negative because $\frac{d\delta^E(K)}{dK} < 0$ and (1.2) implies $CS^{ME}(\delta) > CS^{M\overline{E}}(\delta)$ for all δ. On the other hand, if $K > \pi_i^{ME}(\delta^{ME}), K \neq \pi_i^{ME}(\delta^{M\overline{E}})$,

$$\begin{aligned}
\frac{d}{dK}E_\delta[CS^M(\delta,K)|\delta &> \delta^*(K)] = \frac{d}{dK}\left\{\frac{1}{1-G[\delta^*(K)]}\int_{\delta^*(K)}^{\overline{\delta}} CS^{M\overline{E}}(\delta)g(\delta)d\delta\right\} \\
&= \frac{d\delta^*(K)}{dK}\frac{g[\delta^*(K)]}{(1-G[\delta^*(K)])^2}\int_{\delta^*(K)}^{\overline{\delta}}\left\{\begin{array}{c} CS^{M\overline{E}}(\delta) \\ -CS^{M\overline{E}}(\delta^*(K)) \end{array}\right\} g(\delta)d\delta,
\end{aligned}$$

which is negative for all $K \in (\pi_i^{ME}(\delta^{ME}), \pi_i^{ME}(\delta^{M\overline{E}}))$ and zero for all for all $K \in (\pi_i^{ME}(\delta^{M\overline{E}}), \pi^{\overline{M}}(n)]$ because of (1.4) and $\frac{d}{d\delta}CS^{M\overline{E}}(\delta) > 0$. □

Claim 1. *If demand is $P(X) = a - X$ and the symmetric pre-merger marginal cost is c, entry is never socially efficient, with or without a merger. That is, for all $n \geq 3$,*

$$\begin{aligned}
TS^{M\overline{E}}(\delta,n) &> TS^{ME}(\delta,n) - \pi^{\overline{M}}(n+1), \text{ for all } \delta \in [0,a-c], \quad \textit{(Merger)} \\
TS^{\overline{ME}}(n) &> TS^{\overline{M}E}(n) - \pi^{\overline{M}}(n+1), \quad \textit{(No Merger)}
\end{aligned}$$

where $TS^{\overline{ME}}(n)(= TS^{\overline{M}}(n))$ *denotes total surplus when there is no merger and no entry, and* $TS^{\overline{M}E}(n)(= TS^{\overline{M}}(n+1))$ *denotes total surplus (excluding entry cost) when there is no merger but entry occurs anyway.*

Proof. To see the first inequality, $(Merger)$, note that it can be written equivalently as $0 > A(a-c)^2 + B(a-c)\delta + C\delta^2$, for some terms A, B, C that only depend on n.[48] Multiplying by $n^2(n+1)^2(n+2)^2$ shows that this is equivalent to

$$0 > A'(a-c)^2 + B'(n+2)^2(a-c)\delta + C'(n+2)^2\delta^2, \tag{1.6}$$

for some A', B', C' that also only depend on n, where $\lim_{n\to\infty}\left(\frac{A'}{n^4}, \frac{B'}{n^2}, \frac{C'}{n^2}\right) = (-1,-1,1)$.[49] If $\delta < \frac{a-c}{n+2}$, a sufficient condition for (1.6) to hold is $0 > (A'+C')(a-c)^2$ (as $B' < 0$ for all $n \geq 1$), or $0 > A' + C'$. On the other hand, if $\delta \geq \frac{a-c}{n+2}$, a sufficient condition for (1.6) is $0 > (A'+C')(n+2)^2\delta^2$, or once again, $0 > A'+C'$. It is straightforward to verify $0 > A'+C'$ holds for all $n \geq 3$.

Assuming linear demand and costs, the second inequality, $(NoMerger)$, is equivalent to $\left[\frac{a-c}{n+2}\right]^2 > \frac{(n+1)(n+3)(a-c)^2}{2(n+2)^2} - \frac{n(n+2)(a-c)^2}{2(n+1)^2}$, or $0 > -2n^2 - 2n - 1$, which holds for all $n \geq 1$. □

Lemma 6. *Total surplus from a merger is increasing in the synergy, regardless of whether entry occurs. That is,* $\frac{d}{d\delta}TS^{ME}(\delta) > 0$ *and* $\frac{d}{d\delta}TS^{M\overline{E}}(\delta) > 0$.

[48] $A = \frac{\frac{1}{2}n^2+n}{(n+1)^2} - \frac{\frac{1}{2}(n-1)^2+n-1}{n^2} - \frac{1}{(n+2)^2}$, $B = \frac{n+2}{(n+1)^2} - \frac{n+1}{n^2}$, and $C = \frac{n^2+n-\frac{1}{2}}{(n+1)^2} - \frac{(n-1)^2+n-\frac{1}{2}}{n^2}$.

[49] $A' = (\frac{1}{2}n^2+n)n^2(n+2)^2 - (\frac{1}{2}(n-1)^2+n-1)(n+1)^2(n+2)^2 - n^2(n+1)^2 = 6n + \frac{7}{2}n^2 - n^3 - n^4 + 2$, $B' = (n+2)n^2 - (n+1)^3 = -3n - n^2 - 1$, and $C' = (n^2+n-\frac{1}{2})n^2 - ((n-1)^2+n-1-\frac{1}{2})(n+1)^2 = 2n + n^2 + \frac{1}{2}$.

Proof. I prove the first inequality, where there is no entry after the merger ($M\overline{E}$). Assuming the potential entrant has the same cost function as a non-merging incumbent firm, the only modification to the proof that is needed for the case of entry is to change n to $n+1$ everywhere below.

To prove $\frac{d}{d\delta}TS^{M\overline{E}}(\delta) > 0$, it suffices (by continuity) to show that $\Delta TS^{M\overline{E}} \equiv TS^{M\overline{E}}(\delta')-TS^{M\overline{E}}(\delta) > 0$ for some δ and δ' with $\delta' > \delta$. I will drop the superscript $M\overline{E}$ to simplify the notation (e.g., $x_i(\delta) \equiv x_i^{M\overline{E}}(\delta)$). The change in total surplus can be written as the sum of the changes in consumer surplus and total revenue, minus the change in total production costs (ΔTC). That is,

$$\Delta TS^{M\overline{E}} \equiv \int_{X(\delta)}^{X(\delta')} P(t)dt - \Delta TC,$$

where $\int_{X(\delta)}^{X(\delta')} P(t)dt = \int_{\delta}^{\delta'} P(X(s))\frac{dX(s)}{ds}ds$ (by a change of variables), and

$$\begin{aligned}
\Delta TC &= c^M(x_M(\delta'),\delta') - c^M(x_M(\delta),\delta) + (n-2)[c(x_i(\delta')) - c(x_i(\delta))] \\
&= \int_{\delta}^{\delta'} \left\{ c_x^M(x_M(s),s)\tfrac{dx_M(s)}{ds} + c_\delta^M(x_M(s),s) + (n-2)c_x(x_i(s))\tfrac{dx_i(s)}{ds} \right\} ds.
\end{aligned}$$

Note that the merged firm's marginal cost is uniformly lower than that of an arbitrary non-merging firm: for any (x,δ), $c_x^M(x,\delta) < c_x^M(x,0) = c_x(x/2) < c_x(x)$, where the two inequalities follow from $c_{x\delta}^M < 0$ and $c_{xx} > 0$, respectively. Hence, the Cournot equilibrium first order conditions imply that the merged firm's marginal cost is lower than the marginal cost of an arbitrary non-merging, *at their respective equilibrium output* *levels*: $c_x^M(x_M(s),s) < c_x(x_i(s))$ for all synergies s. This means that

$$\begin{aligned}\Delta TC &< \int_{\delta}^{\delta'} \left\{ c_x(x_i(s)) \left[\tfrac{dx_M(s)}{ds} + (n-2)\tfrac{dx_i(s)}{ds} \right] + c_\delta^M(x_M(s),s) \right\} ds \\ &= \int_{\delta}^{\delta'} \left\{ c_x(x_i(s)) \tfrac{dX(s)}{ds} + c_\delta^M(x_M(s),s) \right\} ds,\end{aligned}$$

and therefore

$$\begin{aligned}\Delta TS^{M\overline{E}} &\equiv \int_{\delta}^{\delta'} P(X(s)) \tfrac{dX(s)}{ds} ds - \Delta TC \\ &> \int_{\delta}^{\delta'} \left\{ [P(X(s)) - c_x(x_i(s))] \tfrac{dX(s)}{ds} - c_\delta^M(x_M(s),s) \right\} ds \\ &> 0.\end{aligned}$$

(The integrand above is always positive because (i) $c_\delta^M < 0$, and (ii) the first order condition of a non-merging firm implies $P(X(s)) - c_x(x_i(s)) = -P'(X(s))x_i(s) > 0$.) □

Proof of Proposition 5. If $K \in (\pi_i^{ME}(\delta^{ME}), \pi^{\overline{M}}(n)]$, $\delta > \delta^*(K)$ implies that $TS^M(\delta, K) \equiv TS^{M\overline{E}}(\delta)$. Thus, because of (1.4), and $\frac{d}{d\delta}TS^{M\overline{E}}(\delta) > 0$ (Lemma 6), we have

$$\begin{aligned}\frac{d}{dK} E_\delta[TS^M(\delta,K)|\delta &> \delta^*(K)] = \frac{d}{dK}\left\{ \frac{1}{1-G[\delta^*(K)]} \int_{\delta^*(K)}^{\bar{\delta}} TS^{M\overline{E}}(\delta) g(\delta) d\delta \right\} \\ &= \frac{d\delta^*(K)}{dK} \frac{g[\delta^*(K)]}{(1-G[\delta^*(K)])^2} \int_{\delta^*(K)}^{c} \left\{ \begin{array}{c} TS^{M\overline{E}}(\delta) \\ -TS^{M\overline{E}}(\delta^*(K)) \end{array} \right\} g(\delta) d\delta \\ &\leq 0, \text{ with strict inequality for all } K \in (K^{ME}, K^{M\overline{E}}).\end{aligned}$$

If $K \in [\pi^{\overline{M}}(n+1), \pi_i^{ME}(\delta^{ME}))$, $\delta > \delta^*(K) \equiv \delta^{ME}$ implies

$$\frac{d}{dK}E_\delta[TS^M(\delta, K)|\delta > \delta^*(K)] = \frac{1}{1-G[\delta^{ME}]}\frac{d}{dK}\left\{\begin{array}{c} \int_{\delta^{ME}}^{\delta^E(K)}[TS^{ME}(\delta) - K]g(\delta)d\delta \\ +\int_{\delta^E(K)}^{c} TS^{M\overline{E}}(\delta)g(\delta)d\delta \end{array}\right\}$$

$$= \frac{1}{1-G[\delta^{ME}]}\left\{\begin{array}{c} -\left[G(\delta^E(K)) - G(\delta^{ME})\right] \\ +\left[TS^{ME}(\delta^E(K)) - TS^{M\overline{E}}(\delta^E(K)) - K\right]g(\delta^E(K))\dfrac{d\delta^E(K)}{dK} \end{array}\right\}.$$

As $K \uparrow \pi_i^{ME}(\delta^{ME})$, we have $\delta^E(K) \downarrow \delta^{ME}$,or $G(\delta^E(K)) - G(\delta^{ME}) \downarrow 0$, so that the first term in $\{\cdot\}$ is negative but approaching zero. When Assumption 4 is satisfied, $TS^{ME}(\delta) - TS^{M\overline{E}}(\delta) - K < 0$ for all δ and all K. Since this term and $g(\delta^E(K))\frac{d\delta^E(K)}{dK}$ are both negative and remain bounded away from zero as $K \uparrow \pi_i^{ME}(\delta^{ME})$, the entire second term in $\{\cdot\}$ remains positive and bounded away from zero as $K \uparrow \pi_i^{ME}(\delta^{ME})$. So the whole expression is positive for all $K \in (\pi_i^{ME}(\delta^{ME}) - \varepsilon, K)$, for some $\varepsilon > 0$ sufficiently small.

Finally, because $E_\delta[TS^M(\delta, K)|\delta > \delta^*(K)]$ is decreasing in K for $K > \pi_i^{ME}(\delta^{ME})$ and increasing in K for K sufficiently close to but less than $\pi_i^{ME}(\delta^{ME})$, it attains a local maximum at $K = \pi_i^{ME}(\delta^{ME})$. To see that this local maximum is also a global maximum, it suffices to show that

$$\max_{K<\pi_i^{ME}(\delta^{ME})} E_\delta[TS^M(\delta, K)|\delta > \delta^*(K)] < E_\delta[TS^M(\delta, \pi_i^{ME}(\delta^{ME}))|\delta > \delta^*(\pi_i^{ME}(\delta^{ME}))].$$

This inequality holds because for all $K < \pi_i^{ME}(\delta^{ME})$,

$$\begin{aligned}
E_\delta[TS^M(\delta, K)|\delta &> \delta^*(K)] = \frac{1}{1-G[\delta^{ME}]}\left(\begin{array}{c} \int_{\delta^{ME}}^{\delta^E(K)}[TS^{ME}(\delta)-K]g(\delta)d\delta \\ +\int_{\delta^E(K)}^{c} TS^{M\overline{E}}(\delta)g(\delta)d\delta \end{array}\right) \\
&\leq \frac{1}{1-G[\delta^{ME}]}\left(\begin{array}{c} \int_{\delta^{ME}}^{\delta^E(K)}[TS^{ME}(\delta)-\pi^{\overline{M}}(n+1)]g(\delta)d\delta \\ +\int_{\delta^E(K)}^{c} TS^{M\overline{E}}(\delta)g(\delta)d\delta \end{array}\right) \\
&< \frac{1}{1-G[\delta^{ME}]}\int_{\delta^{ME}}^{\overline{\delta}} TS^{M\overline{E}}(\delta)g(\delta)d\delta \\
&= E_\delta[TS^M(\delta, \pi_i^{ME}(\delta^{ME}))|\delta > \delta^*(\pi_i^{ME}(\delta^{ME}))].
\end{aligned}$$

(The second inequality is due to (1.5).) □

1.6.2. The Effect of Entry on Outputs

This subsection shows that when non-merging firms have linear symmetric costs, (1.2) is satisfied and (1.3) is satisfied if demand is weakly concave.

1.6.2.1. $X^{ME}(\delta) > X^{M\overline{E}}(\delta)$ for all δ (1.2). Inequality (1.2) can be written as $X^{ME}(\delta, n) > X^{M\overline{E}}(\delta, n)$, where the dependence on n, the number of firms *initially* in the industry (not the number of firms *after* the merger and entry decisions), is made explicit. When the potential entrant is identical to the non-merging incumbents, the only difference between $X^{ME}(\delta, n)$ and $X^{M\overline{E}}(\delta, n)$ is that there is one fewer identical firm if entry does not follow the merger. In other words, $X^{M\overline{E}}(\delta, n) = X^{ME}(\delta, n-1)$. Therefore, it suffices to show $\frac{\partial}{\partial n}X^{ME}(\delta, n) > 0$.

Since $X^{ME} = (n-1)x_i^{ME} + x_M^{ME}$, multiplying the first order condition of an arbitrary non-merging incumbent by $n-1$ and adding to it the first order condition of the merged firm implies

$$P'(X^{ME}(\delta,n))X^{ME}(\delta,n)+nP(X^{ME}(\delta,n))-nc-\delta=0.$$

Differentiating with respect to n yields

$$[P''(X^{ME})X^{ME}+(n+1)P'(X^{ME})]\tfrac{\partial}{\partial n}X^{ME}+P(X^{ME})-c=0,$$

or

$$\tfrac{\partial}{\partial n}X^{ME}=-\frac{P(X^{ME})-c}{P''(X^{ME})X^{ME}+(n+1)P'(X^{ME})}.$$

The first order condition of an arbitrary non-merging incumbent implies $P(X^{ME})-c=-P'(X^{ME})x_i^{ME}>0$. The denominator of $\frac{\partial}{\partial n}X^{ME}$ can be written as

$$[P''(X^{ME})x_M^{ME}+P'(X^{ME})]+(n-1)[P''(X^{ME})x_i^{ME}+P'(X^{ME})]+P'(X^{ME}),$$

which is negative because of (1.1) and $P'(\cdot)<0$. Hence $\frac{\partial}{\partial n}X^{ME}>0$.

1.6.2.2. $x_i^{ME}(\delta)<x_i^{M\overline{E}}(\delta)$ for all δ, for all i (including $i=M$) (1.3). The first order conditions of the merged firm when entry does, and does not, occur imply that

$$\begin{aligned}&P'(X^{ME}(\delta,n))x_M^{ME}(\delta,n)+P(X^{ME}(\delta,n))\\=\;&c-\delta=P'(X^{M\overline{E}}(\delta,n))x_M^{M\overline{E}}(\delta,n)+P(X^{M\overline{E}}(\delta,n)),\end{aligned}$$

while the analogous first order conditions of an arbitrary non-merged firm imply

$$\begin{aligned}&P'(X^{ME}(\delta,n))x_i^{ME}(\delta,n)+P(X^{ME}(\delta,n))\\=\;&c=P'(X^{M\overline{E}}(\delta,n))x_i^{M\overline{E}}(\delta,n)+P(X^{M\overline{E}}(\delta,n)).\end{aligned}$$

Hence, because $P'(\cdot) < 0$ and $X^{ME} > X^{M\overline{E}}$,

$$\begin{aligned} & P'(X^{ME}(\delta,n))x_i^{ME}(\delta,n) - P'(X^{M\overline{E}}(\delta,n))x_i^{M\overline{E}}(\delta,n) \\ = \; & P(X^{M\overline{E}}(\delta,n)) - P(X^{ME}(\delta,n)) > 0, \end{aligned}$$

or

$$x_i^{ME}(\delta,n) < \frac{P'(X^{M\overline{E}}(\delta,n))}{P'(X^{ME}(\delta,n))}x_i^{M\overline{E}}(\delta,n), \text{ for all } i \text{ (including } i = M). \tag{1.7}$$

If $P(\cdot)$ is weakly concave, $X^{ME} > X^{M\overline{E}}$ implies $0 > P'(X^{M\overline{E}}) \geq P'(X^{ME})$, or $\frac{P'(X^{M\overline{E}})}{P'(X^{ME})} \leq 1$. Then (1.7) implies $x_i^{ME}(\delta,n) < x_i^{M\overline{E}}(\delta,n)$ for all i (including $i = M$).

1.6.3. Ex-ante Asymmetric Costs

This subsection discusses how the assumption of ex-ante symmetry can be relaxed without affecting the results in Section 1.3. Suppose that each incumbent firm initially has the cost function $c^i(x)$, $i = 1,...,n$. Since firms have different cost functions to start with, they will have different profits as well even absent a merger. Denote by $\pi_i^{\overline{M}}(n)$, for $i = 1,...,n$, the profit of firm i when there is no merger (and no entry), i.e., the profit in an *asymmetric* Cournot equilibrium with n firms.

If firms 1 and 2 merge, their new cost function is:

$$c^M(x,\delta) \equiv \min_{0\leq x_1,x_2\leq x} \left\{c^1(x_1) + c^2(x_2) - \delta x \mid x_1 + x_2 = x\right\}.$$

or $c^M(x,\delta) = c^1(x_1(x)) + c^2(x_2(x)) - \delta x$, where, for any given output level x of the merged firm, the (interior) cost-minimizing production choices $(x_1(x), x_2(x))$ satisfy $c_x^1(x_1(x)) =$

$c_x^2(x_2(x))$. After a merger, the non-merging incumbents' cost functions remain $c^i(x), i = 3, ..., n$. Let the potential entrant E's cost function be $c^E(x)$, and let $\pi_E^{ME}(\delta)$ denote its profit (excluding entry cost) after a merger with synergy δ.

I show in Section 1.6.4 that regardless of whether entry follows a merger, the merged firm's output and profit, and total output are increasing in δ, while the outputs and profits of the non-merging firms (including the entrant) are decreasing in δ. In particular, this implies that the curve $\delta = \delta^E(K)$ in Figure 1.3 is still downward sloping.[50] If (1.2) and (1.3) are satisfied, a merger will continue to be more profitable when followed by no entry than when there is entry, implying that the minimum synergy required for a merger to be profitable, $\delta^*(K)$, is still non-increasing in the entry cost. Therefore, Figures 1.3 and 1.4, and the accompanying arguments, are exactly the same as before, except for the following modifications.

Suppose the potential entrant is less efficient than all the incumbents, i.e.,

$$c^E(x) \geq \max\{c^i(x) | i = 1, ..., n\} \text{ for all } x \geq 0, \tag{1.8}$$

but that it is not "too inefficient" relative to the (equilibrium) efficiency gain of a merger with no synergies, i.e.,

$$c_x^E(x_E^{ME}(0)) < c_x^1(x_1^{\overline{M}}(n)) + c_x^2(x_2^{\overline{M}}(n)) - c_x^M(x_M^{ME}(0), 0). \tag{1.9}$$

[50]Recall that $\delta^E(K)$ represents the largest synergy that induces entry after a merger. The only difference from before is that since $c^E(\cdot) \neq c^i(\cdot)$ implies $\pi_E^{ME}(\delta) \neq \pi_i^{ME}(\delta)$, $\delta^E(K)$ is now defined by the equation $K = \pi_E^{ME}(\delta^E(K))$ (instead of $K = \pi_i^{ME}(\delta^E(K))$).

Then it can be shown that $\pi_E^{ME}(0) < \min_{i=1,...,n} \pi_i^{\overline{M}}(n)$.[51] Thus, in Figures 1.3 and 1.4 $\pi_i^{ME}(0)$ can simply be replaced with $\pi_E^{ME}(0)$ on the horizontal axis.

Because there are exactly n (asymmetric) firms initially in the industry, the entry cost paid by each of the n incumbents must be at least $\min\limits_{i=1,...,n,E} \pi_i^{\overline{M}}(n+1)$, or else the potential entrant E would have already entered (E would be the last to enter since $\pi_i^{\overline{M}}(n+1) > \pi_E^{\overline{M}}(n+1)$ for all $i = 1, ..., n$[52]), and must be at most $\min\limits_{i=1,...,n} \pi_i^{\overline{M}}(n)$, or else one of the n incumbents would never have entered in the first place. In other words, suppose

$$\min_{i=1,...,n,E} \pi_i^{\overline{M}}(n+1) \leq K \leq \min_{i=1,...,n} \pi_i^{\overline{M}}(n)$$

so that it is an equilibrium for there to be exactly n firms initially in the industry. Therefore, if $\pi^{\overline{M}}(n+1)$ is replaced with $\min\limits_{i=1,...,n,E} \pi_i^{\overline{M}}(n+1)$ and $\pi^{\overline{M}}(n)$ is replaced with $\min\limits_{i=1,...,n} \pi^{\overline{M}}(n)$ in the preceeding analysis, then all the previous results of Section 1.3 continue to hold. The only caveat to note is that in order for Proposition 5 to still hold, firms 1 and 2 would have to be not too inefficient before the merger, so that $\frac{d}{d\delta} TS^{M\overline{E}}(\delta) > 0$ (Lemma 6) is still satisfied.[53]

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[51]For any incumbent $i = 1, ..., n$, observe $\pi_i^{\overline{M}}(n) = P(X^{\overline{M}}(n))x_i^{\overline{M}}(n) - c^i(x_i^{\overline{M}}(n)) \geq P(X^{\overline{M}}(n))x_E^{ME}(0) - c^i(x_E^{ME}(0)) > P(X^{\overline{M}}(n))x_E^{ME}(0) - c^E(x_E^{ME}(0)) > P(X^{ME}(0))x_E^{ME}(0) - c^E(x_E^{ME}(0)) = \pi_E^{ME}(0)$, where the first inequality holds because $x_i^{\overline{M}}(n)$ maximizes $\pi_i^{\overline{M}}(n)$ by definition, the second inequality follows from (1.8), and the third inequality is due to $X^{\overline{M}}(n) < X^{ME}(0)$ (which holds if, holding fixed the outputs of firms $i = 3, ..., n$, at their pre-merger levels, the merged firm and the entrant produce more than the pre-merger combined outputs of firms 1 and 2. This, in turn, is implied by (1.9)).

[52]The argument uses (1.8) and is similar to the one in Footnote 51.

[53]The reason is that if firms 1 and 2 pre-merger marginal costs are very high even for small output levels, then their combined pre-merger output is very small. If their cost synergy δ from merging is small enough, then their post-merger output will continue to be small compared to the output of a non-merging firm. In this case, a small increase in δ may lower total surplus because the new equilibrium would involve shifting production from the larger and more efficient non-merging firms to the smaller and less efficient merged firm.

1.6.4. The Effect of δ on Outputs and Profits

In this subsection, I examine the effects of the merger synergy δ on individual and total output, and on individual profit, when firms have ex-ante (pre-merger) cost functions that may be *asymmetric.*

1.6.4.1. Output. $X^{ME}(\delta), X^{M\overline{E}}(\delta), x_M^{ME}(\delta)$, **and** $x_M^{M\overline{E}}(\delta)$ **are increasing in** δ, **while** $x_i^{ME}(\delta)$ **and** $x_i^{M\overline{E}}(\delta)$ **are decreasing in** δ **for all** $i \neq M$. I only prove these claims for the case when there is merger and entry; the proof for the case of merger but no entry follows exactly the same strategy and does not illuminate any additional effects.

When there is a merger and entry, the first order condition of the merged firm's problem can be written as

$$P'(X^{ME}(\delta))x_M^{ME}(\delta) + P(X^{ME}(\delta)) - c_x^M(x_M^{ME}(\delta), \delta) = 0 \text{ for all } \delta.$$

Differentiating with respect to δ implies

$$(P''x_M^{ME} + P')\frac{d}{d\delta}X^{ME} + (P' - c_{xx}^M)\frac{d}{d\delta}x_M^{ME} = c_{x\delta}^M,$$

or

$$\frac{d}{d\delta}x_M^{ME} = \frac{c_{x\delta}^M}{P' - c_{xx}^M} - \frac{P''x_M^{ME} + P'}{P' - c_{xx}^M}\frac{d}{d\delta}X^{ME}. \tag{1.10}$$

The first order condition of an arbitrary non-merging firm (including the entrant E) can be written as

$$P'(X^{ME}(\delta))x_i^{ME}(\delta) + P(X^{ME}(\delta)) - c_x^i(x_i^{ME}(\delta)) = 0, \ i = 3, ..., n, E$$

Differentiating with respect to δ implies

$$[P''x_i^{ME} + P']\tfrac{d}{d\delta}X_M^{ME} + [P' - c_{xx}(x_i^{ME})]\tfrac{d}{d\delta}x_i^{ME} = 0,\ i = 3, ..., n, E$$

or

$$\frac{d}{d\delta}x_i^{ME} = -\frac{P''x_i^{ME} + P'}{P' - c_{xx}^i}\frac{d}{d\delta}X^{ME},\ i = 3, ..., n, E \tag{1.11}$$

Since $X^{ME} = x_M^{ME} + \sum_{i=3,E}^{n} x_i^{ME}$ when entry follows a merger,

$$\frac{d}{d\delta}X^{ME} = \frac{d}{d\delta}x_M^{ME} + \sum_{i=3,E}^{n}\frac{d}{d\delta}x_i^{ME}. \tag{1.12}$$

Substituting equations (1.10) and (1.11) into (1.12) and rearranging,

$$\frac{d}{d\delta}X^{ME} = \left\{1 + \frac{P''x_M^{ME} + P'}{P' - c_{xx}^M} + \sum_{i=3,E}^{n}\frac{P''x_i^{ME} + P'}{P' - c_{xx}^i}\right\}^{-1}\frac{c_{x\delta}^M}{P' - c_{xx}^M}. \tag{1.13}$$

Because of (1.1), $c_{xx}^M > 0, c_{xx}^i > 0 > P'$, and $c_{x\delta}^M < 0$, (1.13) implies $\frac{d}{d\delta}X^{ME} > 0$. Hence (1.11) implies $\frac{d}{d\delta}x_i^{ME} < 0$, ans so $\frac{d}{d\delta}x_M^{ME} > 0$ follows from (1.12).

1.6.4.2. Profit. $\pi_M^{ME}(\delta)$ and $\pi_M^{M\overline{E}}(\delta)$ are increasing in δ, while $\pi_i^{ME}(\delta)$ and $\pi_i^{M\overline{E}}(\delta)$ are decreasing in δ for all $i \neq M$. Once again, I only prove the case of ME; the case of $M\overline{E}$ is similar. By the envelope theorem, we have for all $i = 3, ..., n, E$,

$$\begin{aligned} \tfrac{d}{d\delta}\pi_i^{ME}(\delta) &= \tfrac{d}{d\delta}\left\{P(X^{ME}(\delta))x_i^{ME}(\delta) - c^i(x_i^{ME}(\delta))\right\} \\ &= P'(X^{ME})x_i^{ME}\left[\sum_{\substack{k=3,E;\\ k\neq i}}^{n}\tfrac{d}{d\delta}x_k^{ME} + \tfrac{d}{d\delta}x_M^{ME}\right], \end{aligned}$$

which is negative because $P' < 0$, and $0 < \frac{d}{d\delta}X^{ME} < \sum_{k=3,E;k\neq i}^{n+1} \frac{d}{d\delta}x_k^{ME} + \frac{d}{d\delta}x_M^{ME}$ (since $\frac{d}{d\delta}x_k^{ME} < 0 \forall k \neq M$). Similarly, the envelope theorem implies that

$$\begin{aligned}\tfrac{d}{d\delta}\pi_M^{ME}(\delta) &= \tfrac{d}{d\delta}\left\{P(X^{ME}(\delta))x_M^{ME}(\delta) - c^M(x_M^{ME}(\delta),\delta)\right\} \\ &= P'(X^{ME})x_M^{ME}\sum_{i=3,E}^{n}\tfrac{d}{d\delta}x_i^{ME} - c_\delta^M,\end{aligned}$$

which is positive because $P' < 0$, $\frac{d}{d\delta}x_i^{ME} < 0$, and $c_\delta^M \leq 0$.

CHAPTER 2

Dynamic Contract Breach

2.1. Introduction

Contracts for the provision of services frequently have cancellation fees that penalize the party who backs out before the contract expires or before the date of performance of the contract. For example, vacation resorts often set two separate fees for cancellation of lodging reservations: an early cancellation fee if the reservation is cancelled with sufficient advanced notice, and a late cancellation fee, which is usually larger, if the reservation is cancelled "at the last minute." Furthermore, the difference between the fees for late cancellation and early cancellation is often larger during the high season, when demand is higher. What causes such variations in breach damages with respect to when a breach is signed and when it is breached? This paper proposes a possibile explanation by allowing for the possibility of contract breach and investment at multiple points in time.

Suppose that when the contract is signed, the buyer is uncertain about the value of his outside option at various future points in time and may therefore breach the contract before his performance (payment) is due. When the seller has multiple opportunities over time to make non-contractible, cost-reducing investments that improve her value from trade, she will want to protect the value of those investments by demanding a higher compensation for breach of contract that occurs later, or closer in time to when performance of the contract is due. Therefore, when the buyer decides whether to breach early or late, he must trade off the option value of not breaching early (and waiting for a potentially cheaper supplier to arrive later) versus the higher penalty associated with later breach.

The law and economics literature on contract breach began by considering the efficiency of standard court-imposed damage measures in a setting where the buyer faces an alternative source of supply that is competitively priced. In particular, Shavell (1980) and Rogerson (1984) considered, respectively, the situations where the incumbent seller and buyer cannot and can renegotiate their initial contract. The common finding in both cases is that standard court-imposed damages generally induce socially excessive investment.

The efficiency of privately stipulated, or liquidated, damages for breach of contract has also been previously addressed, notably by Aghion and Bolton (1987) (assuming no investment or renegotiation), Chung (1992) (allowing for investments but not renegotiation), and Spier and Whinston (1995) (assuming both investments and renegotiation). The common focus of these papers is on the strategic stipulation of socially excessive breach damages when the entrant seller has market power, i.e., when the incumbent seller and buyer's original contract imposes externalities on third parties.[1]

In contrast, I assume that third parties have no bargaining power in their dealings with the incumbent seller and buyer. Instead, the key innovation of this paper is the existence of two potential entrants, and hence multiple opportunities for breach of the

[1]Most of the literature on contract damages, including this paper and those cited above, assumes investments are selfish in that they only directly affect the investing party's payoffs. Che and Chung (1999), however, assume cooperative investments, which directly affect the payoffs of the non-investing party. They show that the relative social desirability of expectation damages, liquidated damages, and reliance damages are different when investments are cooperative instead of selfish.

original contract. Section 3.2 introduces the rest of the model in detail, and Section 2.3 characterizes the ex-ante efficient breach and investment decisions.

In the event of breach, *expectation damages* compensate the breached-against party (in this case, the seller) for the profit that she would have made had breach not occurred, given her *actual* investment decision. By comparison, *efficient* expectation damages compensate the breach-against party for the profit she would have made absent breach – had she chosen the *efficient* investment level. First, assuming renegotiation is not possible, I demonstrate in Section 2.4 that the incumbent parties can implement the efficient breach and investment decisions in both periods by stipulating the efficient expectation damages in their initial contract. This result can be viewed as an extention to multiple periods of the well-known result that the efficient expectation damage is socially efficient when renegotiation is not possible.[2] Furthermore, the efficient expectation damage for late breach exceeds that for early breach.

In a related paper, Chan and Chung (2005) also look at a two-period model of contract breach with sequential investment opportunities. They focus on standard court-imposed breach rememdies and do not allow for renegotiation. In contrast, the main motivation of this paper is to provide explanations for why privately stipulated damages might increase over time as the date of performance approaches. Another related paper is Triantis and Triantis (1998), which studies a continuous time model of contract breach but *assumes* that breach damages are increasing over time. The present paper can be viewed as providing a framework that justifies such an assumption when damages are privately stipulated.

[2]See, for example, Chung (1992) and the references therein.

Another novel feature of this model is the possibility that the seller may find an alternative buyer when the incumbent buyer breaches early but not when he breaches late.[3] In this case, contract law would require the seller to take reasonable measures to reduce, or mitigate, the damages that are owed to her for early breach. Since these damages are decreasing in the probability of trading with an alternative buyer, mitigation in this setting entails efforts to increase this probability of alternative trade. Section 2.5 endogenizes this probability of trading with an alternative buyer and compares the private and social incentives for mitigation of damages. It is shown that unless the incumbent seller has complete bargaining power vis-a-vis the alternative buyer, her private incentives for mitigation are socially insufficient, leading to suboptimal mitigation efforts. However, this result crucially depends upon the implicit assumption that breach is defined as only a function of whether the incumbent buyer refuses trade, or delivery of the good (as opposed to being also a function of whether the incumbent seller is able to trade with an alternative buyer).

Next, I assume in Section 2.6 that the incumbent buyer and seller are able to renegotiate their original contract after the arrival of each perfectly competitive entrant. It is shown that if the incumbent seller has complete bargaining power with the alternative buyer (so that externalities are absent), socially efficient breach and investment decisions can still be implemented with the same contract that induces efficient decisions when renegotiation is not possible. Thus, this paper contributes to the literature on contract

[3]For example, there may be insufficient time to find an alternative buyer if breach occurs late. The qualitative results would continue hold if the probability of finding an alternative buyer upon late breach is positive but still less than the same probability given early breach.

breach by demonstrating that, absent externalities, efficient expectation damages are socially optimal even if breach and renegotiation are possible at multiple points in time.

Finally, 2.7 considers an application of the no-renegotiation version of the model to the lodging industry, and in particular, vacation resorts' policies regarding cancellation of lodging reservations. The model predicts that a resort's opportunity cost of honoring a reservation beyond the early cancellation opportunity is increasing in the likelihood of finding an alternative guest in case early cancellation occurs. Therefore, we should expect the amount by which the late cancellation fee exceeds the early cancellation fee to be larger during periods of high demand than during periods of low demand.

Section 2.8 briefly concludes.

2.2. A Model with Multiple Breach Opportunties

Consider a contract between a buyer and a seller to exchange one unit of an indivisible good or service. The buyer's value for the good, v, is commonly known to both parties.[4] The seller can make sequential cost-reducing investments of r_1 and r_2 to improve her value from trade with the buyer. After the original seller makes each investment r_i, another seller observes her own production cost c_{Ei} and announces a price p_{Ei} that she will charge the buyer if the buyer breaches his contract with the incumbent seller and buys from her, the entrant seller, instead.[5] I study the case where the buyer has all the bargaining

[4]Stole (1992) argues that when the parties are asymmetrically informed, liquidated damages not only provide incentives for efficient breach, but also serve to efficiently screen among different types of buyers and sellers.

[5]Fixed costs of entry for the entrants are not explicitly modeled. Each of them simply observes her production cost and then costlessly shows up to announce a price.

power when dealing with the entrants, so that each entrant sets her price equal to her cost, $p_{\mathrm{Ei}} = c_{\mathrm{Ei}}$, and behaves as if she were perfectly competitive.[6]

The buyer has two opportunities to breach his contract with the incumbent seller: once after each entrant seller arrives and announces p_{Ei}. The entrant's price p_{Ei} and the incumbent's investments r_i are observable by all parties but not verifiable. For now, assume the incumbent seller and buyer cannot renegotiate their contract after each entrant's announcement of p_{Ei} (I examine the case where renegotiation is possible in Section 2.6). So the model is essentially the stage game of Spier and Whinston (1995) repeated twice, with perfectly competitive entrants and with the following additional modification. I assume that if the original buyer breaches early, i.e., immediately after the first entrant sets her price, then with probability θ the seller is able to find an alternative buyer who has the same value v for the good and is charged a price p' by the seller. (Except for the discussion on mitigation of damages in Section 2.5, I will assume throughout the rest of this paper that $p' = v$, so that the alternative buyer has no bargaining power with respect to the incumbent seller.) If the original buyer breaches late, i.e., after the second entrant announces her price, the seller cannot find an alternative buyer. For example, it may be the case that the incumbent seller requires sufficient time to have a chance of finding an alternative buyer.

Because the buyer will have two opportunities to breach, the seller specifies in the contract two liquidated damages, x_1 and x_2, where the buyer must pay x_i to the seller if he cancels the contract after the seller has made her investment r_i. If the buyer never

[6]If an entrant has some bargaining power with respect to the buyer, the damage for breach that the buyer must incur if he were to buy from her would still constrain the entrant's price choice. Since the entrant would make positive profits if she sells to the buyer in this case, the incumbent seller can use (socially excessive) stipulate breach damages to extract surplus from the entrant. See Spier and Whinston (1995).

breaches the contract and buys from the incumbent seller, the only payment that he makes to the seller is a price p, which is paid when the contract is performed in the last period (when the buyer accepts delivery of the good from the seller). In this case, the seller's investment costs are $r_1 + r_2$ and her production cost is $c(r_1, r_2)$, where $c(\cdot, \cdot)$ is strictly decreasing and strictly convex in r_1 and r_2 for all $(r_1, r_2) \gg 0$.[7] I will refer to r_1 as the early investment and r_2 as the late investment. In the event that early breach occurs, $r_2 = 0$.

To summarize, the sequence of events, shown in Figure 2.1 for the case when renegotiation is impossible, is as follows.

t=0 Seller S offers a contract (p, x_1, x_2) to Buyer B. If B rejects, both parties receive a payoff of zero and the game ends. If B accepts, the game continues.

t=1.1 S makes a non-contractible *early investment* $r_1 \geq 0$ to reduce her production costs.

t=1.2 Nature draws Entrant seller E1's cost c_{E1} from a distribution $F(\cdot)$ with support $[0, v]$, and E1 chooses her price p_{E1}.

t=1.3 B decides whether to *breach early* and buy from E1. The cost of the first investment, r_1, is a sunk cost for S at this point, but if B breaches early, S incurs production costs $c(r_1, 0)$ only if she finds an alternative buyer (which occurs with probability θ). Therefore, payoffs for the incumbent buyer, incumbent seller, the

[7]While no functional form assumptions are made with respect to how the seller's production costs depend on her investments, it is assumed that these investments are selfish in the sense that they do not directly affect the *buyer's* payoff.

first entrant, and the alternative buyer in the case of early breach are, respectively,

$$u_B = v - p_{E1} - x_1, \quad u_S = x_1 - r_1 + \theta\left[p' - c(r_1, 0)\right], \quad u_{E1} = p_{E1} - c_{E1}, \quad u_{AB} = \theta[v - p'].$$

The game ends after an early breach. If B does not breach early, $u_{E1} = u_{AB} = 0$ and the game continues.

t=2.1 S makes a non-contractible, relationship-specific *late investment* $r_2 \geq 0$ to further reduce her production costs.[8]

t=2.2 Nature draws Entrant seller E2's cost c_{E2} from $F(\cdot)$, independent of c_{E1}, and E2 chooses her price p_{E2}.[9]

t=2.3 B decides whether to *breach late* and buy from E2. Because I assume that S is unable to find an alternative buyer if breach occurs late, payoffs for the buyer, incumber seller, and second entrant in the case of B breaching late are, respectively,

$$u_B = v - p_{E2} - x_2, \quad u_S = x_2 - r_1 - r_2, \quad u_{E2} = p_{E2} - c_{E2}.$$

If B does not breach, payoffs are

$$u_B = v - p, \quad u_S = p - c(r_1, r_2) - r_1 - r_2, \quad u_{E2} = 0.$$

[8]The seller's late investment r_2 is relationship-specific because it does not improve the her payoff at all if the incumbent buyer breaches late. In contrast, S's early investment r_1 is not completely relationship-specific because it reduces her cost of selling to the alternative buyer, if one is found.

[9]The analysis would clearly be the same if we assumed that there is only one entrant who takes another independent draw of his cost if the buyer does not buy from her at time t=1.3.

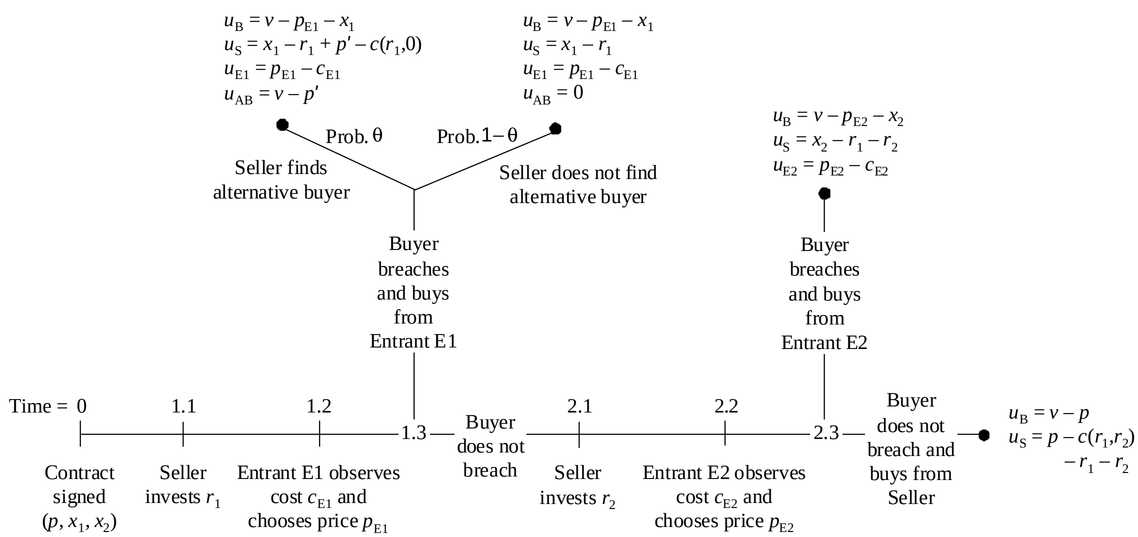

Figure 2.1. Timeline and payoffs when renegotiation is not possible.

2.3. Efficient Investment and Breach

As a benchmark, I identify the investment and breach decisions that maximize expected social surplus, or the sum of payoffs for all parties. Let r_1^* and $r_2^*(r_1^*)$ denote the (ex-ante) efficient investments for the seller.

Proceeding in reverse chronological order, I first characterize the buyer's efficient late breach decision. Assuming no early breach and investments r_1 and r_2, the social surplus (i.e., the sum of payoffs for B, S, and E2) is $v - c_{E2} - r_1 - r_2$ if B breaches and $v - c(r_1, r_2) - r_1 - r_2$ if B does not breach. Thus, given investment levels r_1 and r_2 and no early breach, social surplus is maximized when B breaches late if and only if potential entrant E2 can produce the good at a lower cost than the incumbent seller:

$$c_{E2} \leq c(r_1, r_2). \tag{2.1}$$

In particular, because all investment costs are sunk, they do not have any direct effect on the efficient late breach decision. However, investments indirectly affect the late breach decision through their effects on the seller's production costs.

Next, consider the seller's efficient late investment, $r_2^*(r_1)$, which by definition maximizes expected social surplus given early investment r_1, no early breach, and late breach occurring if and only if $c_{E2} \leq c(r_1, r_2)$. In other words, $r_2^*(r_1)$ is the solution to the problem

$$\max_{r_2 \geq 0} S(r_2|r_1) = \begin{cases} \int_0^{c(r_1,r_2)} [v - c_{E2} - r_1 - r_2] f(c_{E2}) dc_{E2} \\ + \int_{c(r_1,r_2)}^{v} [v - c(r_1, r_2) - r_1 - r_2] f(c_{E2}) dc_{E2}. \end{cases}$$

The seller's efficient late investment $r_2^*(r_1)$, assuming it is positive, is characterized by the first order condition

$$1 = -c_2(r_1, r_2^*(r_1))(1 - F[c(r_1, r_2^*(r_1))]). \tag{2.2}$$

This condition requires that, at its efficient level, the marginal cost of increasing r_2 should equal the expected marginal benefit of increasing r_2, which is the cost reduction from increasing r_2 multiplied by the probability that the cost reduction will be realized (i.e., the probability of late breach not occurring, conditional on early breach not occurring).

Now consider the efficient early breach decision. Social surplus from early breach is $v - c_{E1} - r_1 + \theta[v - c(r_1, 0)]$. Given that the late breach decision is efficient (follows (2.1)) and late investment is efficient (as characterized by (2.2)), expected social surplus from not breaching early is

$$S(r_2^*(r_1)|r_1) = v - F[c(r_1, r_2^*(r_1))]E[c_{E2}|c_{E2} \leq c(r_1, r_2^*(r_1))] \\ - (1 - F[c(r_1, r_2^*(r_1))])[c(r_1, r_2^*(r_1))] - r_1 - r_2^*(r_1).$$

Thus, it is efficient for B to breach early if and only if $v - c_{E1} - r_1 + \theta[v - c(r_1, 0)] \geq S(r_2^*(r_1)|r_1)$, or

$$c_{E1} \leq c^*(r_1) + r_2^*(r_1) + \theta[v - c(r_1, 0)], \tag{2.3}$$

where

$$c^*(r_1) \equiv F[c(r_1, r_2^*(r_1))]E[c_{E2}|c_{E2} \leq c(r_1, r_2^*(r_1))] \\ +(1 - F[c(r_1, r_2^*(r_1))])c(r_1, r_2^*(r_1)) \tag{2.4}$$

is the expected continuation production cost given r_1, and efficient late investment and efficient late breach. So breaching early is efficient if and only if the first entrant's cost, c_{E1}, is lower than the expected *social* cost of continuing with the incumbent seller, given efficient investments and efficient late breach. In other words, in order for the buyer's early breach decision to be efficient, his total expected continuation cost must include not only his private expected continuation cost $c^*(r_1)$, but also internalize the additional investment cost $r_2^*(r_1)$ that the seller will incur once early breach is foregone, as well as the lost expected surplus $\theta[v - c(r_1, 0)]$ that would have been realized had the seller been given the opportunity to find an alternative buyer.

Finally, given the seller's efficient late investment and the buyer's efficient breach decisions as described above, the seller's efficient early investment, r_1^*, should maximize the ex-ante expected social surplus:

$$\max_{r_1 \geq 0} S(r_1) \tag{2.5}$$

$$= \left\{ \begin{array}{c} \int_0^{c^*(r_1)+r_2^*(r_1)+\theta[v-c(r_1,0)]} \{v - c_{E1} - r_1 + \theta[v - c(r_1,0)]\} f(c_{E1}) dc_{E1} \\ + \int_{c^*(r_1)+r_2^*(r_1)+\theta[v-c(r_1,0)]}^{v} \{v - c^*(r_1) - r_2^*(r_1) - r_1\} f(c_{E1}) dc_{E1} \end{array} \right\}$$

$$\iff \max_{r_1 \geq 0} \left\{ \begin{array}{c} v - r_1 + \int_0^{c^*(r_1)+r_2^*(r_1)+\theta[v-c(r_1,0)]} \{-c_{E1} + \theta[v - c(r_1,0)]\} f(c_{E1}) dc_{E1} \\ + \int_{c^*(r_1)+r_2^*(r_1)+\theta[v-c(r_1,0)]}^{v} \{-c^*(r_1) - r_2^*(r_1)\} f(c_{E1}) dc_{E1} \end{array} \right\}$$

In the first version of this problem, the two integrals represent the expected social surpluses when early breach is efficient and when not breaching early is efficient, respectively. The seller's efficient early investment r_1^*, assuming it is positive, can be characterized by the first order condition

$$\begin{aligned} 1 &= -c_1(r_1^*, 0)\theta F[c^*(r_1^*) + r_2^*(r_1^*) + \theta(v - c(r_1^*, 0))] \\ &\quad - \frac{d}{dr_1}[c^*(r_1^*) + r_2^*(r_1^*)]\{1 - F[c^*(r_1^*) + r_2^*(r_1^*) + \theta(v - c(r_1^*, 0))]\}. \end{aligned} \tag{2.6}$$

Because (2.2) implies $\frac{d}{dr_1}[c^*(r_1^*) + r_2^*(r_1^*)] = c_1(r_1^*, r_2^*(r_1^*)) \{1 - F[c(r_1^*, r_2^*(r_1^*))]\}$, (2.6) can be rewritten as

$$(2.7) \quad 1 = -c_1(r_1^*, 0) \cdot \theta F[c^*(r_1^*) + r_2^*(r_1^*) + \theta(v - c(r_1^*, 0))] \\ -c_1(r_1^*, r_2^*(r_1^*)) \cdot \left\{ 1 - F \begin{bmatrix} c^*(r_1^*) + r_2^*(r_1^*) \\ +\theta(v - c(r_1^*, 0)) \end{bmatrix} \right\} \{1 - F[c(r_1^*, r_2^*(r_1^*))]\}$$

Equation (2.7) states that in order for early investment r_1^* to be efficient, its marginal cost must equal its expected marginal benefit. When the buyer (efficiently) breaches early and an alternative buyer is found, an event which occurs with probability $\theta F[c^*(r_1^*) + r_2^*(r_1^*) + \theta(v - c(r_1^*, 0))]$, the marginal benefit of early investment r_1^* is a reduction of the seller's production cost by the amount $-c_1(r_1^*, 0)$. When the buyer (efficiently) never breaches and buys from the incumbent seller, which occurs with probability $(1 - F[c^*(r_1^*) + r_2^*(r_1^*) + \theta(v - c(r_1^*, 0))])\{1 - F[c(r_1^*, r_2^*(r_1^*))]\}$, the marginal benefit of early investment r_1^* is a reduction of the production cost by the amount $-c_1(r_1^*, r_2^*(r_1^*))$. Note that when $\theta = 0$, so that there is no possibility of finding an alternative buyer even if early breach occurs, (2.7) reduces to

$$1 = -c_1(r_1^*, r_2^*(r_1^*))(1 - F[c^*(r_1^*) + r_2^*(r_1^*)]) \{1 - F[c(r_1^*, r_2^*(r_1^*))]\},$$

where the right hand side is the reduction in production cost that results from investment r_1^*, multiplied by the probability that this benefit will actually be realized, i.e., the probability that breach never occurs.

Proposition 7. *The incumbent seller's efficient investments, r_1^* and $r_2^*(r_1^*)$, are characterized by (2.6) and (2.2), respectively. The buyer's efficient breach decision is to breach early if and only if (2.3) is satisfied and (conditional on not breaching early) to breach late if and only if (2.1) is satisfied.*

2.4. Private Contracts Induce Efficient Decisions

In this section, I show that if the incumbent parties' original contract imposes no externalities on third parties,[10] and if renegotiation is not possible, then the incumbent seller and buyer can implement the efficient investment and breach decisions in both periods by stipulating efficient expectation damages. This result has been demonstrated previously for the case of a single breach opportunity.[11]

Suppose the buyer and seller agreed to a contract (p, x_1, x_2) where

$$x_1 = p - c(r_1^*, r_2^*(r_1^*)) - r_2^*(r_1^*) - \theta[v - c(r_1^*, 0)] \tag{2.8}$$

$$x_2 = p - c(r_1^*, r_2^*(r_1^*)) \tag{2.9}$$

Furthermore, assume each entrant Ei sets price equal to cost, $p_{\mathrm{Ei}} = c_{\mathrm{Ei}}$ for $i = 1, 2$, and that the incumbent seller can charge the alternative buyer his value for the good, i.e., $p' = v$. The following analysis shows that this contract will induce the seller to invest efficiently and the buyer to make the efficient breach decision in each period. Note that if a contract satisfies (2.8) and (2.9), then whenever the buyer breaches, the damages that

[10]That is, assume both entrant sellers are perfectly competitive, i.e., constrained to set price equal to cost, and that the incumbent seller has complete bargaining power with respect to the alternative buyer.
[11]See paragraph 4 on p. 186 of Spier and Whinston (1995) for references.

he pays makes the seller as well off as if the contract had been performed, *assuming the seller invested efficiently.* Hence these damages are the *efficient expectation damages.*

Using backwards induction to solve for the subgame perfect Nash equilibrium of the game, consider first B's private incentives for late breach. Given a contract (p, x_1, x_2) that satisfies (2.8) and (2.9), suppose early breach did not occur. B's equilibrium incentive is to breach late if and only if $v - c_{E2} - x_2 \geq v - p$, or $c_{E2} \leq p - x_2 = c(r_1^*, r_2^*(r_1^*))$. Thus, (2.1) implies that B's late breach decision is efficient if S's *equilibrium* investments r_1^e and r_2^e are efficient, i.e., if they equal r_1^* and $r_2^*(r_1^*)$, respectively.

Given this late breach decision by B, an early investment of r_1^e by S, and no early breach, (2.9) can be used to write S's late investment problem as choosing r_2 to maximize her expected continuation payoff:

$$\text{(2.10)} \qquad \max_{r_2 \geq 0} \left\{ \begin{array}{c} \int_0^{c(r_1^*, r_2^*(r_1^*))} [x_2 - r_1^e - r_2] f(c_{E2}) dc_{E2} \\ + \int_{c(r_1^*, r_2^*(r_1^*))}^{v} [p - c(r_1^e, r_2) - r_1^e - r_2] f(c_{E2}) dc_{E2} \end{array} \right\}$$

$$\Longleftrightarrow \quad \max_{r_2 \geq 0} \left\{ -r_2 - \int_{c(r_1^*, r_2^*(r_1^*))}^{v} c(r_1^e, r_2) f(c_{E2}) dc_{E2} \right\}.$$

Then S's equilibrium choice of r_2^e is characterized by the first order condition

$$\text{(2.11)} \qquad 1 = -c_2(r_1^e, r_2^e(r_1^e))(1 - F[c(r_1^*, r_2^*(r_1^*))]).$$

Since $c_{22}(\cdot) > 0$, equations (2.2) and (2.11) imply that $r_2^e(r_1^e) = r_2^*(r_1^*)$ if $r_1^e = r_1^*$. Hence, S's late investment is indeed efficient if her early investment is efficient.

Anticipating the late investment and breach decisions characterized above, B's equilibrium incentive is to breach early if and only if

$$v - c_{E1} - x_1 \geq \int_0^{c(r_1^*, r_2^*(r_1^*))} [v - c_{E2} - x_2] f(c_{E2}) dc_{E2} + \int_{c(r_1^*, r_2^*(r_1^*))}^{v} [v - p] f(c_{E2}) dc_{E2}.$$

By using (2.8)-(2.9) and rearranging, this inequality can be shown to be equivalent to $c_{E1} \leq c^*(r_1^*) + r_2^*(r_1^*) + \theta[v - c(r_1^*, 0)]$, which is the same as (2.3). Therefore, if $r_1^e = r_1^*$ so that S's early investment is efficient, B's early breach decision will be also efficient (as will be the late investment and late breach decisions).

So it remains to show that S's equilibrium early investment is efficient, i.e., $r_1^e = r_1^*$, when breach damages are specified by (2.8) and (2.9). Given that B breaches early if and only if $c_{E1} \leq c^*(r_1^*) + r_2^*(r_1^*) + \theta[v - c(r_1^*, 0)]$, the probability of early breach only depends on the efficient early investment r_1^* and not S's equilibrium choice of r_1. Therefore, S chooses her early investment $r_1 \geq 0$ to maximize

$$\begin{aligned} & F[c^*(r_1^*) + r_2^*(r_1^*) + \theta(v - c(r_1^*, 0))](x_1 - r_1 + \theta[p' - c(r_1, 0)]) \\ & + \{1 - F[c^*(r_1^*) + r_2^*(r_1^*) + \theta(v - c(r_1^*, 0))]\}\Gamma(r_1), \end{aligned}$$

where $x_1 - r_1 + \theta[p' - c(r_1, 0)]$ is S's expected payoff conditional on early breach, and

$$\begin{aligned} \Gamma(r_1) \equiv & \int_0^{c(r_1^*, r_2^*(r_1^*))} [\underbrace{p - c(r_1^*, r_2^*(r_1^*))}_{x_2} - r_1 - r_2^e(r_1)] f(c_{E2}) dc_{E2} \\ & + \int_{c(r_1^*, r_2^*(r_1^*))}^{v} [p - c(r_1, r_2^e(r_1)) - r_1 - r_2^e(r_1)] f(c_{E2}) dc_{E2} \end{aligned}$$

is the maximized value of the first problem in (2.10) when x_1 and x_2 are given by (2.8) and (2.9). That is, $\Gamma(r_1)$ is the continuation payoff for S from choosing early investment r_1 when B does not breach early, S's chooses her late investment according to $r_2^e(\cdot)$, and B breaches late if and only if $c_{E2} \leq c(r_1^*, r_2^*(r_1^*))$. Note that $\Gamma(r_1)$ can be rewritten as

$$\begin{aligned}\Gamma(r_1) &= p - c(r_1^*, r_2^*(r_1^*))F[c(r_1^*, r_2^*(r_1^*))] \\ &\quad -c(r_1, r_2^e(r_1))\{1 - F[c(r_1^*, r_2^*(r_1^*))]\} - r_1 - r_2^e(r_1).\end{aligned}$$

The first order condition for S's equilibrium early investment r_1^e can be written as

$$\begin{aligned}0 &= -F[c^*(r_1^*) + r_2^*(r_1^*) + \theta(v - c(r_1^*, 0))](1 + \theta c_1(r_1^e, 0)) \\ &\quad +\{1 - F[c^*(r_1^*) + r_2^*(r_1^*) + \theta(v - c(r_1^*, 0))]\}\Gamma'(r_1^e),\end{aligned} \tag{2.12}$$

where (2.11) implies that

$$\Gamma'(r_1^e) = -1 - c_1(r_1^e, r_2^e(r_1^e))(1 - F[c(r_1^*, r_2^*(r_1^*))]) \tag{2.13}$$

Substituting (2.13) into (2.12) and rearranging, (2.12) can be written as

$$\begin{aligned}1 &= -c_1(r_1^e, 0) \cdot \theta F[c^*(r_1^*) + r_2^*(r_1^*) + \theta(v - c(r_1^*, 0))] \\ &\quad -c_1(r_1^e, r_2^e(r_1^e)) \cdot (1 - F[c^*(r_1^*) + r_2^*(r_1^*) + \theta(v - c(r_1^*, 0))])\{1 - F[c(r_1^*, r_2^*(r_1^*))]\}\end{aligned}$$

This equation, when combined with (2.11) and compared with (2.7), implies that S's equilibrium early investment is indeed efficient: $r_1^e = r_1^*$ (recall $c_{11}(\cdot) > 0$). Therefore, by the calculations above, S's equilibrium late investment is also efficient ($r_2^e(r_1^e) = r_2^*(r_1^*)$), and both of B's breach decisions are efficient.

The preceeding analysis can be summarized by the following result.

Proposition 8. *Assume that entrants are perfectly competitive, the alternative buyer has no bargaining power, and renegotiation is not possible. Then any contract* (p, x_1, x_2) *with* x_1 *and* x_2 *satisfying (2.8) and (2.9) induces the seller to always invest efficiently and the buyer to always breach efficiently.*

By Proposition 8, a contract satisfying (2.8) and (2.9) maximizes the joint expected payoffs of the seller and buyer. Therefore, such a contract must also maximize the seller's ex-ante expected payoff given that the buyer accepts the contract. Since the seller's original contract proposal is a take-it-or-leave-it offer, she will find it in her interest to offer a contract satisfying (2.8) and (2.9) and choose the price p so that the buyer is just indifferent inbetween accepting or rejecting the contract offer.

Because the alternative buyer and each competitive entrant seller always earn a payoff of zero, a contract satisfying (2.8) and (2.9) also maximizes social surplus. Therefore, assuming all of the assumptions of the model are satisfied, standard court-imposed breach remedies cannot improve welfare. Note that this result crucially depends on the absence of externalities. When an entrant has market power (and the buyer and seller are able to renegotiate after entry), Spier and Whinston (1995) show in a one-period model that "privately stipulated damages are set at a socially excessive level to facilitate the extraction of the entrant's surplus." Presumably, this inefficiency result would continue to hold if entrants have market power and renegotiation is introduced into the above two-period framework.

Note that the intuition behind Proposition 8 can also be seen without resorting to first order conditions. Because the original contract imposes no externalities, the incumbent seller's investments are always efficient *given* the incumbent buyer's breach decisions. Therefore, since efficient expectation damages induce the buyer to make breach decisions that are efficient assuming the seller's investments are ex-ante efficient,[12] such damages will also induce the seller to make (ex-ante) efficient investment decisions.

Subtracting equation (2.8) from (2.9), the following observations are evident.

Corollary 9. *When the entrants are perfectly competitive, the damage payment for breach is higher* after *the second investment has been made than* before *the second investment has been made:*

$$x_2 - x_1 = r_2^*(r_1^*) + \theta[v - c(r_1^*, 0)] > 0.^{13}$$

Furthermore, this difference is increasing in the probability of finding an alternative buyer (if breach occurs early):

$$\frac{d}{d\theta}(x_2 - x_1) = v - c(r_1^*, 0) > 0.$$

The first part of this corollary says that the fee for cancelling the contract increases over time. The relationship $x_2 = x_1 + r_2^*(r_1^*) + \theta[v - c(r_1^*, 0)]$ between the damages for

[12]To see why this is so with sequential breach decisions, first note that with efficient second period investment, the efficient expectation damage for late breach will induce the buyer to make his late breach decision efficiently. Thus, given efficient first period investment, the efficient expectation damage for early breach will also the buyer to make his early early breach decision efficiently (since his continuation payoff from not breaching early is based on efficient second period breach and investment decisions).

[13]This assumes that trade with the alternative buyer is efficient, conditional on efficient early investment.

late and early breach illustrates the intuition. If the buyer does not breach at his first opportunity to do so, the seller will make the investment $r_2^*(r_1^*)$ and forgo an expected surplus of $\theta[v - c(r_1^*, 0)]$ from possible trade with an alternative buyer. Therefore, the penalty for late breach must include the additional cost of the seller's second investment, as well as the lost expected surplus from potential trade with an alternative buyer, in order to induce the buyer to internalize these social opportunity costs of continuing with the contract when making his second breach decision.

Because the opportunity cost $\theta[v-c(r_1^*, 0)]$ of continuing with the contract is increasing in the probability of finding an alternative buyer in case of early breach, the second part of the corollary simply points out the fact that the difference in the penalties between late breach and early breach must also be increasing in this probability.

2.5. Mitigation of Damages

Corollary 9 shows that the amount by which the damages for late breach exceed the damages for early breach is increasing in θ, the probability of finding an alternative buyer. While so far it has been assumed that this probability is exogenous, in reality the incumbent seller frequently has some influence over the likelihood of recouping some of her initial investment, and therefore the damages owed her by the incumbent buyer. When this is the case, contract law stipulates that the seller (i.e., the breached-against party, or promisee) has the responsibility of undertaking (a reasonable amount of) effort to reduce, or mitigate, those damages.[14]

[14] According to Restatement (Second) of Contracts, §350 (p. 127), "As a general rule, a party cannot recover damages for loss that he could have avoided by reasonable efforts." Goetz and Scott (1983) provide a detailed discussion of the general theory of mitigation. Miceli, et al. (2001) consider a specific application to property leases with court imposed damages. They show that whether it is optimal for

Mitigation usually involves effort costs or other opportunity costs, so I modify the previous model by introducing a cost of mitigation for the seller. I demonstrate that the seller's incentive to engage in such mitigation efforts is socially efficient only when she has complete bargaining power vis-a-vis the alternative buyer; otherwise, her mitigation effort is socially insufficient.

2.5.1. Binary Mitigation Decision

First I consider the case where the seller simply makes a binary decision (immediately after early breach occurs) regarding whether or not to mitigate the damage owed to her by the incumbent buyer. Choosing to mitigate implies, as before, encountering an alternative buyer with (fixed) probability θ, and not mitigating implies being unable to find an alternative buyer with certainty. Assume mitigation involves a disutility of $\gamma > 0$ for the incumbent seller.

Suppose that the incumbent seller's early investment is r_1 and that early breach has occurred. The seller's payoff from not mitigating is $x_1 - r_1$, and her payoff from mitigating is $x_1 - r_1 + \theta[p' - c(r_1, 0)] - \gamma$, where recall p' is the price paid by the alternative buyer. Therefore, if there is no legal requirements on the seller's mitigation decision, she will choose to mitigate if and only if

$$\theta[p' - c(r_1, 0)] > \gamma.$$

there to be a duty for the landlord to mitigate damages from tenant breach of contract depends on whether leases fall under the domain of contract law or property law.

That is, effort is expended to search for an alternative buyer when the probability of, or gains from, alternative trade is high, or when the search effort associated with mitigation is not too costly.

How does this compare with the socially efficient mitigation decision? The payoffs of the incumbent buyer and the first entrant seller are independent of whether the incumbent seller mitigates, so they do not influence the socially efficient mitigation decision. Summing the payoffs of the incumbent seller and the alternative buyer, it is straightforward to see that social surplus is maximized with the incumbent seller mitigating if and only if

$$\theta[v - c(r_1, 0)] > \gamma.$$

By comparing the above two inequalities, it can be readily observed that the incumbent seller's private incentives for mitigation of damages is socially insufficient unless $p' = v$, in which case she has complete bargaining power when dealing with the alternative buyer.[15]

2.5.2. Continuous Mitigation Decision

Now consider the more general case where the seller's mitigation effort choice is continuous. Without loss of generality, suppose that the seller directly chooses the probability of finding an alternative buyer, $\theta \in [0, 1]$. In doing so, she incurs an effort cost of $\gamma(\theta)$, where $\gamma(\cdot)$ is strictly increasing and strictly convex in θ, with $\gamma(0) = 0$.

[15] When $p' = v$, the social efficiency of the incumbent seller's mitigation decision follows immediately from the observation that her decision to mitigation can be viewed as an example of a selfish investment.

Given early investment r_1 by the incumbent seller, and early breach by the incumbent buyer, the seller chooses her mitigation effort level θ to maximize her expected payoff:

$$\max_{\theta \in [0,1]} \{x_1 - r_1 + \theta[p' - c(r_1, 0)] - \gamma(\theta)\}.$$

Assuming $p' - c(r_1, 0) > \gamma'(0)$, the first order condition characterizing the interior solution is

$$p' - c(r_1, 0) = \gamma'(\theta^e(r_1)),$$

where $\theta^e(r_1)$ represents the incumbent seller's *equilibrium* choice of mitigation effort. This expression simply states that the privately optimal mitigation effort level equates the marginal private benefit of increasing such effort with the marginal cost.

In contrast, the socially efficient mitigation effort level $\theta^*(r_1)$ satisfies

$$v - c(r_1, 0) = \gamma'(\theta^*(r_1))$$

because the marginal social benefit from increasing the probability of trade with an alternative buyer is the total surplus from such trade, or $v - c(r_1)$. Since this marginal social benefit exceeds the marginal private benefit whenever $v > p'$, or whenever the alternative buyer has some bargaining power, the incumbent seller will tend to choose a socially insufficient mitigation effort level (due to the convexity of her effort costs $\gamma(\cdot)$): $\theta^e(r_1) \leq \theta^*(r_1)$ for all r_1, with equality if and only if $v = p'$.[16]

[16]The same intuition as in footnote 15 above applies here as well.

2.5.3. Contractibility of the Mitigation Decision

Regardless of whether the mitigation choice involves a binary or continuous decision variable, the incumbent buyer usually exerts a socially insufficient amount of effort to mitigate breach damages, and her mitigation decision is socially efficient if and only if she is able to capture all of the gains from trade with the alternative buyer. The intuition for this inefficiency result is analogous to the intuition for inefficient (under-)investment in property rights models with separate ownership: here, unless the seller is able to charge the alternative buyer a price equal to the latter's willingness to pay for the good or service, she (the seller) does not appropriate all of the surplus from trade and therefore has inefficiently weak incentives for mitigation. (Recall that the seller always bears all of the mitigation costs.)

Notice that the above analysis assumes the damage for early breach, x_1, is fixed and unaffected by the mitigation choice. This requires an implicit assumption that while the incumbent seller is able to commit to her choices of damages, she is unable to commit to her mitigation decision when the contract is first signed. This assumption is reasonable to the extent that mitigation effort cannot be contracted upon at the start of the game, and it seems justified as least in the model where the mitigation decision is continuous and assumed to be equivalent to the *probability* of finding an alternative buyer. In such an environment, it is difficult to conceive how the contracting parties may verify to a court the actual mitigation effort level, since it is *possible* that an alternative buyer is found ex-post even though the incumbent seller may have chosen a very small, but positive, mitigation effort level ex-ante. This case would be relevant, for example, when the mitigation effort decision is not publicly observable.[17]

On the other hand, if the mitigation decision is binary, and there really is no chance of finding an alternative buyer upon late breach, it is conceivable that the mitigation decision might be verifiable ex-post and hence contractible ex-ante.[18] The reason is that if, upon early breach, an alternative buyer is indeed found and trade occurs, then the incumbent seller necessarily chose to mitigate damages. However, this logic depends on the assumption that trade with the alternative buyer is verifable. Were this not the case, the incumbent seller would have an incentive to frabricate evidence of trade with an alternative buyer. Nevertheless, this issue is not problematic to the extent that (i) trade with the incumbent buyer is verifiable, so that the original contract is enforable; and (ii) verifiability of trade for the incumbent seller is correlated among buyers.

If the parties truly cannot contract upon the mitigation decision ex-ante, the incumbent seller would no longer have any contractual obligations towards the incumbent buyer once breach has occurred. She would then be free, in the event of early breach, to choose her mitigation decision in any manner she sees fit. In light of this consideration, the legal requirement that breached-against parties take reasonable efforts to mitigate their

[17]If the mitigation effort decision is publicly observable, the question then becomes whether mitigation should be viewed as the mere exertion of effort to search for an alternative buyer, or actual discovery of such an opportunity *and* the consumation of trade with the alternative buyer.

[18]It would be interesting to analyze whether the incumbent seller has private incentives to write a contract that induces socially efficient mitigation effort when this decision is verifiable and included as a part of the original contract. If the incumbent seller has complete bargaining power with respect to both the incumbent and alternative buyers, it may be reasonable to expect that private mitigation efforts will be socially efficient.

damages in the event of breach can be viewed as an attempt to ameliorate the social insufficiency of private mitigation incentives when contracts are incomplete.[19]

2.5.4. The Nature of the Breach Outcome

There is one final observation to make regarding the efficiency of the incumbent seller's mitigation effort. Assuming that she has full bargaining power vis-a-vis the alternative buyer, the preceeding analysis shows that the incumbent seller has socially efficient incentives for mitigation. This result relies on the implicit assumption that whether the contract is breached directly depends upon only the incumbent buyer's action and not the action of the incumbent seller. If whether breach occurs is a function of both party's actions (as is the case in some tort models), the following analysis will show that the incumbent seller's action (mitigation decision) may be socially inefficient, even if she has full bargaining power with respect to the alternative buyer.

The duty to mitigate damages usually arises in situations where breach damages are imposed ex-post by the court, as opposed to being privately stipulated ex-ante. Therefore, to see the importance of the way in which breach is defined, consider the following example, where I assume court-imposed expectation damages.

Suppose there is just one period, with no investment, buyer value v, seller cost c, and a binary mitigation decision for the incumbent seller. Assume the entrant's cost c_E is either c_E^L or c_E^H with $c_E^L < c_E^H \leq v + \theta[v - c] - \gamma$, where γ is the seller's effort cost of mitigation. In particular, if she mitigates upon breach, there is probability θ that she will be able to find an alternative buyer with whom to trade at the price $p' = v$ and cost

[19]See Goetz and Scott (1983).

c. If the incumbent seller does not mitigate after breach, there is zero probability finding an alternative buyer.

First, suppose breach of contract is defined simply as the buyer's refusal to trade with the incumbent seller. As the previous subsection showed, the seller's mitigation decision will be efficient because upon breach, she receives all the expected surplus from trade with the alternative buyer and therefore will decide to mitigate if and only if $\theta[v-c]-\gamma>0$, as required by efficiency.

Now suppose breach of contract is said to occur (and hence breach damage x due) if and only if the incumbent buyer refuses trade *and* the incumbent seller cannot find an alternative buyer.[20] Conditional on the incumbent buyer's refusal of trade, efficiency requires that the seller mitigates, i.e., exerts effort to find an alternative buyer, if and only if $v-c_E+\theta[v-c]-\gamma\geq 0 \iff c_E\leq v+\theta[v-c]-\gamma$.[21] Since $c_E\leq c_E^H\leq v+\theta[v-c]-\gamma$ by assumption, the efficient mitigation decision is to always mitigate (conditional on the incumbent buyer's refusal of trade). However, the seller will never exert mitigation effort. To see this, note that if she does not mitigate, then with probability 1 she does not find an alternative buyer to trade with, and hence by definition breach occurs. So the seller's payoff from not mitigating, given expectation damages, is $x=p'-c=v-c$.[22] The seller's payoff from mitigation is $\theta[v-c]+(1-\theta)x-\gamma=v-c-\gamma$, which is less than

[20]Because the seller's mitigation decision affects her probability of finding an alternative buyer, it also affects the probability that breach is said to occur.

[21]If S does not mitigate after B refuses trade, no surplus is realized because S would not be able to trade with either B or the alternative buyer.

[22]The expectation damage equates the seller's payoff from breach, x, to her payoff from no breach. Conditional on the incumbent buyer's refusal to trade, no breach corresponds to the case in which the seller is able to find an alternative buyer with whom to trade. In this case, the seller receives a payoff of $p'-c=v-c$.

her payoff of $v - c$ from not mitigating.[23,24] Thus, the seller will never choose to mitigate even though it is efficient for her to do so after the buyer's refusal to trade.

The intuition for this result is straightforward. When breach is equivalent to the incumbent buyer's refusal to trade, the seller's mitigation decision does not affect the incumbent buyer's payoff conditional on his refusal to trade. Instead, the mitigation decision only affects the seller's own payoff (recall the alternative buyer always earns zero by assumption), and so her mitigation decision will be efficient. In contrast, if the definition of breach requires not only the buyer's refusal to trade but also the seller's inability to find an alternative buyer, then the seller will not mitigate even when it is efficient to do so. To see this, note that expectation damages ensure that regardless of whether the seller mitigates, she will receive the same gross payoff (excluding any mitigation effort costs) of $v - c$ after the incumbent buyer refuses to trade. Therefore, because mitigation effort is costly, the seller will choose to not mitigate.[25] (This inefficiency result still obtains even if the seller is accurately compensated for her disutility of mitigation effort when no alternative buyer is found. The reason is that while the cost of mitigation is certain, finding an alternative buyer is not. See footnote 24.)

[23]With probability θ, the seller finds and trades with an alternative buyer. In this case, there is no breach and the seller receives $v - c$ from trade with the alternative buyer. With probability $1 - \theta$, the seller is unable to find an alternative buyer, and so by definition breach occurs. The seller receives the breach damage x in this case. Regardless of whether an alternative buyer is found, the seller incurs the effort cost γ if she mitigates.

[24]Note that if the expectation damages were to compensate the seller for her disutility of mitigation effort, then $x = v - c + \gamma$. In this case, the seller's payoff from mitigation is $\theta[v - c] + (1 - \theta)x - \gamma = v - c + (1 - \theta)\gamma - \gamma = v - c - \theta\gamma$, which is still less than her payoff of $v - c$ from not mitigating. Therefore, as long as the court-imposed expectation damage does not grossly over-estimate the seller's disutility of mitigation, she will still prefer to not mitigate.

[25]Alternatively, the intuition for the inefficiency result follows from the observation that when breach depends on both parties' actions, the incubment seller's mitigation decision has an externality on the incumbent seller (even though $p' = v$ implies no externality on the alternative buyer) and therefore will be inefficient.

2.6. Renegotiation

I now examine the situation where the incumbent seller S and buyer B are able to renegotiate their original contract after each entrant seller announces its price p_{Ei} and prior to each breach opportunity. Once again, assume each entrant is perfectly competitive and sets price equal to cost, $p_{\mathrm{Ei}} = c_{\mathrm{Ei}}$, and suppose that S has complete bargaining power vis-a-vis the alternative buyer. Then S and B's contract imposes no externalities on other parties, and so they have joint incentives to induce efficient breach and investment decisions. As Proposition 10 below demonstrates, the efficient breach and investment decisions can in fact be implemented with the same efficient expectation damages as before, when renegotiation was impossible. The logic underlying this argument depends crucially on analyzing the parties' payoffs off the equilibrium path.

Assume Nash bargaining during each renegotation period, so that the renegotiation outcome maximizes the seller and buyer's joint payoffs. The renegotiation surplus, which is split between S and B in the proportions α and $1-\alpha$, is defined as the difference in the sum of payoffs for S and B with and without renegotiation: $s_{reneg} \equiv (u_S + u_B)|_{w/reneg} - (u_S + u_B)|_{w/o\ reneg}$. Hence, the payoffs after each stage of renegotiation are $u_S|_{w/o\ reneg} + \alpha \cdot s_{reneg}$ for the seller and $u_B|_{w/o\ reneg} + (1-\alpha) \cdot s_{reneg}$ for the buyer. If B is indifferent between buying from an entrant or S, assume B buys from the entrant, regardless of whether the indifference arises before or after renegotiation.

Suppose that early and late investment are complementary, i.e.,

$$c_{12}(r_1, r_2) \leq 0 \text{ for all } (r_1, r_2). \tag{2.14}$$

Then S's privately optimal, or equilibrium, late investment $r_2^e(r_1)$ is increasing in her early investment r_1. Finally, assume

$$1 - \max\{F[c(r_1, r_2^e(r_1))], F[c(r_1^*, r_2^*(r_1^*))]\} \geq \theta \text{ for all } r_1, \tag{2.15}$$

which can be shown to imply that: (i) when r_1 is less than r_1^*, the private value of early investment for S exceeds its social value assuming early breach occurs; and (ii) when r_1 is greater than r_1^*, the private value of early investment for S is less than its social value assuming early breach does not occur.

Proposition 10. *Suppose S and B can renegotiate after each competitive entrant arrives and that (2.14) and (2.15) are satisfied. Then the ex-ante efficient breach and investment decisions (as characterized in Section 3) can be implemented by the same contract that implements the efficient outcome when renegotiation is not possible, i.e., any contract* (p, x_1, x_2) *where* x_1 *and* x_2 *are the effficient expectation damages and satisfy (2.8) and (2.9).*

The intuition for this result is as follows. When $r_1 < r_1^*$, early renegotiation causes early breach to occur (but not absent early renegotiation) for intermediate realizations of the early entrant's cost. In this case, S's private incentive to increase r_1 slightly exceeds the social marginal benefit of increasing r_1. (To see this, suppose no alternative buyer exists. Then a social planner would not value early investment at all given that early breach occurs. However, S obtains a share of the early renegotiation surplus, which is *increasing* in S's early investment.[26]) Similarly, when $r_1 > r_1^*$, early renegotiation causes early breach to not occur (but it does occur absent early renegotiation) for intermediate realizations of the early entrant's cost. Here, assumption (2.15) implies that S has a smaller private incentive to increase r_1 relative to the social marginal benefit. Together, these two observations will induce S to choose the efficient early investment r_1^*.

Given that S chooses the efficient early investment r_1^*, early renegotiation implies that B's early breach decision will be (ex-ante) efficient as well. It can also be shown that S's privately optimal late investment, $r_2^e(r_1)$, coincides with the efficient late investment $r_2^*(r_1)$ when $r_1 = r_1^*$. In other words, given that S's early investment is efficient, so is her late investment (see Lemma 12 below). Late renegotiation then leads to the efficient late breach decision. (These observations also imply that no renegotiation occurs on the equilibrium path.)

The rest of this section details the proof of this proposition.[27] Using backwards induction, I first look at B's late breach decision, then S's late investment decision, then B's early breach decision, and finally S's early investment decision.

2.6.1. Late Breach Decision

First consider B's late breach decision. Given there is no early breach and that x_2 satisfies (2.9), B has a private incentive to breach late absent renegotiation if and only if

[26] If trade with an alternative buyer is possible and $r_1 < r_1^*$, assumption (2.15) implies that S's private marginal benefit from increasing early investment continues to exceed the social marginal benefit, given that early breach occurs.

[27] Readers who are either uninterested in the technical details underlying Proposition 10 or more interested in a concrete application of this model may wish to skip ahead to Section 2.7.

$v - c_{E2} - x_2 \geq v - p$, i.e.

$$c_{E2} \leq p - x_2 = c(r_1^*, r_2^*(r_1^*)).$$

On the other hand, conditional on S having actually chosen investment levels r_1 and r_2, renegotiation after the second entrant arrives (what I will sometimes refer to as "late renegotiation") leads to late breach if and only if $v - c_{E2} \geq v - c(r_1, r_2)$, i.e.,

$$c_{E2} \leq c(r_1, r_2).$$

Given (r_1, r_2), this is the ex-post efficient breach decision. Since ex-ante efficiency requires late breach to occur exactly when $c_{E2} \leq c(r_1^*, r_2^*(r_1^*))$, late renegotiation implies that B's late breach decision is ex-ante efficient *if* S's early and late investments are ex-ante efficient, i.e., if $(r_1, r_2) = (r_1^*, r_2^*(r_1^*))$.

2.6.2. Renegotiation Payoffs in the Second Period

Before examining S's late investment decision, we must first consider the (renegotiation-induced) payoffs of S (and B) for all possible realizations of the second entrant's price/cost c_{E2}, as well as for all possible early and late investments (r_1, r_2) that S might make (including those off the equilibrium path).

When $c_{E2} \leq \min\{p - x_2, c(r_1, r_2)\}$, B breaches late regardless of whether late renegotiation is possible, and so payoffs are $u_S = x_2 - r_1 - r_2$ and $u_B = v - c_{E2} - x_2$. On the other hand, when $c_{E2} > \max\{p - x_2, c(r_1, r_2)\}$, B does not breach late regardless of whether late renegotiation is possible, and so payoffs are $u_S = p - c(r_1, r_2) - r_1 - r_2$ and $u_B = v - p$.

If $p - x_2 < c_{E2} \leq c(r_1, r_2)$, B does not breach late absent late renegotiation because $p - x_2 < c_{E2}$. But since the second entrant can produce the good at a lower cost than S in this case, renegotiation will induce B to breach and allow the parties to share the renegotiation surplus $c(r_1, r_2) - c_{E2} \geq 0$. Disagreement payoffs are those associated with the no-breach outcome, i.e., $p - c(r_1, r_2) - r_1 - r_2$ for S and $v - p$ for B, and so the renegotiation payoffs are $u_S = p - c(r_1, r_2) - r_1 - r_2 + \alpha[c(r_1, r_2) - c_{E2}]$ and $u_B = v - p + (1 - \alpha)[c(r_1, r_2) - c_{E2}]$.

On the other hand, if $c(r_1, r_2) < c_{E2} \leq p - x_2$, B breaches late absent late renegotiation because $c_{E2} \leq p - x_2$. But late renegotiation will cause B to not breach and allow the parties to share the renegotiation surplus $c_{E2} - c(r_1, r_2) > 0$ (in this case, S has the lower cost). Disagreement payoffs are therefore those associated with the breach outcome, i.e., $x_2 - r_1 - r_2$ for S and $v - c_{E2} - x_2$ for B, and so the renegotiation payoffs are $u_S = x_2 - r_1 - r_2 + \alpha[c_{E2} - c(r_1, r_2)]$ and $u_B = v - c_{E2} - x_2 + (1 - \alpha)[c_{E2} - c(r_1, r_2)]$.

To summarize:

Lemma 11. *If early breach does not occur and S's investments are (r_1, r_2), payoffs after late renegotiation (excluding investment costs) for the incumbent seller S and buyer*

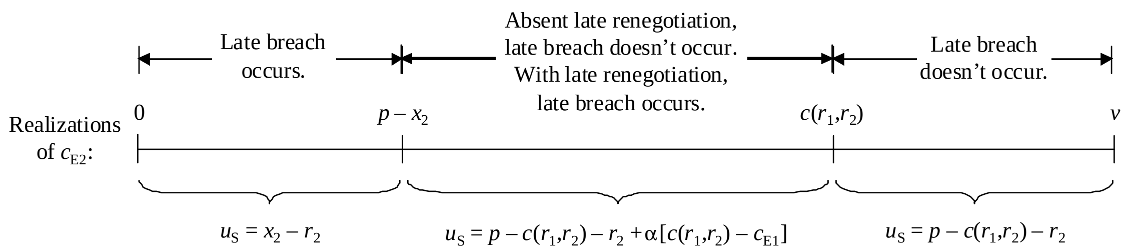

Figure 2.2. Seller S's ex-post payoffs after late renegotiation, for the case when $p - x_2 \leq c(r_1, r_2)$.

B, respectively, are given by:

$$\begin{array}{ll}
\{x_2, v - c_{E2} - x_2\} & \textit{if } c_{E2} \leq \min\{p - x_2, c(r_1, r_2)\}; \\
\{p - c(r_1, r_2), v - p\} & \textit{if } c_{E2} > \max\{p - x_2, c(r_1, r_2)\}; \\
\left\{ \begin{array}{c} p - c(r_1, r_2) + \alpha[c(r_1, r_2) - c_{E2}], \\ v - p + (1 - \alpha)[c(r_1, r_2) - c_{E2}] \end{array} \right\} & \textit{if } p - x_2 < c_{E2} \leq c(r_1, r_2); \\
\left\{ \begin{array}{c} x_2 + \alpha[c_{E2} - c(r_1, r_2)], \\ v - c_{E2} - x_2 + (1 - \alpha)[c_{E2} - c(r_1, r_2)] \end{array} \right\} & \textit{if } c(r_1, r_2) < c_{E2} \leq p - x_2.
\end{array}$$

2.6.3. Late Investment Decision

Now consider S's late investment decision given that she chose r_1 in period 1. First, suppose S chooses r_2 such that $p - x_2 \leq c(r_1, r_2)$. Conditional on early breach not occurring, Figure 2.2 summarizes S's ex-post payoff after late renegotiation (from Lemma 11) as a function of the second entrant's price offer $p_{E2} = c_{E2}$.

In this case, S's expected payoff (exclusive of her early investment cost) is

$$\begin{aligned}
\pi_L(r_1, r_2) \quad &= \quad F[p - x_2]x_2 + \int_{p-x_2}^{c(r_1 r_2)} \{p - c(r_1, r_2) + \alpha[c(r_1, r_2) - c_{E1}]\} f(c_{E1}) dc_{E1} \\
&\quad + (1 - F[c(r_1, r_2)])[p - c(r_1, r_2)] - r_2.
\end{aligned}$$

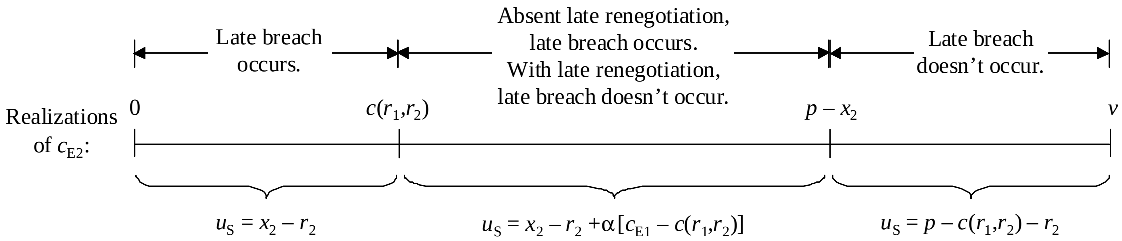

Figure 2.3. Seller S's ex-post payoffs after late renegotiation, for the case when $p - x_2 \geq c(r_1, r_2)$.

On the other hand, if S chooses r_2 such that $p - x_2 \geq c(r_1, r_2)$, Figure 2.3 depicts her ex-post payoff after late renegotiation as a function of the second entrant's price.

For these values of r_1 and r_2, S's expected payoff (exclusive of early investment cost) is

$$\begin{aligned} \pi_H(r_1, r_2) &= F[c(r_1, r_2)]x_2 + \int_{c(r_1 r_2)}^{p-x_2} \{x_2 + \alpha[c_{E1} - c(r_1, r_2)]\} f(c_{E1}) dc_{E1} \\ &\quad + (1 - F[p - x_2])[p - c(r_1, r_2)] - r_2. \end{aligned}$$

Note that $\pi_H(r_1, r_2)$ can be rewritten as

$$\begin{aligned} \pi_H(r_1, r_2) &= F[p - x_2]x_2 + \int_{c(r_1 r_2)}^{p-x_2} \alpha[c_{E1} - c(r_1, r_2)] f(c_{E1}) dc_{E1} \\ &\quad + (1 - F[p - x_2])[p - c(r_1, r_2)] - r_2 \\ &= \pi_L(r_1, r_2), \end{aligned}$$

where the second inequality follows from (i) switching the bounds of integration in the second term and multiplying the integrand by -1; and (ii) writing $1 - F[p - x_2]$ in the third term as $(1 - F[c(r_1, r_2)]) + (F[c(r_1, r_2)] - F[p - x_2])$ and then rearranging. Thus, given r_1, simply denote S's expected payoff from chosing r_2 (exclusive of early investment cost) by

$$\pi(r_1, r_2) \equiv \pi_L(r_1, r_2) = \pi_H(r_1, r_2) \text{ for all } (r_1, r_2).$$

Let $r_2^e(r_1)$ denote S's privately optimal, or equilibrium, late investment choice, given that her early investment is r_1. It is characterized by the first order condition

$$\begin{aligned} (2.16) \qquad 0 &= \pi_2(r_1, r_2^e(r_1)) \text{ for all } r_1 \\ &= -c_2(r_1, r_2^e(r_1))\{1 - \alpha F[c(r_1, r_2^e(r_1))] - (1-\alpha)F[p - x_2]\} - 1 \end{aligned}$$

Lemma 12. *If S's early investment is efficient, her late investment is efficient as well:*

$$r_2^e(r_1^*) = r_2^*(r_1^*).$$

Proof. To see this, observe that since $r_2^*(r_1^*)$ maximizes social surplus given r_1^*, we have $S'(r_2^*(r_1^*)|r_1^*) \gtreqless 0$ for all $r_2 \lesseqgtr r_2^*(r_1^*)$. Thus, if $p - x_2 = c(r_1^*, r_2^*(r_1^*)) \leq c(r_1^*, r_2)$, then $r_2 \leq r_2^*(r_1^*)$ (as $c_2 < 0$). In this case, (2.9) implies $\pi_2(r_1^*, r_2) \geq -c_2(r_1^*, r_2)\{1 - F[c(r_1^*, r_2)]\} - 1 = S'(r_2|r_1^*) \geq 0$. Similarly, $p - x_2 = c(r_1^*, r_2^*(r_1^*)) \geq c(r_1^*, r_2)$ implies $r_2 \geq r_2^*(r_1^*)$ and hence $\pi_2(r_1^*, r_2) \leq -c_2(r_1^*, r_2)\{1 - F[c(r_1^*, r_2)]\} - 1 = S'(r_2|r_1^*) \leq 0$. □

This result is analogous to Proposition 1 in Spier and Whinston (1995), where efficient expectation damages lead the seller to invest efficiently. (As in their Proposition 1, I also assume renegotiation and a perfectly competitive (late) entrant.) The intuition is the same as well. When the seller's late investment is less than efficient (given r_1^*), late renegotiation allows her to capture a share of the return on her cost reduction for realizations of c_{E2} that ultimately lead to late breach (see the middle interval in Figure

2.2). Since a social planner only values late investment when S actually produces the good, the seller's incentive to increase her late investment exceeds that of a social planner when r_2 is less than efficient (given r_1^*). Similarly, when r_2 is more than efficient (given r_1^*), the seller's incentive to increase her late investment is less than that of a social planner. Hence, the seller chooses the efficient late investment (given early investment r_1^*).

Finally, assuming the second order condition is satisfied, (2.14) implies that $r_2^e(r_1)$ is increasing in r_1.[28] Hence, because $c_1 < 0, c_2 < 0$, we have $\frac{d}{dr_1}c(r_1, r_2^e(r_1)) < 0$. Therefore Lemma 12 implies that

$$r_1 \lesseqgtr r_1^* \iff p - x_2 = c(r_1^*, r_2^*(r_1^*)) \lesseqgtr c(r_1, r_2^e(r_1)) \tag{2.17}$$

with equality if and only if $r_1 = r_1^*$.

2.6.4. Early Breach Decision

Absent Early Renegotiation.

Absent early renegotiation, the incumbent buyer B obtains a payoff of $v - c_{E1} - x_1$ if he breaches early to buy from the first entrant. Now consider B's expected payoff from not breaching early, with late renegotiation still possible.

Given S's early investment r_1, B will anticipate S's late investment choice of $r_2^e(r_1)$. First, suppose $r_1 \leq r_1^*$, which is equivalent to $p - x_2 \leq c(r_1, r_2^e(r_1))$ by (2.17). Lemma 11

[28] A sufficient condition for the second order condition to be satisfied is that $\pi_{22} = -c_{22}(r_1, r_2)\{1 - \alpha F[c(r_1, r_2)] - (1-\alpha)F[c(r_1^*, r_2^*(r_1^*))]\} + c_2(r_1, r_2)^2 f[c(r_1, r_2)] < 0$ at $r_2 = r_2^e(r_1)$ for all r_1. Given (2.14), $\pi_{21} = -c_{21}\{1 - \alpha F[c] - (1-\alpha)F[p - x_2]\} + \alpha c_1 c_2 f(c) > 0$, and so $r_2^{e\prime}(r_1) = -\pi_{21}/\pi_{22} > 0$ at $(r_1, r_2^e(r_1))$.

and (2.9) imply that B's expected payoff from not breaching early is

$$\begin{aligned}
&\int_0^{p-x_2}(v-c_{E2}-x_2)f(c_{E2})dc_{E2}+(1-F[c(r_1,r_2^e(r_1))])(v-p)\\
&+\int_{p-x_2}^{c(r_1,r_2^e(r_1))}(v-p+(1-\alpha)[c(r_1,r_2^e(r_1))-c_{E2}])f(c_{E2})dc_{E2}\\
=\ &v-p-\int_0^{p-x_2}(c_{E2}-c(r_1^*,r_2^*(r_1^*))f(c_{E2})dc_{E2}\\
&+\int_{p-x_2}^{c(r_1,r_2^e(r_1))}(1-\alpha)[c(r_1,r_2^e(r_1))-c_{E2}]f(c_{E2})dc_{E2}.
\end{aligned}$$

Since (2.9) implies $c^*(r_1^*) = \int_0^{p-x_2} c_{E2}f(c_{E2})dc_{E2} + \int_{p-x_2}^{v} c(r_1^*, r_2^*(r_1^*))f(c_{E2})dc_{E2}$ (recall (2.4), the definition of $c^*(r_1)$), B's expected payoff from not early early can be further rewritten as $v - \psi(r_1) - x_2$, where

$$\psi(r_1) \equiv c^*(r_1^*) - \int_{c(r_1^*,r_2^*(r_1^*))}^{c(r_1,r_2^e(r_1))}(1-\alpha)[c(r_1,r_2^e(r_1))-c_{E2}]f(c_{E2})dc_{E2}.$$

If $r_1 \geq r_1^*$ instead, i.e., $p-x_2 \geq c(r_1, r_2^e(r_1))$, B's expected payoff from not breaching early is

$$\begin{aligned}
&\int_0^{c(r_1,r_2^e(r_1))}(v-c_{E2}-x_2)f(c_{E2})dc_{E2}+(1-F[p-x_2])(v-p)\\
&+\int_{c(r_1,r_2^e(r_1))}^{p-x_2}(v-c_{E2}-x_2+(1-\alpha)[c_{E2}-c(r_1,r_2^e(r_1))])f(c_{E2})dc_{E2}.
\end{aligned}$$

It turns out that this expression can also be written as $v - \psi(r_1) - x_2$.

So for any r_1, B breaches early absent early renegotiation if and only if $v - c_{E1} - x_1 \geq v - \psi(r_1) - x_2$, or equivalently,

$$c_{E1} \leq \psi(r_1) + x_2 - x_1. \tag{2.18}$$

Since $\psi'(r_1) = (1-\alpha)\frac{dc(r_1,r_2^e(r_1))}{dr_1}(F[c(r_1^*, r_2^*(r_1^*))] - F[c(r_1, r_2^e(r_1))])$, (2.17) implies that

$$\psi'(r_1) \gtreqless 0 \text{ for all } r_1 \lesseqgtr r_1^*, \tag{2.19}$$

with equality only at r_1^*.

Finally, $\psi(r_1^*) = c^*(r_1^*)$ follows from Lemma 12. So *if* S's early investment is efficient, (2.8) and (2.9) imply that B will breach early absent early renegotiation if and only if $c_{E1} \leq \psi(r_1^*) + x_2 - x_1 = c^*(r_1^*) + r_2^*(r_1^*) + \theta[v - c(r_1^*, 0)]$, which is the efficient early breach decision.

With Early Renegotiation.

With early renegotiation, B will breach early to buy from the first entrant if and only if expected social surplus is higher from his breaching early. Absent early breach, surplus is $u_S + u_B = v - \psi(r_1) - x_2 + \pi(r_1, r_2^e(r_1)) - r_1$. With early breach, $u_S + u_B = v - c_{E1} + \theta[v - c(r_1, 0)] - r_1$. Thus, early renegotiation leads to early breach if and only if

$$\begin{aligned} c_{E1} &\leq \phi(r_1) + \theta[v - c(r_1, 0)], \\ \text{where } \phi(r_1) &\equiv \psi(r_1) + x_2 - \pi(r_1, r_2^e(r_1)), \end{aligned} \tag{2.20}$$

(which is the efficient breach decision given r_1).

Recall from Section 4 that when renegotiation is never possible, early breach is efficient given r_1 if and only if

$$c_{E1} \leq c^*(r_1) + r_2^*(r_1) + \theta[v - c(r_1, 0)] \tag{2.21}$$

(compare with (2.3) for the case $r_1 = r_1^*$). It can be verified that $c^*(r_1) + r_2^*(r_1)$ and $\phi(r_1)$, and hence the right hand sides of (2.20) and (2.21), are not equal unless $r_1 = r_1^*$. Therefore, the efficient early breach decisions when renegotiation is and is not possible do not coincide with each other unless S's early investment is efficient. *In other words, the possibility of renegotiation does not alter the efficient early breach decision on the equilibrium path but does affect it off the equilibrium path.*

Since B's early breach decision (with early renegotiation) is ex-ante efficient given r_1^*, it remains to show that S's early investment is indeed efficient.

2.6.5. Renegotiation Payoffs in the First Period

Before analyzing S's early investment decision, we first derive the payoffs of S (and B) after early renegotiation for all possible realizations of the first entrant's price/cost c_{E1} and all levels of S's early investment r_1. Recall that absent early renegotiation, B breaches early if and only if (2.18) holds, while with early renegotiation early breach occurs if and only if (2.20) is satisfied.

When $c_{E1} \leq \min\{\psi(r_1) + x_2 - x_1, \phi(r_1) + \theta[v - c(r_1, 0)]\}$, B breaches early regardless of whether early renegotiation is possible, and so payoffs are $u_S = x_1 + \theta[v - c(r_1, 0)] - r_1$ and $u_B = v - c_{E1} - x_1$. On the other hand, when $c_{E2} > \max\{\psi(r_1) + x_2 - x_1, \phi(r_1) + \theta[v - c(r_1, 0)]\}$, B does not breach early regardless of whether early renegotiation is possible, and so payoffs are $u_S = \pi(r_1, r_2^e(r_1)) - r_1$ and $u_B = v - \psi(r_1) - x_2$.

If $\psi(r_1) + x_2 - x_1 < c_{E1} \leq \phi(r_1) + \theta[v - c(r_1, 0)]$, B does not breach early absent early renegotiation because $\psi(r_1) + x_2 - x_1 < c_{E1}$. But early renegotiation induces B to breach early and allow the parties to share the renegotiation surplus $s_{reneg}^L \equiv \phi(r_1) + \theta[v -$

$c(r_1, 0)] - c_{E1} \geq 0$. Disagreement payoffs are those associated with the no-early-breach outcome, i.e., $\pi(r_1, r_2^e(r_1)) - r_1$ for S and $v - \psi(r_1) - x_2$ for B, and so the renegotiation payoffs are $u_S = \pi(r_1, r_2^e(r_1)) - r_1 + \alpha \cdot s_{reneg}^L$ and $u_B = v - \psi(r_1) - x_2 + (1-\alpha) \cdot s_{reneg}^L$.

If $\phi(r_1) + \theta[v - c(r_1, 0)] < c_{E1} \leq \psi(r_1) + x_2 - x_1$, B breaches early absent early renegotiation because $c_{E1} \leq \psi(r_1) + x_2 - x_1$. However, early renegotiation induces B to not breach early and allow the parties to share the renegotiation surplus $s_{reneg}^H \equiv c_{E1} - \phi(r_1) - \theta[v - c(r_1, 0)] \geq 0$. Disagreement payoffs are those associated with the early breach outcome, i.e., $x_1 + \theta[v - c(r_1, 0)] - r_1$ for S and $u_B = v - c_{E1} - x_1$ for B, and so the renegotiation payoffs are $u_S = x_1 + \theta[v - c(r_1, 0)] - r_1 + \alpha \cdot s_{reneg}^H$ and $u_B = v - c_{E1} - x_1 + (1-\alpha) \cdot s_{reneg}^H$.

To summarize:

Lemma 13. *If S's early investment is r_1, the expected payoffs after early renegotiation (excluding early investment costs) for S and B, respectively, are given by:*

$$\begin{cases} \{x_1 + \theta[v - c(r_1, 0)], v - c_{E1} - x_1\} & \text{if } c_{E1} \leq \min \left\{ \begin{array}{c} \psi(r_1) + x_2 - x_1, \\ \phi(r_1) + \theta[v - c(r_1, 0)] \end{array} \right\}; \\ \{\pi(r_1, r_2^e(r_1)), v - \psi(r_1) - x_2\} & \text{if } c_{E1} > \max \left\{ \begin{array}{c} \psi(r_1) + x_2 - x_1, \\ \phi(r_1) + \theta[v - c(r_1, 0)] \end{array} \right\}; \\ \left\{ \begin{array}{c} \pi(r_1, r_2^e(r_1)) + \alpha \cdot s_{reneg}^L, \\ v - \psi(r_1) - x_2 + (1-\alpha) \cdot s_{reneg}^L] \end{array} \right\} & \text{if } \psi(r_1) + x_2 - x_1 < c_{E2} \leq \phi(r_1) + \theta[v - c(r_1, 0)]; \\ \left\{ \begin{array}{c} x_1 + \theta[v - c(r_1, 0)] + \alpha \cdot s_{reneg}^H, \\ v - c_{E1} - x_1 + (1-\alpha) \cdot s_{reneg}^H \end{array} \right\} & \text{if } \phi(r_1) + \theta[v - c(r_1, 0)] < c_{E2} \leq \psi(r_1) + x_2 - x_1; \end{cases}$$

where $s_{reneg}^L \equiv -s_{reneg}^H \equiv \phi(r_1) + \theta[v - c(r_1, 0)] - c_{E1}$.

2.6.6. Early Investment Decision

Given the preceeding analysis, to prove Proposition 10 it suffices to show that S's privately optimal early investment is indeed at the efficient level r_1^*. Define $\Pi(r_1)$ to be S's ex-ante expected payoffs from choosing r_1. Recall that ex-ante expected social welfare given r_1 is denoted by $S(r_1)$ and, by definition, is maximized at r_1^*. We will show that

$$\begin{aligned} \Pi'(r_1) &\geq S'(r_1) \geq 0, \forall r_1 \leq r_1^*, \text{ and} \\ \Pi'(r_1) &\leq S'(r_1) \leq 0, \forall r_1 \geq r_1^*. \end{aligned}$$

It will then follow that S's privately optimal early investment (the value of r_1 that maximizes $\Pi(r_1)$) is indeed the efficient one, r_1^*. (Note that similar to the proof of Lemma 12 above, this part of the proof of Proposition 10 also follows the strategy of the proof of Proposition 1 in Spier and Whinston (1995). The complicating factor in this model is that because there is a second period if early breach does not occur, one must replace the (final) renegotiation payoffs derived in Spier and Whinston's Lemma 1 with the (interim) renegotiation payoffs given by Lemma 13 above.)

First of all, observe that given assumption (2.15),

$$r_1 \lesseqqgtr r_1^* \iff \psi(r_1) + x_2 - x_1 \lesseqqgtr \phi(r_1) + \theta[v - c(r_1, 0)],$$

with equality only at $r_1 = r_1^*$. To see this, note that $\psi(r_1) + x_2 - x_1 \lesseqqgtr \phi(r_1) + \theta[v - c(r_1, 0)]$ is equivalent to

$$0 \lesseqqgtr \theta[v - c(r_1, 0)] + x_1 - \pi(r_1, r_2^e(r_1)),$$

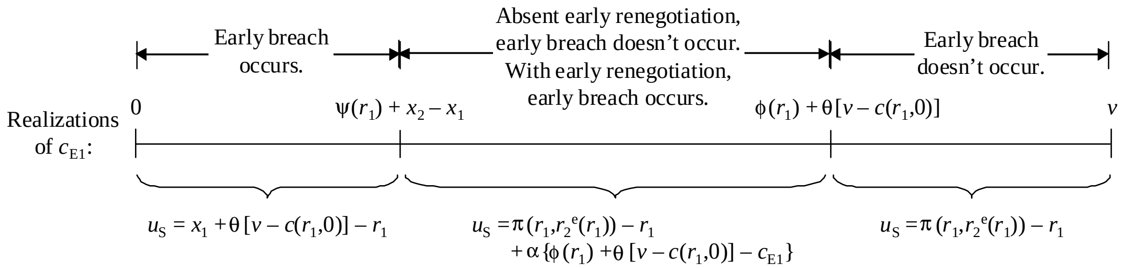

Figure 2.4. Seller's payoffs after early renegotiation in Case (A), where $r_1 \leq r_1^*$.

which is satisfied for all $r_1 \lesseqgtr r_1^*$ because the right hand side of this expression is zero at r_1^* (by (2.8) and (2.9)) and strictly decreasing in r_1 for all r_1 (by assumption (2.15)).[29,30]

Case (A). Suppose $r_1 \leq r_1^*$, which implies $\psi(r_1) + x_2 - x_1 \leq \phi(r_1) + \theta[v - c(r_1, 0)]$. There are three subcases to consider for different realizations of c_{E1}, and Figure 2.4 shows S's payoffs in each subcase.

(i) If $c_{E1} \leq \psi(r_1) + x_2 - x_1$, early breach always occurs. Social surplus is $v - c_{E1} + \theta[v - c(r_1, 0)] - r_1$ for these realizations of c_{E1}, so the marginal net social return from increasing r_1 slightly is $-\theta c_1(r_1, 0) - 1$. Since S's private payoff is $x_1 + \theta[v - c(r_1, 0)] - r_1$ in this range, her marginal net private return from increasing r_1 corresponds to the net social return.

(ii) If $\psi(r_1) + x_2 - x_1 < c_{E1} \leq \phi(r_1) + \theta[v - c(r_1, 0)]$, early breach still occurs because of early renegotiation, and so the marginal social return from increasing r_1 is still

[29] $\frac{d}{dr_1}\{\theta[v - c(r_1, 0)] + x_1 - \pi(r_1, r_2^e(r_1))\} = -\theta c_1(r_1, 0) - \pi_1(r_1, r_2^e(r_1))$, which is negative for all r_1 by assumption (2.15).

[30] Recall that excluding early investment cost and absent early renegotiation, S's earns a payoff of $x_1 + \theta[v - c(r_1, 0)]$ from early breach occurring and $\pi(r_1, r_2^e(r_1))$ from early breach not occurring. Therefore, $\pi(r_1, r_2^e(r_1)) \lesseqgtr x_1 + \theta[v - c(r_1, 0)]$ for all $r_1 \lesseqgtr r_1^*$ implies that (i) when r_1 is less than r_1^*, early investment is more valuable to S if early breach occurs; and (ii) when r_1 is greater than r_1^*, early investment is more valuable to her if early breach does not occur.

$-\theta c_1(r_1, 0) - 1$. For these realizations of c_{E1}, however, S's private expected payoff given early renegotiation is $\pi(r_1, r_2^e(r_1)) - r_1 + \alpha\{\phi(r_1) + \theta[v - c(r_1, 0)] - c_{E1}\}$ (Lemma 13), and so her marginal private return is

$$\begin{aligned} & \pi_1(r_1, r_2^e(r_1)) + \alpha\{\psi'(r_1) - \pi_1(r_1, r_2^e(r_1)) - \theta c_1(r_1, 0)\} - 1 \\ = \; & \alpha\{\psi'(r_1) - \theta c_1(r_1, 0)\} + (1 - \alpha)\pi_1(r_1, r_2^e(r_1)) - 1. \end{aligned}$$

The marginal private return of S from increasing r_1 exceeds the marginal social return, $-\theta c_1(r_1, 0) - 1$, if and only if $\alpha\psi'(r_1) + (1 - \alpha)\{\pi_1(r_1, r_2^e(r_1)) + \theta c_1(r_1, 0)\} \geq 0$, which is indeed satisfied because $r_1 \leq r_1^*$ and (2.19) imply $\psi'(r_1) \geq 0$ while $r_1 \leq r_1^*$ and footnote 29 imply $\pi_1 + \theta c_1 \geq 0$.

(iii) If $\phi(r_1) + \theta[v - c(r_1, 0)] < c_{E1}$, early breach never occurs. The continuation social surplus is $v - \psi(r_1) - x_2 + \pi(r_1, r_2^e(r_1)) - r_1$ from these realizations of c_{E1}, and the marginal social return from increasing r_1 is $\pi_1(r_1, r_2^e(r_1)) - \psi'(r_1) - 1$, which is less than $\pi_1(r_1, r_2^e(r_1)) - 1$, i.e. S's marginal private return (recall $r_1 \leq r_1^*$ and (2.19) implies $\psi'(r_1) \geq 0$).

So to summarize case (A), when $r_1 \leq r_1^*$, S's marginal net private return from increasing r_1 slightly is weakly greater than the marginal net social return for all realizations of c_{E1}. Hence $\Pi'(r_1) \geq S'(r_1) \geq 0$ when $r_1 \leq r_1^*$.

Case (B). If $r_1 \geq r_1^*$, then $\psi(r_1) + x_2 - x_1 \geq \phi(r_1) + \theta[v - c(r_1, 0)]$. S's payoffs for all possible realizations of c_{E1} are depicted in Figure 2.5.

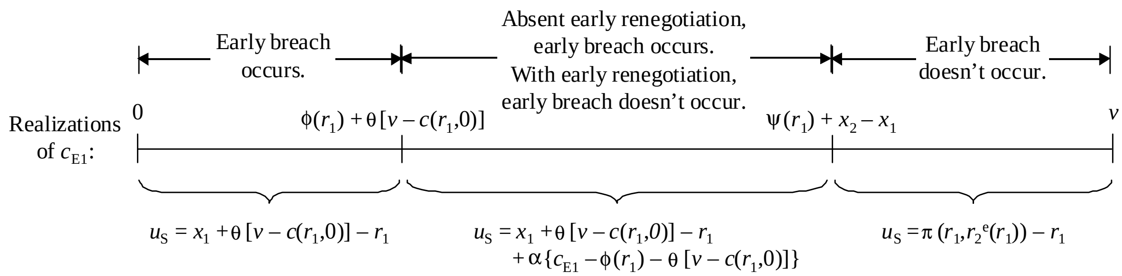

Figure 2.5. Seller's payoffs after early renegotiation in Case (B), where $r_1 \geq r_1^*$.

Similary to the previous case, it can be shown that S's marginal net private return to increasing r_1 slightly is weakly less than the marginal net social return for all c_{E1}. Therefore $\Pi'(r_1) \leq S'(r_1) \leq 0$ for all $r_1 \geq r_1^*$.

Hence, given any a contract (p, x_1, x_2) where x_1 and x_2 are the efficient expectation damages (satisfying (2.8) and (2.9)), S's privately optimal early investment (the value of r_1 that maximizes $\Pi(r_1)$) is indeed the efficient one, r_1^*.

This concludes the proof of Proposition 10.

To summarize, any contract (p, x_1, x_2) where x_1 and x_2 are the efficient expectation damages specified in (2.8) and (2.9) will induce S to choose the ex-ante efficient early investment r_1^*. The work above shows that early renegotiation then leads to B making the efficient early breach decision, S making the efficient late investment, and B making the efficient late breach decision.

Proposition 10 says that the same contract that implements the efficient outcome when renegotiation is not possible also implements the efficient outcome when renegotiation is possible. Therefore, renegotiation will not occur on the equilibrium path. Nevertheless, it is crucial in establishing Proposition 10 to consider the payoffs of the parties from choices made off the equilibrium path.

2.7. An Application

Consider once again the model without renegotiation or mitigation effort (so that the probability of finding an alternative buyer is exogenous). One application of this model is to study the way in which hotels structure their fees for cancellation of a reservation. There are usually different cancellation policies for reservations during the high season versus the low season. For example, the following is a summary of the deposit and cancellation policies of The Lodge at Vail, a ski resort in Vail, Colorado.[31]

> *Deposit Policies*: In the winter season, a 50% deposit is due at the time of booking. The remaining balance of the deposit is then due 45 days prior to the arrival. In spring, summer, and fall seasons, no deposit is required.
>
> *Cancellation Policies*: In the winter season, a full refund, less the first night's room and tax, will be given if reservations are cancelled more than 45 days prior to arrival. However, there will be a full forfeiture of the entire deposit if cancelled within 45 days of arrival. In spring, summer, and fall seasons, one night's deposit will be forfeited if cancellation occurs within 24 hours of arrival.[32]

In the case of The Lodge at Vail, their penalities for breach of contract (cancelling the reservation) are increasing as one approaches the date of performance (start of the

[31] See http://lodgeatvail.rockresorts.com. For the cancellation policy, see http://lodgeatvail.rockresorts.com/info/rr.fees.asp.

[32] Even though no deposit it required at the time a reservation is made in the spring, summer, or fall season, one night's deposit is still charged to the guest if cancellation occurs within 24 hours of arrival.

reserved stay), regardless of the time of the year. However, due to presumably higher demand in the winter season for ski resorts, the difference in their penalties between between cancelling late and cancelling early is larger during the winter than during other times of the year (ignoring the seasonal difference in the definitions of what constitutes a late breach). This choice of breach damages is consistent with the assumption that it is impossible (or in general, more difficult) to find an alternative buyer if breach occurs late, and the fact that it is easier (by definition) to find an alternative buyer in case of early breach during the high season than low season.

In order to precisely apply the model to this lodging industry example, the parameter θ should, strictly speaking, be interpreted as the probability of finding an alternative buyer/guest (upon early breach) to fill the *same* room that was vacated by the incumbent buyer/guest who breached the original contract. (For example, the seller/hotel may be booked to capacity at the time that the original contract is breached.) Otherwise, without a binding capacity constraint, the seller may be able to accommodate another buyer even if early breach does not occur.

Note that the seller/hotel is less likely to be booked to capacity during the low season than during the high season, which is consistent with θ being lower during the low season. Furthermore, whether breach is considered late or early in the low season depends on whether it occurs within 24 hours prior to arrival; whereas during the high season breach is considered late if it occurs within 45 days prior to arrival. The shorter prior notice requirement for early breach during the low season is also consistent with θ being lower during the low season.

To formalize the connection between the Lodge at Vail example and the model, suppose that the price of the entire reserved stay can be written as np^s, where p^s is the price per night, with $s \in \{H, L\}$ denoting the season, and n is the number of nights. Assume that the price is higher during the high season than during the low season, or $p^H > p^L$ (presumably, short-run supply in the lodging industry is fixed), and that the stay is for at least $n > \frac{p^L}{p^H} + 1$ nights. Then the Lodge at Vail's policy is such that during the high (winter) season, $x_2^H - x_1^H = np^H - p^H = (n-1)p^H$, which exceeds the analogous difference $x_2^L - x_1^L = p^L - 0 = p^L$ during the low season. Thus this example is consistent with the second inequality in Corollary 9. Note that the Lodge at Vail's policy also satisfies $x_2^H = np^H > p^L = x_2^L$, i.e., the penalty for cancelling a reservation at the last minute is larger in the high season than in the low season. If the model formally accounts for seasonal variations in the contract price, then this observation would again be consistent with the model's predicted efficient expectation damages for late breach. (This claim follows from replacing p with p^H and p^L in (2.9) and noting that $(r_1^*, r_2^*(r_1^*))$ do not depend on p).

Finally observe that both results in Corollary 9 could have been obtained even if the seller does not make any investments, or if she only invests before the first breach decision. If the seller only invests before the first breach decision, efficient investment and breach decisions can be induced by $x_2 = p - c(r_1^*)$ and $x_1 = p - c(r_1^*) - \theta[v - c(r_1^*)]$ so that $x_2 - x_1 = \theta[v - c(r_1^*)] > 0$. Similarly, if the seller does not make any investments ($r_1^* \equiv 0$), efficient breach decisions can still be induced with (x_1, x_2) satisfying $x_2 - x_1 = \theta[v - c(0)] > 0$. Therefore, an empirical investigation is necessary to determine whether, and how, a seller's investments affect the difference in her chosen penalties for late breach versus early breach in reality. However, regardless of whether, and when, the seller makes investments, the models predict that the difference in the penalties for late breach versus early breach, $x_2 - x_1$, is increasing in θ, the likelihood of finding an alternative buyer if breach occurs early.

2.8. Conclusion

This paper studies optimal liquidated damages when breach of contract is possible at multiple points in time. It suggests that when the potentially breached-against party makes sequential investment decisions, efficient breach damages should increase over time so as to make the potentially breaching party internalize those increasing opportunity costs. This provides an intuitive explanation for why fees for cancelling some service contracts, such as hotel reservations, tend to increase as the time for performance approaches.

Furthermore, when the investing party may be able to find an alternative trading partner when breach occurs early but not when breach occurs late, it is shown that the amount by which the damage for late breach exceeds the damage for early breach is increasing in the probability of finding an alternative trading partner. This provides one possible explanation for why hotels tend to charge larger penalties for late cancellation of high-season reservations than late cancellation of low-season reservations.

When an incumbent seller, as the potentially breached-against party, can affect the probability of finding an alternative buyer, her private incentives to mitigate breach damages are shown to be socially insufficient whenever she does not have full bargaining power vis-a-vis the alternative buyer. This is because while mitigation costs are always borne entirely by the incumbent seller, the benefits of mitigation are shared whenever the alternative buyer has some bargaining power. However, if breach is defined as not only a function of whether the incumbent buyer refuses trade, but also a function of whether the incumbent seller is able to trade with an alternative buyer, then the incumbent seller's mitigation incentives may be insufficient even if she has full bargaining power with the alternative buyer.

Finally, it is shown that when the incumbent buyer and seller are able to renegotiate their original contract after the arrival of each perfectly competitive entrant, the socially efficient breach and investment decisions can still be implemented with the same efficient expectation damages that implement the first best outcome absent renegotiation.

CHAPTER 3

Dynamic Merger Enforcement

3.1. Introduction

The private and social desirability of horizontal mergers have traditionally been analyzed in static environments.[1] In reality, however, horizontal mergers proposals are not made in isolation but rather are often put forth in response to previous mergers or in anticipation of future mergers. Thus, to the extent that firms and antitrust authorities take into account future profits and welfare, any static model in which mergers are proposed and evaluated in isolation will necessarily be incomplete. Moreover, merger enforcement policies have dynamic implications as well, since changes in policies may induce changes in the set of firms that propose to merge in the future. The goal of this paper is to understand how these dynamic considerations affect firms' merger proposal decisions and an antitrust authority's optimal merger enforcement policy.

Although the literature on horizontal mergers is immense, far less research has been done on horizontal mergers in dynamic environments. Among those papers that do model horizontal mergers using dynamic models, most focus primarily on equilibrium merger decisions (see, for example, Kamien and Zang (1990), Nilssen and Sorgard (1998), Gowrisankaran (1999), Fauli-Oller (2000), and Pesendorfer (2005)).

[1] See, for example, Salant, Switzer, and Reynolds (1983), Davidson and Deneckere (1985), and Farrell and Shapiro (1990).

Recently, however, some attention has been paid to optimal merger enforcement policies. Motta and Vasconcelos (2005), for example, compare the merger policy implications of two types of antitrust authorities: myopic ones that only consider a proposed merger's consumer-surplus consequences in the current period, and forward-looking ones that anticipate future market structures when evaluating merger proposals.

Unlike Motta and Vasconcelos, who consider a model of Cournot competition with linear demand and four firms initially, Nocke and Whinston (2007) assume an arbitrary number of firms and a more general demand function. Nocke and Whinston show that when the set of possible mergers is disjoint, a completely myopic consumer-surplus-based approval policy maximizes discounted consumer surplus for every possible realization of the set of feasible mergers.

Both of the above two papers derive their results analytically. In contrast, Gowrisankaran (1997 and 1999) and this paper use computational techniques to study Markov perfect equilibria in an infinitely repeated game with mergers, exit, entry, and investment. Although both of Gowrisankaran's papers endogenize the merger formation process, only Gowrisankaran (1997) addresses the issue of merger enforcement, doing so by restricting attention to concentration thresholds as the only policy instrument. While concentration threshold policies are widely used in practice and relatively easy to model, it is not clear why they should fully characterize the optimal merger enforcement rule in a dynamic environment. Therefore, one goal of this paper is to characterize the optimal enforcement policy in such a dynamic model.

Section 3.2 presents the details of the model, which closely follows that of Gowrisankaran (1999). Firms make merger, exit, entry, and investment decisions every period (in that order) in an infinitely repeated game. The solution concept is assumed to be Markov perfect Nash equilibrium, so firms' actions are restricted to be functions of payoff-relevant state variables. The main differences between Gowrisankaran (1999) and this paper lies in the merger process. Gowrisankaran endogenizes the merger formation process at the start of each period by allowing firms to bid for the right to acquire one another. In comparison, I simplify the merger process by assuming that two randomly chosen active firms are given the opportunity to merge each period. Although this assumption sacrifices realism, I adopt it in order to focus more attention on the characterization of merger enforcement policies (which are considered by Gowrisankaran's 1997 paper but not his 1999 paper).

I also study how the set of firms that propose to merge changes when there are changes in the welfare standard and/or merger enforcement policy. In a static model, if proposing a merger is costless, then the choices of the welfare standard and merger enforcement policy have no effect on which mergers are profitable, i.e., the set of firms that propose to merge. In a dynamic model, however, where future mergers may be proposed, a change in the welfare standard, or more generally, a change in the merger enforcement policy, *does* have an effect on the set of firms that propose to merge in any given period. This is because whether a merger is profitable (and hence proposed) today depends on payoffs in future periods, which depend on any future mergers that may occur, which in turn depends on the future merger enforcement policy. Therefore, merger enforcement decisions in the future have a dynamic feedback effect on the set of mergers that are proposed in the current period, an effect which is missing in any static model of merger proposals and enforcement.

Section 3.3 contrasts the dynamic (forward-looking) merger enforcement policy with the myopic merger enforcement policy. The details of how the model is computed is described in Section 3.4. In particular, I assume no commitment by the antitrust authority, and compute the equilibrium policies and value functions of the firms and antitrust authority using value function iteration. Finally, Section 3.5 uses simulation results to (i) compare the implications of different enforcement policies for industry evolution and merger proposal and approval decisions, (ii) quantify the welfare differences between the different policies, and (iii) perform comparative dynamics exercises. Section 3.6 briefly concludes.

In its current state, this paper should be thought of as a first attempt at understanding merger enforcement policies in a dynamic environment. While some of the simulation and comparative dynamics results are intuitive, others findings are not and require further investigation. In addition, the process by which potential merging firms are identified has not been endogenized, so the predictions of the model may not be valid to the extent that potential merging parties in an industry are not randomly chosen. In order for this model to meaningfully inform our understanding of the dynamic implications of merger enforcement policies, more work is needed to narrow the gap between the assumptions of the model and the realities of dynamic oligopolistic competition with endogenous mergers.

3.2. Model

The model is based on Gowrisankaran (1999), which in turn builds on the framework of Ericson and Pakes (1995) for modeling Markov perfect industry dynamics. Time is discrete with an infinite horizon. The merger subgame I consider is simpler than that of

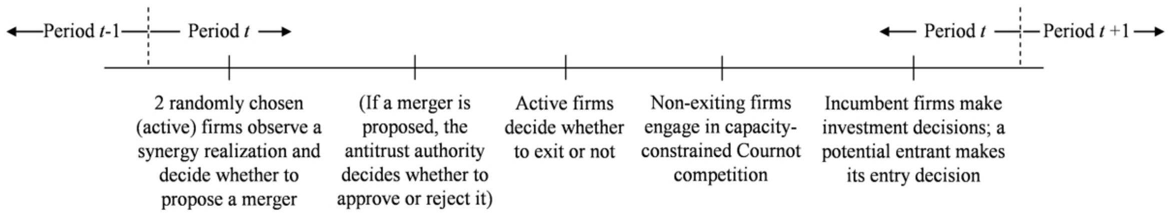

Figure 3.1. Within-period time line.

Gowrisankaran (1999), and the other parts of each stage game (entry, investment, and exit decisions) are the same. However, unlike Gowrisankaran (1999), I attempt to characterize the optimal merger enforcement policy.

I assume two randomly chosen (active) firms decide at the beginning of each period whether to propose a merger after observing the synergy realization. If they propose a merger, an antitrust authority then decides whether to approve or reject the merger, with the merger occurring immediately if it is approved. Merging firms combine their capacities, and their post-merger value reflects an idiosyncratic shock, or synergy, which will be detailed below in Section 3.2.4. Each firm then decides whether to exit or not, with non-exiting firms playing a static game of capacity-constrained Cournot competition (with complete information). Finally, incumbent firms make investment decisions to increase their capacity next period, and a potential entrant makes its entry decision. See Figure 3.1 for a summary of the within-period time line.

Although the identities of the merging parties in each period are chosen exogenously, firms take into account the possibility that they (or their competitors) may be able to merge in the future when calculating their value from being in any state. Denote by $N < \infty$ the maximum number of active firms in any period, and let $\Omega = [0, 1, ..., \omega_{\max}]$

denote the set of possible states for each firm. The state vector in any period can then be written as $\omega = (\omega_1, \omega_2, ..., \omega_N)$, where firm i's capacity is given by $\tau \cdot \omega_i$, with $\tau > 0$ being an exogenous parameter.

I now examine each part of the stage game in greater detail, proceeding in reverse chronological order.

3.2.1. Entry and Investment

The evolution of each firm's state is determined by a Markov process that depends on its current state, its current investment, and an exogenous industry-wide shock. More specifically, if a firm i has a capacity of $\tau \cdot \omega_{i,t}$ in period t, its capacity in period $t+1$ is given by $\tau \cdot \omega_{i,t+1}$, with

$$\omega_{i,t+1} \equiv \omega_{i,t} + v_{i,t} - \overline{v}_t,$$

where $v_{i,t}$ represents firm i's random return to its investment $x_{i,t}$ and has the distribution

$$v_{i,t} = \begin{cases} 1, & \text{with probability } \frac{\alpha x_{i,t}}{1+\alpha x_{i,t}} \\ 0, & \text{with probability } \frac{1}{1+\alpha x_{i,t}}. \end{cases}$$

The variable $\overline{v}_t$ represents an industry-wide shock to every firm's efficiency level in period t and is distributed

$$\overline{v}_t = \begin{cases} 1, & \text{with probability } \theta \\ 0, & \text{with probability } 1-\theta. \end{cases}$$

Parameters α and θ are exogenously specified.

If there are $J < N$ firms active in period t, then at the same time that incumbents make investment decisions $(x_1(\omega), ..., x_J(\omega))$ at the end of the period, a potential entrant makes its entry decision. The entry cost is drawn from a uniform distribution on $[x_E^{MIN}, x_E^{MAX}]$ and is observed by the potential entrant before it decides whether to enter. If it decides to pay the entry cost in period t, the potential entrant enters in period $t+1$ with capacity $\tau \cdot (\omega_E - \overline{v}_t)$, where $\omega^E \geq 1$ is an exogenously chosen parameter.

Let $V_E(\omega)$ be the potential entrant's gross value from entry, excluding the entry cost. Let $V(\omega, i)$ denote the value to an incumbent firm from being in position i in state ω. Finally, let η be a function (mapping $\Omega \times \{1, 2, ..., N\}$ into itself) which takes a state vector ω that is not necessarily in weakly descending order and the index of a firm, and maps them into the weakly descending version of ω and the new index of the same firm. Then, assuming $J < N$ so that there is room for an entrant, we have

$$V_E(\omega) = \beta \sum_{v_{1t},...,v_{Jt},\overline{v}_t \in \{0,1\}} \Pr[v_{1t}, ..., v_{Jt}, \overline{v}_t | x_1(\omega), ..., x_J(\omega)] \\ \times V(\eta[\omega_1 + v_{1t} - \overline{v}_t, ..., \omega_J + v_{Jt} - \overline{v}_t, 0, ..., 0, \omega_E - \overline{v}_t], N).$$

Let $\chi_E(\omega)$ denote the probability of entry. Recall that there is room for entry whenever $J < N$, or equivalently, $\omega_N = 0$. Since the entry cost is drawn uniformly from $[x_E^{MIN}, x_E^{MAX}]$, we have

$$\chi_E(\omega) = \begin{cases} 0, & \text{if } \omega_N > 0 \\ \max\left\{0, \min\left\{1, \frac{V_E(\omega) - x_E^{MIN}}{x_E^{MAX} - x_E^{MIN}}\right\}\right\}, & \text{if } \omega_N = 0. \end{cases} \tag{3.1}$$

(Note that when the firm with the lowest capacity – firm N – is active, i.e., has a positive capacity $\tau \cdot \omega_N > 0$, the probability of entry is zero because there is no room for the entrant to enter in this period.) Define a random variable e_t that equals 1 if entry occurs and 0 otherwise. So $\Pr[e_t = 1|\omega] = \chi_E(\omega)$.

Each of the J active incumbents solves the following optimal investment problem:

(3.2)

$$x_i(\omega) = \arg\max_{\widehat{x}_i \geq 0} \left\{ \begin{array}{c} -\widehat{x}_i + \beta \sum_{v_{1t},...,v_{Jt},\overline{v}_t,e_t \in \{0,1\}} \Pr[v_{1t},...,v_{Jt,}\overline{v}_t, e_t | x_1(\omega),...,x_{i-1}(\omega), \\ \widehat{x}_i, x_{i+1}(\omega),...,x_J(\omega),\chi_E(\omega)] \\ \times V(\eta[\omega_1 + v_{1t} - \overline{v}_t,...,\omega_J + v_{Jt} - \overline{v}_t, 0,...,0, e_t(\omega_E - \overline{v}_t)], i). \end{array} \right\}$$

In order to solve for an analytic solution to (3.2), it is helpful to define the values that a firm in position i in state ω obtains conditional on its own investment being successful or not as $V(\omega, i|v_{it} = 1)$ and $V(\omega, i|v_{it} = 0)$, respectively. Then

$$\begin{aligned} V(\omega, i|v_{it} &= 1) = \sum_{\substack{v_{1t},...,v_{i-1,t},,v_{i+1,t},\\ ...,v_{Jt,}\overline{v}_t,e_t \in \{0,1\}}} \Pr\left[\begin{array}{c} v_{1t},...,v_{i-1,t,},v_{i+1,t,},...,v_{Jt,}\overline{v}_t, e_t \\ |x_1(\omega),...,x_{i-1}(\omega),x_{i+1}(\omega),...,x_J(\omega),\chi_E(\omega)] \end{array} \right] \\ &\times V\left(\begin{array}{c} \eta[\omega_1 + v_{1t} - \overline{v}_t,...,\omega_{i-1} + v_{i-1,t} - \overline{v}_t, \omega_i + 1 - \overline{v}_t, \\ \omega_{i+1} + v_{i+1,t} - \overline{v}_t,...,\omega_J + v_{Jt} - \overline{v}_t, 0,...,0, e_t(\omega_E - \overline{v}_t)], i \end{array} \right), \end{aligned}$$

and

$$\begin{aligned} V(\omega, i|v_{it} &= 0) = \sum_{\substack{v_{1t},...,v_{i-1,t},,v_{i+1,t},\\ ...,v_{Jt,}\overline{v}_t,e_t \in \{0,1\}}} \Pr\left[\begin{array}{c} v_{1t},...,v_{i-1,t,},v_{i+1,t,},...,v_{Jt,}\overline{v}_t, e_t \\ |x_1(\omega),...,x_{i-1}(\omega),x_{i+1}(\omega),...,x_J(\omega),\chi_E(\omega) \end{array} \right] \\ &\times V\left(\begin{array}{c} \eta[\omega_1 + v_{1t} - \overline{v}_t,...,\omega_{i-1} + v_{i-1,t} - \overline{v}_t, \omega_i - \overline{v}_t, \\ \omega_{i+1} + v_{i+1,t} - \overline{v}_t,...,\omega_J + v_{Jt} - \overline{v}_t, 0,...,0, e_t(\omega_E - \overline{v}_t)], i \end{array} \right). \end{aligned}$$

By the law of total probability, (3.2) can then be written as

$$x_i(\omega) = \arg\max_{\widehat{x}_i \geq 0} \left\{ -\widehat{x}_i + \beta \left(\tfrac{\alpha \widehat{x}_i}{1+\alpha \widehat{x}_i} V(\omega, i | v_{it} = 1) + \tfrac{1}{1+\alpha \widehat{x}_i} V(\omega, i | v_{it} = 0) \right) \right\}. \tag{3.3}$$

The first order condition implies that the optimal investment for firm i in state ω is

$$x_i(\omega) = \frac{p}{\alpha(1-p)},$$

where

$$p \equiv \max \left\{ 0, 1 - \sqrt{\frac{1}{\alpha\beta \left\{ V(\omega, i | v_{it} = 1) - V(\omega, i | v_{it} = 0) \right\}}} \right\}$$

is the equilibrium probability that firm i's investment $x_i(\omega)$ is successful.

3.2.2. Static Competition

In each period, before entry and investment decisions are made, active incumbent firms play a static game of Cournot competition with capacity constraints, under complete information. Given a state vector $\omega = (\omega_1, \omega_2, ...\omega_N)$, firm i's capacity is given by $\tau \cdot \omega_i$, with $\tau > 0$ common to all firms. Every firm's constant marginal cost is given by $mc > 0$. The (inverse) demand function is assumed to be linear and given by $P(Q) = D - Q$, where $D > 0$ is a market-size parameter and $Q = \sum_j q_j$ is total output. Suppose every firm has the same fixed cost $f > 0$.

Because quantity is not a payoff-relevant state variable, restricting attention to Markov perfect equilibria ensures that each firm i will play its static Cournot equilibrium strategy, i.e., choose its quantity q_i to maximize its static profits:

$$\max_{0 \leq q_i \leq \tau \cdot \omega_i} \left[\left(D - \sum_j q_j \right) - mc \right] q_i - f.$$

Let $q_i^*(\omega)$ denote the unique solution to this problem for each active firm $i = 1, ..., J$, and let $p^*(\omega)$ be the associated market clearing price. The payoff for each active firm $i = 1, ..., J$ in this static game is therefore

$$\pi(\omega, i) = [p^*(\omega) - mc] \cdot q_i^*(\omega) - f.$$

By definition, $q_i^*(\omega) \equiv 0$ for each inactive firm, so $\pi(\omega, i) \equiv 0$ for all $i = J + 1, ..., N$. Let $\Pi_i(\omega) \equiv \pi(\omega, i)$ for all $i = 1, ..., N$, so that $(\Pi_1(\omega), ..., \Pi_J(\omega), 0, ..., 0)$ denotes static profits, where the zeros appear whenever $J < N$, or there are fewer active firms (J) than the maximum possible number of firms (N).

3.2.3. Exit Decisions

Immediately before the static competition stage, each incumbent firm decides whether to exit. A firm will choose to exit if and only if its scrap value from exiting, ϕ, exceeds its expected discounted value from continuing. Define $(\chi_{x_1}(\omega), .., \chi_{x_J}(\omega))$ to be the vector of exit decisions, where $\chi_{x_i}(\omega) = 1$ if firm i remains active in state ω and $\chi_{x_i}(\omega) = 0$ if firm i exits. Given the equilibrium investment and entry strategies $(x_1(\cdot), ...x_J(\cdot), x_E(\cdot))$

and equilibrium static profits $(\Pi_1(\cdot), ..., \Pi_N(\cdot))$, the equilibrium exit decisions satisfy

$$\chi_{x_i}(\omega) = \begin{cases} 1 & \omega_i > 0, \text{ and} \left\{ \begin{array}{c} -x_i(\overline{\omega}^i) + \Pi_i(\overline{\omega}^i) \\ +\beta \left[\begin{array}{c} \frac{\alpha x_i(\overline{\omega}^i)}{1+\alpha x_i(\overline{\omega}^i)} V(\overline{\omega}^i, i | v_{it} = 1) \\ + \frac{1}{1+\alpha x_i(\overline{\omega}^i)} V(\overline{\omega}^i, i | v_{it} = 0) \end{array} \right] \end{array} \right\} > \phi \\ 0 & \text{otherwise,} \end{cases} \tag{3.4}$$

where $\overline{\omega}^i \equiv (\chi_{x_1}(\omega), ..., \chi_{x_{i-1}}(\omega), 1, \chi_{x_{i+1}}(\omega), ..., \chi_{x_J}(\omega)) \cdot \omega$. Recall that within the stage game, the exit decision precedes static competition, which precedes the investment decision. Therefore, unlike its investment decision (3.3), a firm takes into account its payoff from static competition when making its exit decision (3.4). The *post-merger-decision* value function $V^{PM}(\omega, i)$ for a firm in position i in state ω can then be written as

$$\begin{aligned} V^{PM}(\omega, i) &= \left[1 - \chi_{x_i}(\omega)\right] \cdot \phi + \chi_{x_i}(\omega) \\ &\times \left\{ \begin{array}{c} -x_i(\overline{\omega}) + \Pi_i(\overline{\omega}) \\ +\beta \left[\frac{\alpha x_i(\overline{\omega})}{1+\alpha x_i(\overline{\omega})} V(\overline{\omega}, i | v_{it} = 1) + \frac{1}{1+\alpha x_i(\overline{\omega})} V(\overline{\omega}, i | v_{it} = 0) \right] \end{array} \right\}, \end{aligned}$$

where $\overline{\omega} \equiv \chi_x(\omega) \cdot \omega$ is the state after exit decisions have been made.

3.2.4. Merger Decision

At the start of each period, two randomly chosen firms are given the opportunity to merge. If firms i and j merge in state ω, their post-merger state becomes $\omega_M(\omega, i, j) \equiv \min\{\omega_i + \omega_j, \omega_{\max}\}$, and their joint capacity becomes $\tau \cdot \omega_M(\omega, i, j)$.

If firms ι and j are given the opportunity to merge in state ω, they will chose to do so if and only if it is profitable, i.e., if and only if their value as a merged firm exceeds their combined expected discounted values from continuing to the next period as separate firms. I assume that the merged firm's value reflects not only the expected discounted value of the merged firm $V^{PM}(\eta[\omega_M(\omega,i,j),\min\{i,j\}])$,[2] but also an additive term $s \cdot \delta$. The scaling parameter s is specified ex-ante. The value of δ is drawn from the discrete uniform distribution with support $\mathbb{K} \equiv \{-K, -(K-1), ..., -1, 0, 1, ..., K-1, K\}$, for some integer $K > 0$, and it is assumed to be commonly observed by all the firms before the merger decision is made.[3]

Let $m(\omega, i, j, \delta) \in \{0, 1\}$ represent the merger proposal decision of firms i and j in state ω given δ. That is, $m(\omega, i, j, \delta) = 1$ ("propose") if and only if a merger between firms i and j in state ω is profitable given δ, i.e.,

$$V^{PM}(\eta[\omega_M(\omega,i,j),\min\{i,j\}]) + s\cdot\delta > V^{PM}(\omega,i) + V^{PM}(\omega,j),$$

where $\omega_M(\omega, i, j)$ represents the state vector after firms i and j have merged in state ω. (Recall that η maps a state vector that is not necessarily in weakly descending order, and the index of a firm, into a state vector that is weakly descending, and the index of the same firm.) Assuming $i < j$, the (unordered) post-merger state vector $\omega_M(\omega, i, j)$ can be expressed as

$$\omega_M(\omega,i,j) = (\omega_1, ..., \omega_{i-1}, \omega_i + \omega_j, \omega_{i+1}, ..., \omega_{j-1}, 0, \omega_{j+1}, ..., \omega_J, 0, ..., 0). \tag{3.5}$$

[2] I assume, without loss of generality, that the merging firm with the smaller pre-merger capacity (firm $\max\{i,j\}$) shuts down and transfers its capacity to the merging firm with the larger pre-merger capacity (firm $\min\{i,j\}$). (Recall that state vectors are ordered in weakly descending order.) So the merged firm's state is the element that appears in the $\min\{i,j\}$ position of the state vector $\omega_M(\omega,i,j)$. Applying the function $\eta(\cdot,\cdot)$ reorders the post-merger state in descending order.

[3] For simplicitly, I assume δ is independent of the state ω and the identities of the merging firms i and j. Alternatively, one might imagine a situation where firms with lower (higher) pre-merger capacities are more likely to realize a large synergy from merging.

(If $j > i$, the expression for $\omega_M(\omega, i, j)$ is the same as in (3.5), except the places of i and j are switched.)

Let $\sigma(\omega, i, j, \delta) \in \{0, 1\}$ denote the antitrust authority's merger enforcement decision. It equals one if a proposed merger between firms i and j in state ω with δ will be approved, and zero if it will be rejected. See Section 3.3 below for the types of enforcement policies that will be considered. For now, it suffices to note that the merger enforcement policy is chosen by the antitrust authority at date $t = 0$, and that all firms observe this choice.

If firm i in state ω is chosen as one of the two potential merging parties, its expected discounted value from being in that position, before the identity of the other merging party has been realized (and so before the δ has been realized and before the merger proposal decision has been made), is given by[4]

$$V^{In}(\omega, i) = \sum_{j \in \{1,\dots,J\} \setminus \{i\}} \frac{1}{J-1} \sum_{\delta \in \mathbb{K}} \frac{1}{|\mathbb{K}|} \left\{ \begin{array}{c} m(\omega, i, j, \delta)\sigma(\omega, i, j, \delta) \\ \times \frac{1}{2}\{V^{PM}(\eta[\omega_M(\omega, i, j), \min\{i, j\}]) + s \cdot \delta\} \\ +[1 - m(\omega, i, j, \delta)\sigma(\omega, i, j, \delta)]V^{PM}(\omega, i) \end{array} \right\}$$

where it is assumed that each merging party receives half of the total value of the merged firm if the merger is proposed and approved. Obviously, in order for the gains from the merger to be realized, the merger must not only be proposed ($m(\omega, i, j, \delta) = 1$) but also approved ($\sigma(\omega, i, j, \delta) = 1$).

[4]Note that the term $\frac{1}{J-1}$ represents the probability of any of the other firms $j \neq i$ being chosen as the potential merging partner of firm i. The term $|\mathbb{K}|$ is the probability of any synergy value $\delta \in \mathbb{K}$ being realized.

If firm i in state ω is not chosen as one of the two potential merging parties, its expected discounted value from being in that position, before the identities of the potential merging parties have been realized, is

$$
\begin{aligned}
V^{Out}(\omega,i) &= \sum_{j,k\in\{1,\ldots,J\}\setminus\{i\},j\neq k} C(J-1,2)^{-1} \\
&\quad \sum_{\delta\in\mathbb{K}} \frac{1}{|\mathbb{K}|}\left\{\begin{array}{c} m(\omega,j,k,\delta)\sigma(\omega,j,k,\delta)V^{PM}(\eta[\omega_M(\omega,j,k),i]) \\ {[1-m(\omega,j,k,\delta)\sigma(\omega,j,k,\delta)]V^{PM}(\omega,i)} \end{array}\right\}
\end{aligned}
$$

where $C(J-1,2)\equiv\frac{(J-1)!}{2!(J-1-2)!}$ is the number of ways of randomly choosing 2 firms j and k among the set of $J-1$ active firms (excluding i), with order not mattering (so the probability of any two firms j and k being randomly selected is $\frac{1}{C(J-1,2)}=\frac{2!(J-3)!}{(J-1)!}=\frac{2}{(J-1)(J-2)}$).[5]

The expected discounted value of a firm i in state ω, before the identities of the two potential merging parties have been realized and assuming $J\geq 2$, is given by

$$V^{BM}(\omega,i)=\frac{2}{J}V^{In}(\omega,i)+\left(1-\frac{2}{J}\right)V^{Out}(\omega,i),$$

where $\frac{2}{J}$ is the probability that firm i is chosen as one of the two potential merging parties (among the J active firms).[6]

[5]Note that the expression for $V^{Out}(\omega,i)$ assumes $J\geq 3$. If $J=2$, both firms would certainly be "in," i.e., certain to be given the chance to merge (with each other). If $J=1$, no merger is possible and so $V^{Out}(\omega,i)\equiv V^{PM}(\omega,i)$.

[6]If $J=1$, a merger is not possible and so $V^{BM}(\omega,i)=V^{Out}(\omega,i)=V^{PM}(\omega,i)$.

3.3. Merger Enforcement Policies

In any period, if two firms wish to merge, they must obtain approval from an antitrust authority by making a merger proposal. Given a merger proposal, the enforcement decision is made before incumbent firms make their exit decisions. Suppose the antitrust authority makes its decision on a case-by-base basis, basing its approval/rejection decision on the current state of the world and the identities of the merging parties. Formally, a merger enforcement policy is a mapping σ from $\Omega \times \{1, 2, ..., J\}^2 \times \mathbb{K}$ to $\{0, 1\}$, where $\sigma(\omega, i, j, \delta) = 1$ if a proposed merger between firms i and j in state ω with synergy δ is approved, and $\sigma(\omega, i, j, \delta) = 0$ if the merger is rejected. If the firms chosen to make the merger decision do not propose to merge, set $\sigma(\omega, \emptyset, \emptyset, \delta) \equiv 0$.

3.3.1. Myopic Enforcement Policies

First, suppose the antitrust authority is myopic in that approval/rejection decisions are based solely on a merger's effect on consumer welfare or aggregate welfare in *the current period*. I call this decision rule the *myopic enforcement policy* and denote it by σ^M. It can be written as

$$\sigma^M(\omega, i, j, \delta) \equiv \arg \max_{\sigma \in \{0,1\}} (1 - \sigma) \cdot S(\omega) + \sigma \cdot S(\omega_M(\omega, i, j), \delta),$$

where $S(\omega_M(\omega, i, j), \delta)$ is welfare – either consumer surplus or total surplus – in the current period if the merger between firms i and j with δ in state ω is approved, and $S(\omega)$ is welfare in the current period if the merger is rejected.

3.3.2. Dynamic Enforcement Policies

Now assume the antitrust authority is far-sighted and takes into account future merger proposals. Let welfare from competition in period t be discounted by the same factor $\beta \in (0,1)$ with which firms discount their expected future payoffs. In this case, the antitrust authority's *dynamic enforcement policy* is to approve a proposed merger if and only if the expected discounted welfare is higher from approving the merger than from rejecting the merger.

Denote by $\sigma^D(\omega, i, j)$ the dynamic enforcement policy. Let $W^{\sigma}_{ij}(\omega, \delta)$ be the antitrust authority's expected discounted payoff from being in state ω and using some policy σ in the future, *after* it has been confronted with a merger proposal between firms i and j with δ.[7] Finally, let $W^{\sigma}(\omega)$ denote the antitrust authority's expected discounted payoff from being in state ω and using some policy σ in the future, *prior* to observing the merger proposal decision. We then have

$$\sigma^D(\omega, i, j, \delta) \equiv \arg \max_{\sigma \in \{0,1\}} \left\{ \begin{array}{c} (1-\sigma) \cdot [S(\omega) + \beta E_{\omega'} \{W^{\sigma}(\omega')|\omega\}] \\ +\sigma \cdot [S(\omega_M(\omega, i, j), \delta) + \beta E_{\omega'} \{W^{\sigma}(\omega')|\omega_M(\omega, i, j)\}] \end{array} \right\},$$

where $S(\omega_M(\omega, i, j), \delta)$ is welfare – either consumer surplus or total surplus – in the current period if the merger between firms i and j with δ in state ω is approved, and $S(\omega)$ is welfare in the current period if the merger is rejected.

[7] I assume the synergy δ is observable to the antitrust authority before it makes its enforcement decisions. If this were not the case, there is an inference issue which needs to be addressed: namely, by observing a merger proposal, the asymmetrically informed antitrust authority would infer that the value of δ is such that it is profitable for the merging parties to want to merge. See Chapter 1 for a static model that deals with this issue.

Upon being confronted with a merger proposal between firms i and j with δ in state ω, the expected discounted welfare given a policy σ can be written as

$$\begin{aligned} W_{ij}^{\sigma}(\omega,\delta) &= (1-\sigma)\cdot\left[S(\omega)+\beta E_{\omega'}\left\{W^{\sigma}(\omega')|\omega\right\}\right] \\ &\quad +\sigma\cdot\left[S(\omega_M(\omega,i,j),\delta)+\beta E_{\omega'}\left\{W^{\sigma}(\omega')|\omega_M(\omega,i,j)\right\}\right]. \end{aligned}$$

Finally, the antitrust authority's expected discounted payoff from being in state ω and using the policy σ, prior to observing the merger proposal decision, can be written as

$$W^{\sigma}(\omega)=\sum_{i,j\in\{1,\ldots,J\},i\neq j} C(J,2)^{-1}\left\{\frac{1}{|\mathbb{K}|}\sum_{\delta\in\mathbb{K}}\Pr[m(\omega,i,j,\delta)=1]W_{ij}^{\sigma}(\omega,\delta)\right\},$$

where recall that $C(J,2)^{-1}$ is the probability of any two firms being randomly selected (to have the opportunity to merge).

Note that this policy assumes no commitment by the antitrust authority, because I am implicitly holding firms' strategies fixed when computing the antitrust authority's optimal dynamic policy. In other words, holding fixed the antitrust authority's welfare standard, changes in its policy do not induce changes in the firms' strategies.

3.4. Computing the Model

The model in this paper is computed numerically using programs written in Matlab 7, several of which build upon the work of Gautam Gowrisankaran. The programs are based on the Pakes and McGuire algorithm for computing Markov perfect Nash equilibria.[8] The computational burden of these types of algorithms increase very quickly in the size of the

[8]See http://post.economics.harvard.edu/faculty/pakes/program.html for the publically available version of the code, which does not model mergers or merger enforcement.

Table 3.1. Parameter Values of the Benchmark Model

Parameter	Description	Value
N	Maximum number of active firms	3
$\omega_{\max}$	Maximum state for each firm	2
K	Maximum merger synergy	1
s	Synergy scaling factor	0.4
α	Investment efficiency parameter	3.5
θ	Probability of negative industry shock	0.5
f	Fixed cost	0.01
D	Demand function's vertical intercept	2.5
ϕ	Scrap value	0
mc	Marginal cost	1
τ	Capacity parameter	0.1
ω_E	State at which entrants enter	1
$[x_E^{MIN}, x_E^{MAX}]$	Range of entry costs	[0, 0.6]
β	Discount factor	0.925

state space and the maximum number of firms. I have therefore limited myself to a few firms and possible states in order to reduce computation time. (Recent advances have been made that reduce the time it takes to compute equilibria of dynamic stochastic games. These works have employed techniques such as continuous-time models and approximation methods. See Doraszelski Judd (2006) for an example of the former approach and Weintraub, Benkard, and Van Roy (2007) for an example of the latter.)

Before compiling the program, the exogenous parameters of the model have to be specified. Table 3.1 presents the parameter values of the benchmark model.

The first step in computing the model is to calculate the firms' profits from static competition as functions of the state vector. (Recall that the state vector affects firms' payoffs because they are mapped into firms' capacity constraints.) During this step, the program also computes, for every possible state, each firm's output, market share, price-cost margin, investment level, and the probability of entry. Since the outcomes associated with static competition do not depend on firms' value functions, these calculations are performed only once and reused during each iteration described below.

Next, given a merger enforcement policy – i.e., whether the antitrust authority is myopic or farsighted, and whether the welfare standard is consumer surplus or total surplus – the equilibrium policies and value functions of the firms and antitrust authority are computed using value function iteration. Under this method, the program first assigns initial guesses to the value functions and policies and then repeatedly updates them given the old value functions and old policies. During each iteration, while taking as given the old value functions and policies from the previous iteration, the program computes the firms' and antitrust authority's new policies – the entry and investment decisions, exit decisions, and merger proposal and approval decisions, respectively – for each state using the framework described in Sections 3.2 and 3.3. These new policies are then used to compute the new value functions. The program terminates, i.e. is said to converge to an equilibrium, when two consecutive iterations of the value functions are sufficiently close to each other.[9]

As in other models of this type, a multiplicity of equilibria cannot be ruled out. And even if a unique equilibrium exists, convergence cannot be guaranteed for all values of the model parameters. Because only appropriately chosen values for the model's parameters will result in convergence to a single equilibrium, sacrificing robustness of the model's predictions is unavoidable to some extent if any results are to be obtained. (See the discussion at the beginning of Section 3.5.4 below for more on this trade-off.)

[9]See Pakes, Gowrisankaran, and McGuire (1993) for additional computational details.

Finally, the model is simulated for a large number of periods in order to (i) characterize firms' merger, exit, entry, and investment decisions and the industry evolution, (ii) compare the welfare differences implied by different merger enforcement policies, and (iii) perform comparative statics exercises on the effects of changes in parameter values.

When discussing the simulation results below, it is important to remember that this computational algorithm assumes no commitment by the antitrust authority. First, equilibrium strategies for firms and the antitrust authority – i.e., firms' entry, exit, and investment decisions, and the antitrust authority's approval/rejection decisions – are computed as described above. These equilibrium strategies are then used to simulate the industry evolution by taking many repeated draws of the model's random variables.[10] Section 3.5 below characterizes the industry evolution and welfare implied by this simulation process. (In contrast, for the case of commitment by the antitrust authority, one would have to (1) fix a policy for the antitrust authority and calculate firms' equilibrium strategies given that policy, (2) simulate the equilibrium many times and compute the implied mean expected discounted welfare, and (3) repeat this procedure for every possible enforcement policy to determine which policy results in the highest mean expected discounted welfare.)

3.5. Results

3.5.1. Proposed and Approved Mergers

Consider the myopic total surplus (TS) policy and the dynamic TS policy. Given each merger enforcement policy, we can calculate, for every merger that may be proposed, the

[10]These random variables include the merger synergy δ, individual investment shocks $v_{i,t}$, aggregate efficiency shock $\overline{v}_t$, and the entry cost.

Table 3.2. Proposed and Approved Mergers

State	Firm A	Firm B	Synergy	Myopic TS		Dynamic TS	
				Propose?	Approve?	Propose?	Approve?
(1,1,0)	1	2	0	0	0	1	0
(1,1,0)	1	2	1	1	1	1	0
(1,1,1)	1	2	1	1	1	1	0
(1,1,1)	1	3	1	1	1	1	0
(1,1,1)	2	3	1	1	1	1	0
(2,1,1)	2	3	1	1	0	1	1

equilibrium approval/rejection decisions of the antitrust authority. Table 3.2 illustrates, for each merger that is proposed under either the myopic TS policy or the dynamic TS policy, the state in which it is proposed, the firms' identities, the merger synergy (δ), and the approval/rejection decision under each policy.

First of all, note that no merger proposals are made when the merger synergy is negative, regardless of whether the antitrust authority is myopic or far-sighted. In these cases, the increased market power from a merger is more than off set by the negative shock to the merged firm's value from a negative merger synergy.[11]

The myopic TS policy approves more mergers than the dynamic TS policy. It is interesting to note that among the five state-firms-synergy combinations for which a merger is *proposed* under both policies, there is no single combination of state-firms-synergy for which a merger is *approved* under both policies. In particular, under the myopic policy, a proposed merger with positive synergy is approved only if no firm has reached its maximum capacity. In contrast, under the dynamic policy, a proposed merger (between firms 2 and 3) is approved only if the non-merging firm (firm 1) has reached its maximum capacity. These differences in the approval decisions of these two policies

[11]Of course, this interpretation and any subsequent ones should be qualified as being applicable only for the chosen set of initial parameter values (see Table 3.1).

reflect the fact that the dynamic policy, by accounting for future payoffs and the possibility of future mergers, recognizes that expected discounted total surplus from approving a proposed merger is lower than from rejecting it whenever the non-merging firm is still able to grow in capacity (see first 5 rows of Table 3.2 above).

Given a larger maximum capacity for each firm ($\omega_{\max}$) or larger number of potentially active firms (N), it maybe be possible to further characterize the merger enforcement policies. For example, fixing the states of non-merging parties, is each enforcement policy (weakly) monotonic in the state of each of the merging parties? Or, fixing the states of all other firms, is each enforcement policy (weakly) monotonic in the state of a non-merging firm? With more efficient computational resources (or more efficient computational algorithms), it may be possible to investigate these questions.

3.5.2. Characterization of Industry Evolution

The results from simulating the no-commitment equilibrium for 10,000 periods is shown in Table 3.3 for the myopic and dynamic TS enforcement policies. In order to properly compare the implications of the merger enforcement policy on industry evolution, the same draws of the model's random variables were used for both policies in every period.

Table 3.3 shows that when the myopic TS policy is compared to the dynamic TS policy, there are more periods with any given (positive) number of active firms and more entry and exit under the former policy than under the latter. Although slightly more than half as many mergers are proposed under the myopic policy (113) as compared to the dynamic policy (211), the number of proposed mergers that are approved under the former policy (107) is more than twenty-one times the number of approvals under the

Table 3.3. Industry Characterization, 10,000 Periods

	Myopic TS	Dynamic TS
Periods w/ 0 firm active	7493	8031
Periods w/ 1 firms active	1883	1506
Periods w/ 2 firms active	415	317
Periods w/ 3 firms active	209	146
Periods w/ exit	2573	2103
Periods w/ entry	2572	2102
Periods w entry & exit	539	384
Periods w/ proposed merger	113	211
Periods w/ approved merger	107	5

latter (5). This difference in the numbers of proposed and approved mergers induced by the two enforcement policies seems reasonable given the state-firms-synergy combinations for which a merger is (i) proposed and (ii) approved under each policy (see Table 3.3), and the frequency distribution of the states actually visited, as shown in Table 3.4.[12] Finally, note that a merger is proposed in approximately 1-2 % of the periods, and the myopic policy approves a proposed merger in 1.07 % of the periods. The later number is roughly consistent with Gowrisankaran (1999), who finds that approximately 1.54 % of the periods in his simulation involve at least one merger occurring in his model.

Table 3.4 shows the number of periods spent in each state, out of a total number of 10,000 simulated periods. Evidently, for any given state with at least one active firm, the frequency with which it is visited is higher under the myopic TS policy than under the dynamic TS policy.

One reason why the frequency distribution of states visited is more skewed towards the state (0,0,0) under the dynamic policy is that firms make smaller (capacity-increasing)

[12] In particular, more mergers are proposed under the dynamic policy because the set of state-firms-synergy combinations that involve a merger proposal under the myopic policy is a strict subset of those state-firms-synergy combinations that involve a proposal under the dynamic policy.

Table 3.4. Frequency Distribution of States Visited, 10,000 Periods

State	Percent of periods in each state (%) Myopic TS	Dynamic TS
(0,0,0)	74.93	80.31
(1,0,0)	16.60	14.44
(1,1,0)	3.59	2.48
(1,1,1)	0.71	0.57
(2,0,0)	1.34	0.62
(2,1,0)	1.11	0.53
(2,1,1)	0.80	0.51
(2,2,0)	0.16	0.11
(2,2,1)	0.39	0.27
(2,2,2)	0.37	0.16

Table 3.5. Investments and Probability of Entry

	Myopic TS				Dynamic TS			
State	Investments			Prob. of Entry	Investments			Prob. of Entry
(0,0,0)	0.00	0.00	0.00	0.23	0.00	0.00	0.00	0.20
(1,0,0)	0.04	0.00	0.00	0.35	0.02	0.00	0.00	0.24
(1,1,0)	0.08	0.08	0.00	0.48	0.06	0.06	0.00	0.40
(1,1,1)	0.13	0.13	0.13	0.00	0.09	0.09	0.09	0.00
(2,0,0)	0.00	0.00	0.00	0.31	0.00	0.00	0.00	0.25
(2,1,0)	0.00	0.12	0.00	0.50	0.00	0.07	0.00	0.43
(2,1,1)	0.00	0.15	0.15	0.00	0.00	0.11	0.11	0.00
(2,2,0)	0.00	0.00	0.00	0.52	0.00	0.00	0.00	0.43
(2,2,1)	0.00	0.01	0.17	0.00	0.00	0.00	0.14	0.00
(2,2,2)	0.05	0.06	0.06	0.00	0.03	0.03	0.03	0.00

investments with the dynamic policy than with the myopic policy. Table 3.5 shows, for each state and each enforcement policy, the investment levels of the firms and the probability of entry occurring.

As this table shows, firms invest more in capacity in every state under the myopic policy than under the dynamic policy. This finding is consistent with entry being more likely under the myopic policy than the dynamic policy, since a firm is better able to

Table 3.6. Mean Welfare for Different Enforcement Policies

	Myopic TS	Dynamic TS
Mean consumer surplus	0.089 (0.101)	0.061 (0.084)
Mean producer surplus	-0.078 (0.383)	0.003 (0.340)
Mean total surplus	0.011 (0.463)	0.064 (0.413)

(Standard deviations in parentheses)

compete with an extra entrant when that firm has a larger capacity (which is more likely when that firm invests more).

The investment levels for the state (2,2,2) should be zero because every active firm has achieved its maximum capacity already. The fact that they are not suggests that the maximal state should be increased for future investigations. However, the impact of the non-negative investments at (2,2,2) is likely negligible because the state is visited with very low frequency during simulations, and because no merger is proposed or approved in this state for any synergy level and any merger enforcement policy.

3.5.3. The Welfare Differences of the Enforcement Policies

In order to compare the welfare implications of the two types of merger enforcement policies, the model was simulated 100 times, with 100 periods per simulation. As before, the same draws of the model's random variables were used for both types of policies in every period. Table 3.6 presents the simulation results, which suggest that the dynamic TS policy, when compared to the myopic TS policy, implies higher expected discount producer and total surpluses and lower expected discounted consumer surplus.

Despite the large standard deviations of the mean welfare, Table 3.6 shows that the myopic policy implies a lower expected discounted total surplus than the dynamic policy. This ordering reflects the fact that the myopic TS policy (by definition) does not account for the effects of mergers on future payoffs for firms and consumers, while the dynamic TS policy does.

In the future, it would be worthwhile to study the welfare implications of myopic and dynamic merger enforcement policies with consumer surplus as the welfare standard by which proposed mergers are evaluated. Furthermore, it may be interesting to investigate how merger enforcement policies based on concentration thresholds (as in Gowrisankaran (1997)) compare with the policies studied in this paper.

3.5.4. Comparative Dynamics

There is a trade-off between robustness of the model and convergence when performing comparative dynamics exercises. It is a fact that the model converges for some parameter values but not others (because of the existence of multiple equilibria or the lack of existence of any equilibria). Therefore, when analyzing how the model's predictions change when various parameters are systematically changed, one is constrained to consider only those parameter values for which the model converges. When this constraint limits one to consider only small portions of the parameter space, obtaining convergence invariably involves sacrificing some robustness of the results.

With this trade-off in mind, I examine how changes in the model's parameters affect the evolution of industry dynamics and the nature of the merger enforcement policies. First consider changes in the cost of entry for an entrant. Recall that in the benchmark

Figure 3.2. Comparative Dynamics: Effects of Different Entry Costs, 10,000 Periods

	Myopic TS			Dynamic TS		
$[x_E^{MIN}, x_E^{MAX}] =$	$[0, 0.55]$	$[0, 0.6]$	$[0, 0.65]$	$[0, 0.55]$	$[0, 0.6]$	$[0, 0.65]$
Periods w/ 0 firm active	7004	7335	7515	7690	7902	8093
Periods w/ 1 firms active	2038	1926	1931	1657	1552	1522
Periods w/ 2 firms active	561	484	359	416	387	256
Periods w/ 3 firms active	397	255	195	237	159	129
Periods w/ exit	2944	2556	2326	2339	2115	1888
Periods w/ entry	2944	2556	2325	2339	2115	1887
Periods w entry & exit	718	492	444	458	342	300
Periods w/ proposed merger	158	140	114	274	250	173
Periods w/ approved merger	146	127	107	7	6	4
Mean firm capacity	0.018	0.015	0.013	0.012	0.010	0.009
Mean total investment	0.028	0.020	0.015	0.014	0.011	0.008
Mean consumer surplus *	0.091	0.089	0.064	0.046	0.061	0.047
	(0.141)	(0.101)	(0.114)	(0.067)	(0.084)	(0.101)
Mean producer surplus *	-0.187	-0.078	-0.109	-0.098	0.003	-0.051
	(0.354)	(0.383)	(0.390)	(0.321)	(0.340)	(0.398)
Mean total surplus *	-0.096	0.011	-0.045	-0.052	0.064	-0.004
	(0.465)	(0.463)	(0.493)	(0.377)	(0.413)	(0.490)

* Welfare results are from 100 simulations with 100 periods per simulations .
(Standard deviations in parentheses)

model, the entry cost is assumed to be drawn from the uniform distribution with support $[x_E^{MIN}, x_E^{MAX}] = [0, 0.6]$. Figure 3.2 (a table) shows the effects of changing the mean entry cost by changing x_E^{MAX}, for the myopic TS and the dynamic TS policies, respectively.

Clearly, as it becomes more costly to enter, there are fewer periods with entry, and so fewer firms that are active, regardless of the merger enforcement policy. Moreover, fewer mergers are proposed (and approved) when entry becomes more costly (again, irrespective of the enforcement policy). Intuitively, if entry becomes less likely, then incumbent firms are less pressured to combine their capacities in order to better compete with an extra firm, and therefore less likely to propose a merger. Under the myopic TS policy, consumers are worse off when entry becomes more costly; Producer surplus and total surplus, however, are not monotonic in the entry cost. Under the dynamic TS policy, none of the three surplus measures are monotonic in the entry cost. Finally, note that for any given support for the entry cost, consumer surplus is higher under the myopic TS policy while producer surplus and total surplus are higher under the dynamic TS policy (which is consistent with the findings in Table 3.6).

Next, consider changes in the effectiveness of the investment technology, α. (Recall that the probability that a firm's investment is successful is given by $\frac{\alpha x}{1+\alpha x}$, where x is the investment level.) Therefore, the larger (smaller) is α, the more (less) likely it is that any given level of investment will be successful, i.e., the more (less) efficient is the investment technology. In the benchmark model, $\alpha = 3.5$. Figure 3.3 (a table) shows the effects of changing the efficiency of the investment technology (by changing the parameter α), for the myopic TS and dynamic TS policies.

Surprisingly, a more efficient the investment technology implies a *lower* mean total surplus (for a given merger enforcement policy). This result is somewhat counterintuitive and also the opposite of what Gowrisankaran (1999) finds in Table 4 of his paper. (Consumer surplus, however, appears to be higher with a more efficient investment technology.)

Obviously, the entry cost and the investment technology are not the only parameters with which comparative dynamics exercises can be performed. For example, increasing

Figure 3.3. Comparative Dynamics: Effects of Different Investment Technologies, 10,000 Periods

	Myopic TS			Dynamic TS		
α =	3.4	3.5	3.6	3.4	3.5	3.6
Periods w/ 0 firm active	7537	7335	7189	8106	7902	7886
Periods w/ 1 firms active	1797	1926	1976	1471	1552	1557
Periods w/ 2 firms active	437	484	487	310	387	349
Periods w/ 3 firms active	229	255	348	113	159	208
Periods w/ exit	2417	2556	2709	2011	2115	2159
Periods w/ entry	2416	2556	2709	2010	2115	2159
Periods w entry & exit	441	492	588	312	342	372
Periods w/ proposed merger	109	140	134	193	250	220
Periods w/ approved merger	102	127	127	3	6	5
Mean firm capacity	0.013	0.015	0.017	0.009	0.010	0.011
Mean total investment	0.016	0.020	0.026	0.007	0.011	0.013
Mean consumer surplus*	0.090	0.089	0.106	0.057	0.061	0.062
	(0.130)	(0.101)	(0.154)	(0.099)	(0.084)	(0.110)
Mean producer surplus*	-0.018	-0.078	-0.131	0.013	0.003	-0.051
	(0.406)	(0.383)	(0.385)	(0.379)	(0.340)	(0.343)
Mean total surplus*	0.072	0.011	-0.026	0.070	0.064	0.011
	(0.517)	(0.463)	(0.513)	(0.468)	(0.413)	(0.442)

* Welfare results are from 100 simulations with 100 periods per simulations .
(Standard deviations in parentheses)

(decreasing) the scaling factor s for merger synergies seems to increase (decrease) the frequency of merger proposals. Also, increasing (decreasing) the probability, θ, of the industry shock being negative decreases (increases) the frequency with which any state with at least one active firm is is reached. The effects of the other initial parameters of the model (see Table 3.1) can be investigated in a similar fashion.

3.6. Conclusion

This paper makes a first attempt at understanding horizontal merger enforcement policies in a dynamic model with merger, exit, entry, and investment decisions, using Markov perfect equilibrium as the solution concept. The setup, which is based on the work of Gowrisankaran (1999), assumes that exactly one pair-wise merger may be considered in every period. An antitrust authority approves or rejects each merger proposal, basing its decision on the effects of the proposed merger on either current period payoffs alone for the firms and consumers – a myopic policy – or expected discounted future payoffs as well – a dynamic policy.

First, I considered how firms' efficiency levels and the model's parameters affect both myopic and forward-looking merger enforcement decisions using total surplus (TS) as the welfare standard. There are more situations in which a proposed merger is approved under the myopic TS policy than under the dynamic TS policy, and each of those situations are realized more often than the scenario that involves approval under the dynamic policy. However, more mergers are proposed under the dynamic policy.

Next, the model was simulated for a large number of periods in order to examine the implications of the myopic and dynamic policies for industry structure and welfare. There are more active firms, more entry, and more exit under the myopic TS policy than under the dynamic TS policy. While more mergers are proposed under the dynamic policy, more approvals are made under the myopic policy. Furthermore, firms make more capacity-enhancing investments in every state under the myopic policy than under the dynamic policy, while expected discounted total surplus is higher with the dynamic TS policy than with the myopic policy. Finally, comparative dynamics exercises were performed to study the effects of changes in the model's parameters on industry structure and welfare. While changes in entry cost had the expected effects on industry structure and welfare, changes in the efficiency of the investment technology did not.

If the positive and normative implications of this model are to be taken seriously, a better understanding is needed of the effects of the model's assumptions and parameters on the equilibrium outcomes. Furthermore, the trade-off between guaranteeing convergence to a unique equilibrium and restrictions on the parameter space needs to be better understood. In future work, it would be instructive to examine the implications of alternative merger enforcement policies and merger formation processes in such a dynamic environment.

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