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MODELLING SERVICE-DIFFERENTIATED DEMAND FOR FREIGHT TRANSPORTATION: Theory, Regulatory Policy Analysis, Demand Estimation*

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NORTHWESTERN UNIVERSITY EVANSTON • ILLINOIS

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ABSTRACT

A model of the demand for freight transportation is developed. It incorporates service differentiation, multiple modes, and multiple markets in a spatial price equilibrium framework. Included in service differentiation are modal characteristics such as transport rate, speed of delivery, handling and loading costs, loss and damage, equipment availability and variation in scheduled transit times.

The model is applied to the following problems:

- The characterization of the shipper's optimal mode-market patterns of distribution.
- The assessment of a regulatory policy (flexible transport rates) on long run intermodal competition.
- The examination of two econometric models for estimating the demand for grain transportation (regression models and qualitative choice models).

The analysis indicates the following: It is generally optimal for shippers to ship by more than one mode to more than one market; Service differentiation tends to ameliorate the effects of increasing returns-to-scale on intermodal competition; The application of choice models to freight demand estimation is a viable alternative to standard regression models and appears to be easier to apply.

1. Introduction

It has long been recognized that in its simplest form the demand for freight transportation is derived in part from a process in which spatially separated markets engage in trade that results in the physical movement of goods from one market to another. The equilibrium concept that has evolved to explain the behavior of the flow of trade and the goods-prices in such markets is called spatial price equilibrium. This concept can be extended to reflect the interdependence between spatially separated markets and the transport sector.

The basic theory of spatially separated markets has generally treated the transport sector as though it provides a homogeneous service that is available at a constant transport rate. Examples of this approach include Enke [6], Samuelson [14], Smith [16], Takayama-Judge [18], and Silberberg [15]. This view is obviously unrealistic. Rather, the transport sector is comprised of alternative modes that offer differentiated transport services (e.g., speed of delivery, reliability, damage, etc.) at different transport rates.

A notable departure from the view that transport is a homogeneous service available at constant prices is Stucker's econometric model of transport demand [17]. In this study he extends the basic spatial transport model to include several modes with differentiated service characteristics. His analysis deals mainly with the two mode case in which service differences are absorbed into the transport costs incurred by the shipper.

In section 2 of this paper we present a theory of demand for freight transportation. It incorporates service differentiation in a multi-mode, multi-market model. Included in service differentiation are modal characteristics such as supply price, speed of delivery, handling and loading costs, loss and damage, equipment availability and variation in scheduled transit times.

In this section we show that in general, it is optimal for a shipper to split modes and markets in his distribution pattern. In other words it is usually not optimal for shippers to send shipments by only one mode to one market. This implies that demand for transport, which is derived, is complex and embodies not only a modal rate but characteristics of all the modes and markets. Modal demand is derived and conditions for market equilibrium are examined.

In section 3 the model is used to analyze a regulatory policy: flexible transport rates. A dynamic model shows that differences in the quality of service provide conditions under which stability of intermodal competition may obtain in the face of increasing returns-to-scale in a monopolized mode. Thus, econometric studies concerned with such issues must include degree of service differentiation along with measures of the extent of returns-to-scale in order to actually assess market power.

Finally, section 4 examines two econometric models for estimating the demand for grain transportation that can be derived from the theory in section 2. The first model, using regression, embodies mode-market splitting. It is argued that this approach faces serious data problems and that misspecification error is highly likely. A second approach, based on qualitative choice models (i.e. logit) is examined. The approach reduces the data acquisition problem and avoids some of the misspecification. issues inherent in regression analysis.

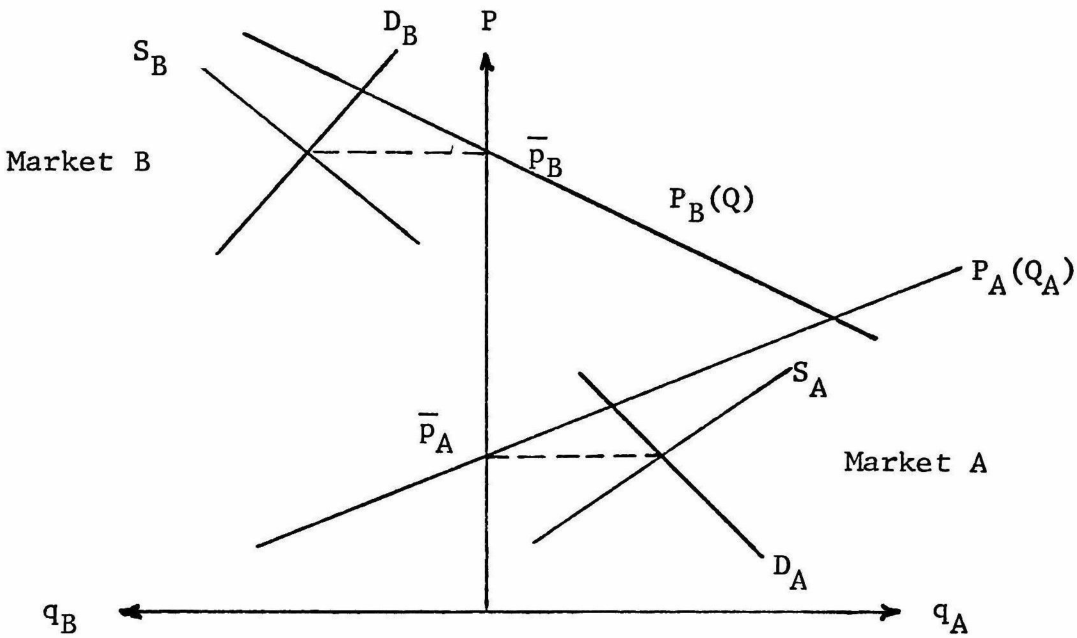
2. Multi-modal Spatial Price Equilibrium

2.1 The Two-Market, One-Mode Case

Consider a single homogeneous commodity that is bought and sold in either of two spatially separated markets, A and B. Let $S_A(p_A)$, $D_A(p_A)$, $S_B(p_B)$ and $D_B(p_B)$ be the supply and demand function for the two markets where p_A is the price in market A and p_B is the price in market B. As shown in Figure 1, we assume that $\bar{p}_B > \bar{p}_A$ where $S_A(\bar{p}_A) = D_A(\bar{p}_A)$ and $S_B(\bar{p}_B) = D_B(\bar{p}_B)$, i.e. $\bar{p}_A$ and $\bar{p}_B$ are prices that clear their respective markets. Now let $Q_A(p_A) = S_A(p_A) - D_A(p_A)$ and $Q_B(p_B) = D_B(p_B) - S_B(p_B)$, i.e. $Q_A(p_A)$ is the quantity of the good exported by market A at price p_A while $Q_B(p_B)$ is the quantity of the good imported by market B at price p_B. Given that the demand and supply functions in both markets are continuously differentiable, monotonic and that $S_A' > D_A'$, $S_B' > D_B'$ (i.e. Walrasian stable) then we can invert $Q_A(p_A)$ and $Q_B(p_B)$. We will label the new functions $P_A(Q_A)$ and $P_B(Q_B)$. Clearly, $P_A' > 0$, $P_B' < 0$.

Figure 1 is a standard representation of the supply and demand curves in the two markets (Samuelson [14]). We can determine the level of exports and imports and their corresponding prices from the excess supply price and demand price curves $P_A(Q_A)$, $P_B(Q_B)$. Finally, to avoid pathological cases we assume that there exists some quantity $\tilde{Q}$ such that P_A and P_B intersect, i.e. $P_A(\tilde{Q}) = P_B(\tilde{Q})$.

If only one type of homogeneous transport service exists between the two markets then the analysis is straight-forward. Let $Q = S_T(p_T)$ be the supply function for transport service,



MARKETS FOR THE COMMODITY

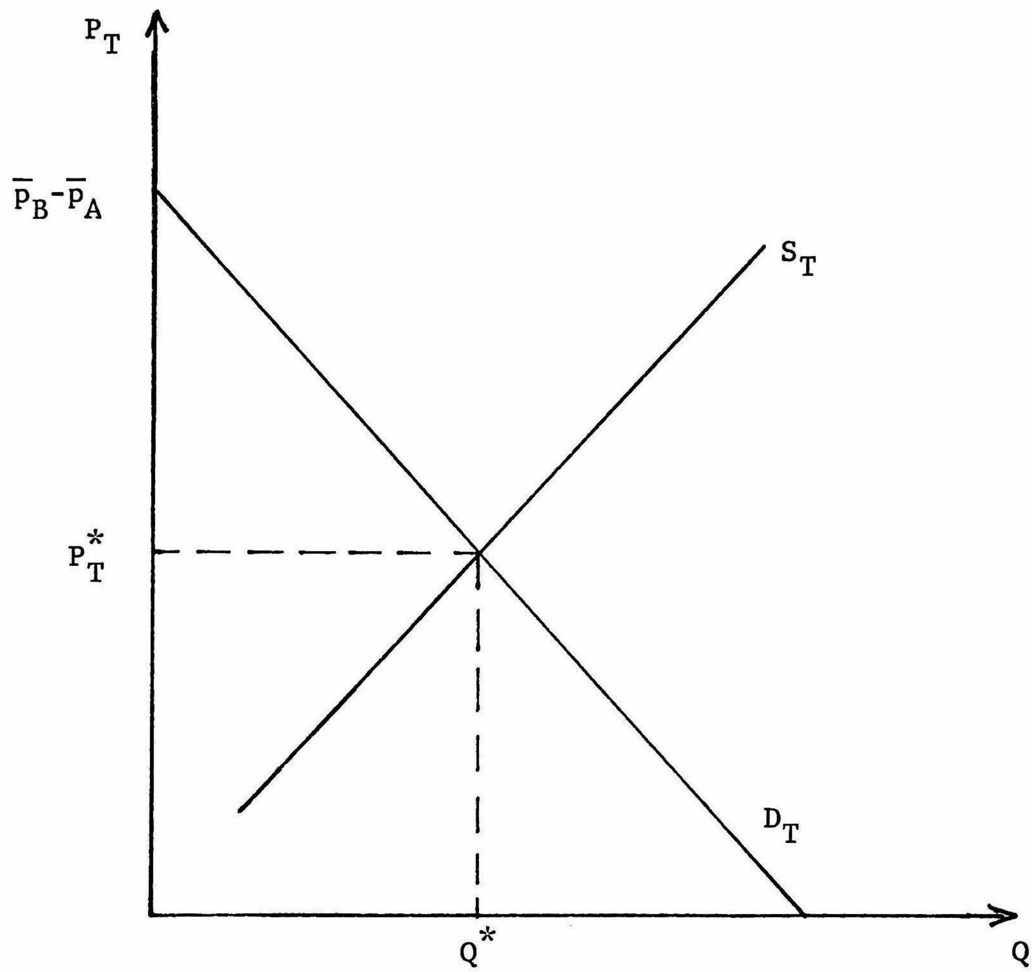
Figure 1

i.e. Q units will be shipped from A to B at rate p_T. If $p_T = 0$ then from above we know that the demand will be for $\tilde{Q}$ to be shipped. On the other hand, if $p_T = \bar{p}_B - \bar{p}_A$ then there will be no demand for transport service. In general the demand for transport service is found by subtracting $P_A(Q)$ from $P_B(Q)$ (since $Q_A = Q = Q_B$). Figure two displays the resulting transport sector market equilibrium solution. In equilibrium the transport rate is p_T^*. Thus Q^* units are transported from A to B. This means that the price in A is $P_A(Q^*)$ and the price in B is $P_B(Q^*)$. Therefore we can find the quantities of goods produced and sold in all markets. Given elasticities of the supply and demand functions in the two markets one could calculate the elasticity of transport demand (Kobe [7]). One could also pose the supply of transport as monopolistic (Orr [12]). Later in the paper we will examine a problem in partial monopoly. First, however, we will extend the above to multiple modes. This extension is critical since transport service is not homogeneous but is provided by many forms of technology and is differentiated in its provision. Initially we will assume that modes differ in speed of delivery and have different supply functions. Later we will add other service characteristics.

2.2 The Two-Market, Multi-mode Case

2.2.1 The Goods-Producers' Problem

Assume now that every shipper in A can choose to ship goods to B by M different modes. We will assume that the modes are differentiated in terms of their supply price function



TWO MARKET, ONE MODE EQUILIBRIUM

Figure 2

($p_m(q_m)$ for shipping q_m from A to B) and speed of delivery (T_m, m=1,...,M). Speed of delivery is a service characteristic; the faster the mode the more valuable the service.
In particular, since a shipment sent to B is sold T_m time units later than it could have been sold in A, we will assume that the return from B is discounted when comparing it to A. The nth goods producer in A takes as given the commodity prices P_A, P_B, the transport rates on each mode, p_m, and the interest rate r. His problem is to produce total output q^n at a cost $C^n(q^n)$, sell q_A^n in A and ship q_m^n to B by mode m. Thus the optimization problem for the nth firm is

$$\max \quad P_A q_A^n + \sum_{m=1}^{M} P_B e^{-rT_m} q_m^n - \sum_{m=1}^{M} p_m q_m^n - C^n(q^n)$$

$$\text{S.T.} \quad \text{(a)} \quad q^n = q_A^n + \sum_{m=1}^{M} q_m^n$$

$$\text{(b)} \quad q_m^n \geq 0$$

It can be shown (see [3]) that the firm's optimal distribution conditions are (the optimal production condition is $C^n(q^n) = P_A$):

$$\left.\begin{array}{ll} (1) & P_B e^{-rT_m} q_m^n = P_A q_m^n + p_m q_m^n \\ (2) & P_B e^{-rT_m} \leq P_A + p_m \\ (3) & q_m^n \geq 0 \end{array}\right\} \quad m=1,\ldots,M$$

Condition (1) states that a profit maximizing goods producer uses the mth mode up to the point at which the discounted revenue from B just equals the opportunity costs of not selling in A$(P_A q_m^n)$ plus transport costs $p_m q_m^n$. Condition (2) is a marginal condition such that any incremental quantity shipped on mode m yields opportunity costs plus transport costs (P_A+p_m) at least as great as discounted returns, $P_B e^{-rT_m}$. If (2) does not hold then profits can always be increased by shipping an additional unit by the mth mode.

2.2.2. Market Equilibrium Conditions and the Demand for Transport Service

In this section we show that conditions analogous to (1)-(3) can be used to define an equilibrium for the whole economy. Let q_m be the amount carried by mode m. Clearly the total flow $Q(=Q_A=Q_B)$ is simply $\sum_m q_m$. We will say that $q_1,\ldots,q_M$ are equilibrium flows if they satisfy the following conditions:

$$(4) \quad \sum_{m=1}^{M} P_B(Q)e^{-rT_m}q_m = P_A(Q)Q + \sum_{m=1}^{M} p_m(q_m)q_m$$

$$(5) \quad P_B(Q)e^{-rT_m} \leq P_A(Q) + p_m(q_m)$$

$$(6) \quad q_m \geq 0$$

Assuming that $p_m(q_m)$ arises from profit maximization in the transport firm and that it is monotonically increasing and continuous then it can be shown ([3]) that (4)-(6) has a unique solution. When this solution is found it sets prices so that each goods producer can maximize profits. Thus all possible gains are arbitraged away and the solution is an equilibrium.

Let Q be <u>any</u> total flow between A and B and let

$$R_m(Q) = P_B(Q)e^{-rT_m} - P_A(Q)$$

$R_m(Q)$ is the unit differential return associated with a unit sold in B versus A. We can now derive the demand for transportation by mode m. Under perfect competition (in the goods market) $R_m(Q)$ would be the goods producer's demand price (p_m^D) for mode m transport. In response to this offer, the h^{th} transport firm offering mode m service carries q_m^{h*} units which is the solution to

$$\max \; p_m^D q_m^h - C_m^h(q_m^h)$$

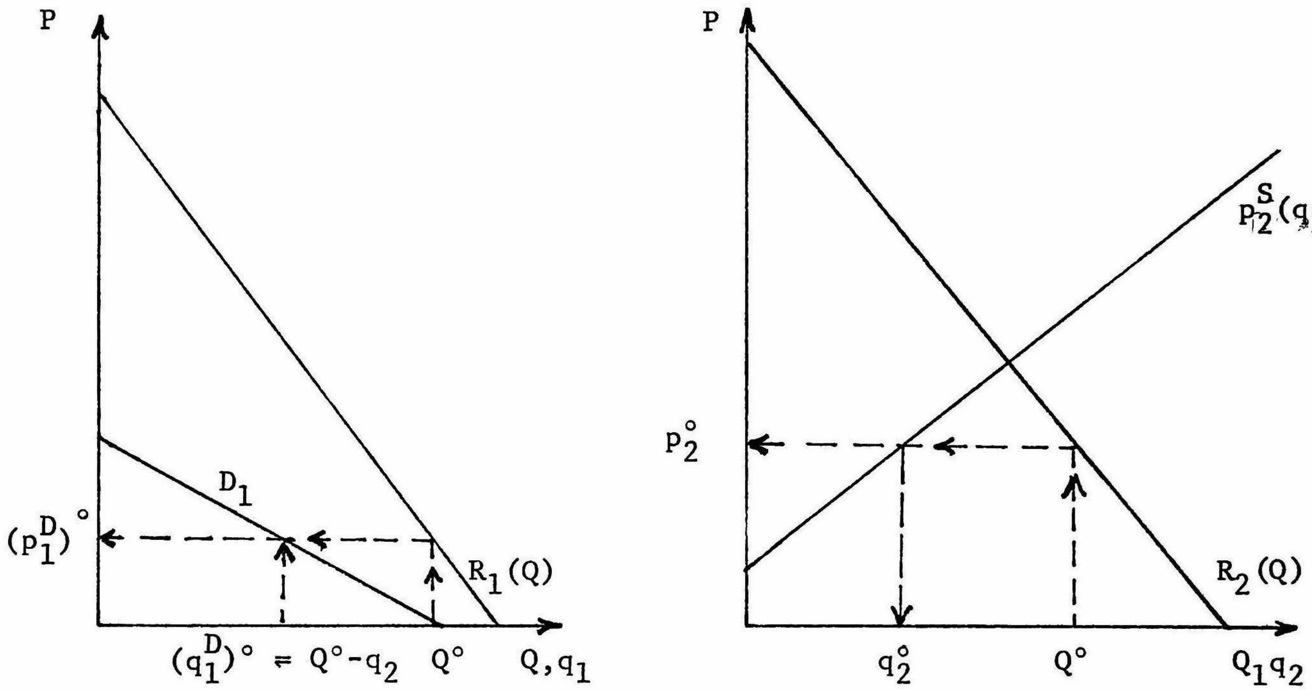
with $C_m^h(\cdot)$ a standard (increasing marginal cost) cost function. Thus <u>total</u> modal supply is

$$q_m^S = \sum_h q_m^{h*}$$

i.e. it is the sum of optimal quantities supplied by all firms offering mode m service. Thus, the quantity demanded of any mode i is simply the difference between the total flow Q and the total that would be shipped at the respective demand price (p_m^D) by all modes other than <u>i</u>, i.e.;

$$q_i^D \equiv \max\{0, Q - \sum_{\substack{m \neq i \\ m=1}}^{M} q_m^S\}$$

Figure 3 depicts the method of finding price-quantity pairs (p_i^D, q_i^D). We will call the set of price-quantity pairs (p_i^D, q_i^D) that have been defined above the <u>residual demand curve for mode i</u>.



To obtain mode one demand curve solve ($R_1(Q)$, $R_2(Q)$, $p_2^S(q_2)$ given):

$$p_1^D(q_1)=R_1(q_1+q_2)$$

$$p_1^S(q_2)=R_2(q_1+q_2)$$

To obtain mode two demand curve (not shown) solve ($R_1(Q)$, $R_2(Q)$, $p_1^S(q_1)$ given):

$$p_1^S(q_1)=R_1(q_1+q_2)$$

$$p_2^D(q_2)=R_2(q_1+q_2)$$

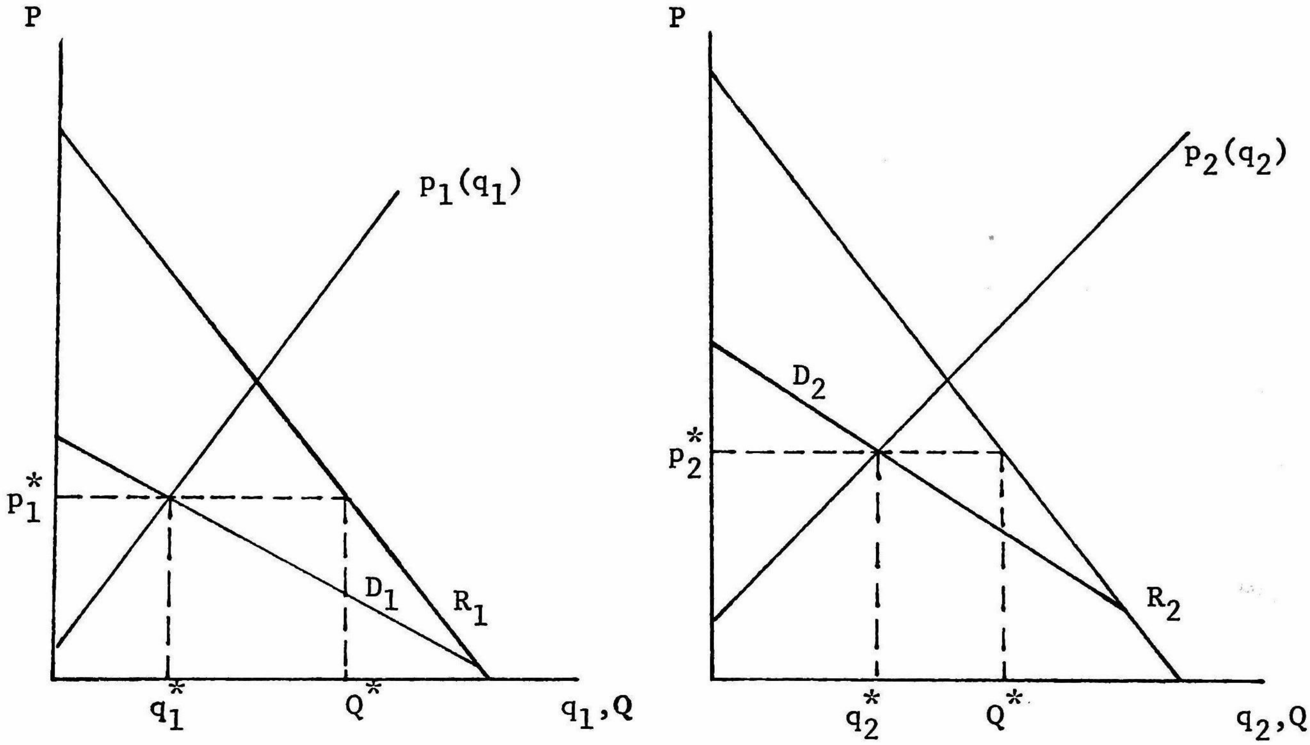
DERIVING MODE ONE'S DEMAND CURVE

Figure 3

It should be emphasized that this demand relationship was not derived in the standard fashion in that prices on the modes were allowed to vary (as Q varied). The approach is valid since the choice of mode by a goods-producer is assumed to depend only on a profit criterion, i.e. the mode that is best (including speed of delivery considerations) is the one that is used. In ([3]) it is shown that the resulting residual demand curves are downward sloping as one would expect. Figure 4 depicts the demand and supply functions for a two-mode model. The starred quantities and prices are the equilibrium solution.

2.2.3 Two-Market, Multi-mode Case Summary

Before proceeding to the multi-market case and the inclusions of other service characteristics, it is best to pause and summarize the last few sections. The development of a multi-mode model required the inclusion of service characteristics. We have used speed of delivery as a surrogate and have incorporated it in a simple and direct manner. The result is that demand curves are significantly more complex. They embody not only the characteristics of the mode in question, but of the other modes. This is a crucial point. Transport can not be approached in the same way we estimate commodity demand. Estimation procedures must account for alternative mode characteristics and prices as well as commodity characteristics.



MARKET EQUILIBRIUM USING
RESIDUAL DEMAND CURVES

Figure 4

2.3 The Multi-Market, Multi-Mode Case with Firm-Level Modal Splitting

2.3.1 The Goods-Producers Problem

In this section we will expand the model to allow for multiple markets, other service characteristics such as physical reliability (e.g. loss and damage) and schedule reliability (e.g. late arrival) and to include special costs such as loading and handling costs. The overall result is that an individual firm will in general find modal and market splitting optimal. Thus, in general it will be optimal to send shipments to different markets and by different modes rather than ship all output to one market by one mode. This result is in contrast to the standard, traditional assumption that in perfect competition the individual goods-producing firm will choose one market and one mode to use.

Formulation of the goods producers problem requires us to expand the notation from before. Let P_j be the price for the good in market j (j=1,...,J) with i representing the home market. Let q^n_{ijm} be the amount sent from i to j by firm n at i by mode m which services the (i,j) pair. Let T_{ijm} be the time it takes to go by mode m from i to j $(T_{iim}=0)$. In a slight change of notation, let t_{ijm} be the transport rate charged for sending a unit of the good from i to j by mode m $(t_{iim}=0)$. The reason for this change will become apparent later. Let q^n_i be the total amount of good produced by firm n at location i; we assume that the associated cost function $C^n_i(\cdot)$ has the standard neoclassical properties.

If we were to formulate the goods producers problem at this point it would be exactly like the formulation in section 2.2.1 save for some extra subscripts. We are in a position, however, to consider more service characteristics than simply the speed of delivery. Specifically we will incorporate costs of loading and handling and costs associated with schedule and physical reliability.

In the case of loading and handling we take such costs to be represented by a function of the amount shipped i.e. q^n_{ijm}. The function notation will be $H^{nL}_{ijm}(\cdot)$. We assume that such a function is strictly convex with only positive derivatives, i.e. $(H^{nL}_{ijm})' > 0$, $(H^{nL}_{ijm})'' > 0$. This function gives the cost associated with taking a finished product and loading it on a specified mode to go to a specified market. The strict convexity reflects the fact that as more is loaded we typically observe congestion costs being imposed. We emphasize that strict convexity is not required for what follows.

In the case of physical and schedule reliability a more detailed mathematical treatment is warranted. By physical reliability we are referring to loss and damage. By schedule reliability we mean the effect of equipment availability and transit time variance on the ability of a goods-producer to deliver a shipment to a receiver on a promised date. Both types of reliability introduce the notion of risk into the decision as to where to ship and by what mode. One could, in fact, view the selection of market and mode as a portfolio problem of investment in risky assets. Extending the analogy we see that modal and market splitting is then to be expected, since diversification will help ameliorate the risk.

To account for risk we will describe a function $H^R_{ijm}(\cdot)$ of the amount sent from i to j by firm n on mode m. $H^R_{ijm}(\cdot)$ is the <u>risk</u> function and reflects the cost of an insurance policy, purchased in a perfectly competitive insurance market, that pays all penalties for late arrivals and reimburses a recipient for loss and damage sustained by the goods. In agricultural terms loss and damage is sometimes referred to as shrink, i.e. a certain fraction of a corn shipment is lost due to settling, etc. In general, the insurance policy reflects the cost associated with transporting the commodity between two locations by some mode. Clearly such a cost is a function of the (i,j) pair and the mode.

We shall now show that $H^R_{ijm}(\cdot)$ is strictly convex with positive first derivative. We will first introduce some notation which reflects the mode-market attributes and then supress the subscripts in our proof of the properties of $H^R_{ijm}(\cdot)$. Let θ_{ijm} be a random variable reflecting the percentage loss and damage incurred by shipping a unit of good from i to j by mode m. For simplicity of exposition we assume that it takes on two possible values:

$$\theta_{ijm} = \begin{cases} \bar{\theta}_{ijm} \text{ with probability } 1 - d_{ijm} \\ \qquad\qquad (0 \leq \bar{\theta}_{ijm} \leq 1) \\ 0 \text{ with probability } d_{ijm} \end{cases}$$

Thus if y^n_{ijm} is the amount of goods received at j that were sent by firm n at location i by mode m then

$$y^n_{ijm} = (1 - \bar{\theta}_{ijm})q^n_{ijm}$$

Furthermore let K_j be the per unit penalty cost for lateness of arrival. Such lateness of arrival can come about from failure of modal equipment (railroad cars, barges, etc.) to be available when promised or from variance in scheduled modal transit times. We assume simply that a shipment is either on time (with probability f_{ijm}) or late (with probability $1-f_{ijm}$) in which case the per unit penalty K must be paid. Thus while the premium, $H^R_{ijm}(q^n_{ijm})$ for shipping q^n_{ijm}, is paid with certainty, there are two types of losses that the insurance company might incur. The first is that they might have to cover the market value of the loss and damage, namely $P_j(q^n_{ijm}-y^n_{ijm})$. The second potential loss is the penalty cost $K_jq^n_{ijm}$.

Insurance companies (one of which might represent the goods-producer himself as a self-insurer) are assumed to be risk-averse. In other words, we assume that they can be represented by a neo-classical utility function which is increasing and concave as a function of money. Let $U(\cdot)$ be such a function. Thus $U'(\cdot) > 0$, $U''(\cdot) < 0$. Thus the insurance company is risk-averse. We assume that the insurance company maximizes expected utility i.e. it solves:

$$\max \quad E\{U(H(q) - P(q-y) - Kq)\}$$

where subscripts have been dropped to facilitate the analysis. If the insurance market is perfect then the company will be indifferent between the maximum expected utility above and the utility of no transaction U(0). Thus we have that in equilibrium:

$$(7)\quad U(0) = f\cdot d\cdot U(H(q)) + f(1-d)\cdot U(H(q) - P(q-y)) + (1-f)\cdot d\; U(H(q) - Kq) + (1-f)(1-d)U(H(q) - P(q-y) - Kq)$$

If we now totally differentiate (7) and solve for $H'(q)$ we find that it is positive:

$$H'(q) = \frac{P\bar{\theta}f(1-d)U_2' + (1-f)dU_3'\cdot K + (1-f)(1-d)U_4'\cdot(P\theta+K)}{fdU_1' + f(1-d)U_2' + (1-f)dU_3' + (1-f)(1-d)U_4'} > 0$$

where to simplify the notation we have used:

$$U_1 = U(H(q))$$
$$U_2 = U(H(q) - P(q-y)) = U(H(q) - P\bar{\theta}q)$$
$$U_3 = U(H(q) - Kq)$$
$$U_4 = U(H(q) - P(q-y) - Kq) = U(H(q) - P\bar{\theta}q - Kq)$$

Thus, as the shipment size increases, the premium increases. Totally differentiating (7) a second time yields the result that $H''(q)$ is also positive. Thus, in general, $H^R_{ijm}(\cdot)$ is strictly convex in q^n_{ijm}.

We are now finally in a position to formulate the goods-producers problem. For firm n in location i choosing among modes and markets, the producer will find (q^n_i, q^n_{ijm}) that solves (given P_j, r, T_{ijm} and t_{ijm}):

$$(8) \quad \max \quad P_i q_{ii}^n + \sum_{j \neq i} \sum_m [(P_j e^{-rT_{ijm}} - t_{ijm}) q_{ijm}^n - H_{ijm}^n (q_{ijm}^n)] - C_i^n (q_i^n)$$

$$(9) \quad \text{S.T.} \qquad q_i^n = \sum_{j \neq i} \sum_m q_{ijm}^n + q_{ii}^n \geq 0$$

$$(10) \qquad q_{ijm}^n \geq 0 \qquad \begin{matrix} j=1,\ldots,J \\ j \neq i \end{matrix} \quad m=1,\ldots,M$$

where $H_{ijm}^n(q_{ijm}^n) \equiv H_{ijm}^{nL}(q_{ijm}^n) + H_{ijm}^R(q_{ijm}^n)$

Note that production for the home market q_{ii}^n is unconstrained. Under some conditions we will choose to sell in the home market while in others we might choose to <u>buy</u> (and ship). The Kuhn-Tucker conditions are (after substituting (9) into (8))

$$q_i^n(P_i - C_i^{n\prime}(q_i^n)) = 0$$

$$q_{ijm}^n((P_j e^{-rT_{ijm}} - t_{ijm} - H_{ijm}^{n\prime}(q_{ijm}^n) - P_i) = 0 \qquad \forall j,m$$

$$q_i^n - (q_{ii}^n + \sum_{j \neq i} \sum_m q_{ijm}^n) = 0$$

$$q_{ijm}^n \geq 0 \quad \forall j,m \qquad q_i^n \geq 0$$

$$P_j e^{-rT_{ijm}} - t_{ijm} - H_{ijm}^{n\prime}(q_{ijm}^n) - P_i \leq 0$$

Thus, for positive q_{ijm}^n we have

$$(11) \quad P_j e^{-rT_{ijm}} - t_{ijm} - H_{ijm}^{n\prime}(q_{ijm}^n) = P_i = C_i^{n\prime}(q_i^n) \qquad \forall j,m$$

$$(12) \qquad q_{ii}^n = q_i^n - \sum_{j \neq i} \sum_m q_{ijm}^n$$

The firm can be thought of as being involved in the following sequence of activities. First, it decides a production level by solving $P_i = C_i^{n\prime}(q_i^n)$. Then it equates marginal revenues and costs among the various mode-market pairs. Finally it decides how much to sell or buy in the home market so as to complete its plans. Notice that the left side of (11) is downward sloping due to the nature of H_{ijm}^R. A graphical solution is shown in Figure 5. As long as the mode-market marginal curve intersects the price axis above P_i, then flow is positive. If any marginal intersects the axis below P_i then the associated variable will be zero and its associated Kuhn-Tucker multiplier will be positive. Clearly, unless one mode-market pair completely dominates all others, mode-market splitting will occur.

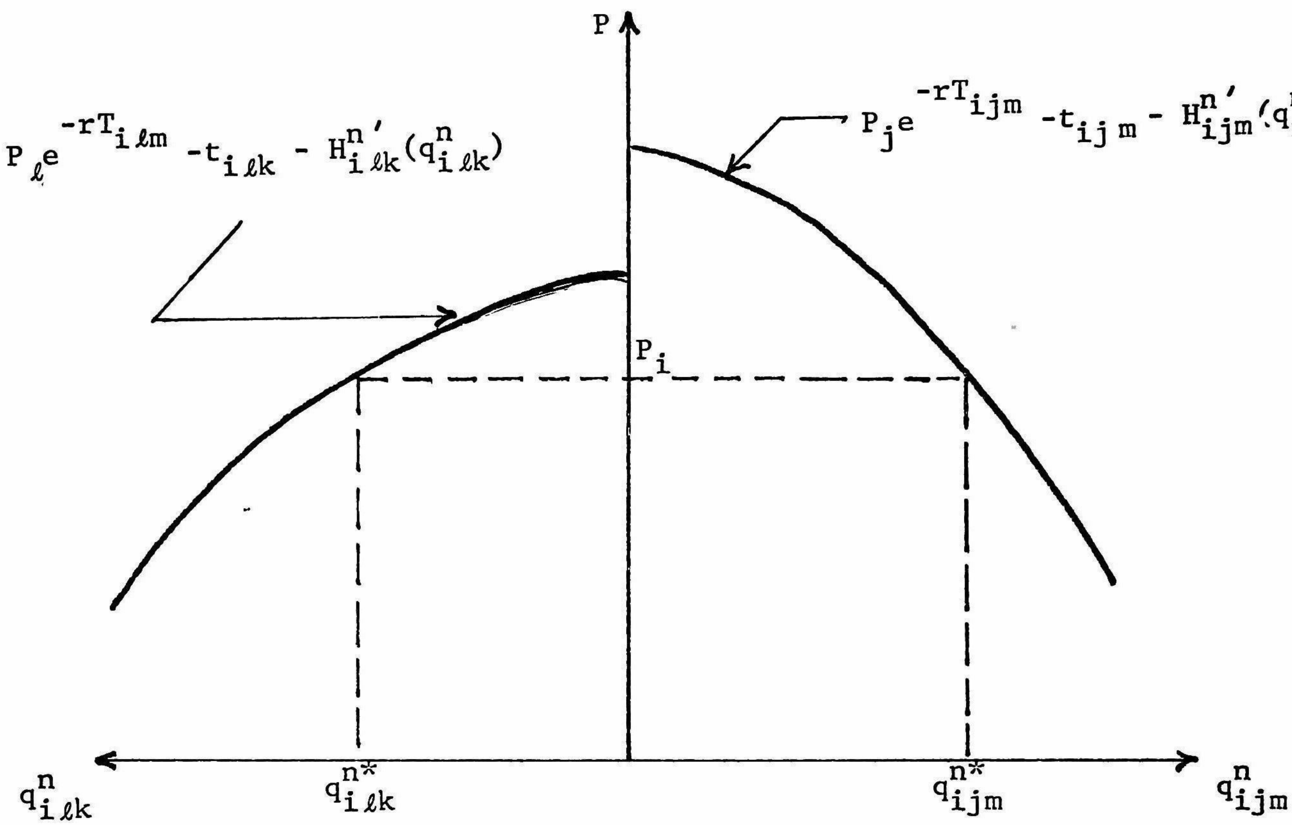
2.3.2. Market Equilibrium Conditions

It is now quite simple to write down the market equilibrium conditions. All conditions are written on the assumption that we have a given home location, namely market i. Then $q_i = (q_{i11}, \ldots, q_{iJM})$ is an equilibrium vector of flows if it satisfies $(\forall j, \forall m)$

$$q_{ijm}[P_j(Q_j)e^{-rT_{ijm}} - t_{ijm}(q_{ijm}) - H'_{ijm}(q_{ijm}) - P_i(Q_i)] = 0$$

$$P_j(Q_j)e^{-rT_{ijm}} - P_i(Q_i) \leq t_{ijm}(q_{ijm}) + H'_{ijm}(q_{ijm})$$

$$q_{ijm} \geq 0$$



INDIVIDUAL MODAL SPLITTING IN A MULTI-MARKET,

MULTI-MODE FRAMEWORK

Figure 5

We have assumed that one can aggregate (by summing inverses) $H^n_{ijm}(q^n_{ijm})$ over firms to produce $H_{ijm}(q_{ijm})$. It should be noted that $H^R_{ijm}(\cdot)$ is already at an aggregate level and it only remains to aggregate $H^{nL}_{ijm}(\cdot)$ over all firms. We note also that the above conditions are of the same form as the previous model if we set $p_{ijm}(q_{ijm}) \equiv t_{ijm}(q_{ijm}) + H'_{ijm}(q_{ijm})$, i.e. we associate the direct transport costs (t_{ijm}) with the indirect imposed costs, for convenience of exposition. Finally the demand curve for using mode m service to ship goods from i to j is found by solving:

$$p^D_{ijm}(q_{ijm}) = R_{ijm}(Q_i, Q_j)$$

$$p^S_{i\ell k}(q_{i\ell k}) = R_{i\ell k}(Q_i, Q_\ell) \qquad \forall_\ell \forall_k \text{ such that } (\ell,k) \neq (j,m)$$

where

$$R_{ijm}(Q_i, Q_j) = P_j(Q_j)e^{-rT_{ijm}} - P_i(Q_i)$$

$$p^S_{ijm}(q_{ijm}) \equiv p_{ijm}(q_{ijm})$$

$$Q_i = \sum_n (q^n_i - q^n_{ii})$$

$$Q_j = \sum_{i,m} q_{ijm}$$

3. An Application to a Policy Question

3.1 Introduction

In this section we will examine a proposal for regulatory change using the analytical structure developed in the previous section. We will assume that two modes exist to transport goods between two spatially separated markets that trade in one commodity. One of these modes will represent a single-firm industry (e.g. a railroad) while the other will consist of perfectly competitive firms (e.g. owner-operator trucks). For simplicity we will assume that the service differentiation is sufficiently summarized by speed of delivery.

Our basic policy question is whether long run intermodal allocation of freight can be maintained when all modal prices are flexible. While we do not provide a simple answer to this question, our analysis does provide some insights. Specifically, differences in the quality of service provide conditions under which stability of intermodal competition may obtain in the face of increasing returns-to-scale in the monopolized mode.

3.2 Short and Long Run Equilibrium

As stated above, the first mode is represented by a single firm while the second mode consists of a large number of small competitive firms. Let x be the number of mode two firms. In the aggregate they haul q_2. Thus we can write the system of equations describing the demand for mode 1 service as

$$p_1^D(q_1) = R_1(q_1+q_2)$$
$$p_2^S(q_2,x) = R_2(q_1+q_2)$$

We assume that

$$p_{2,q}^S \equiv \frac{\partial p_2^S}{\partial q_2} > 0$$

$$p_{2,x}^S \equiv \frac{\partial p_2^S}{\partial x} < 0$$

Thus additional firms entering the market to provide mode two service shift the supply curve to the right, lowering supply price. In the short run the number of competitive firms is fixed. Thus a short run equilibrium is conditioned on the number of mode two firms. Formally, the short run equilibrium E(x) is the equilibrium set of transport prices and quantities

$$E(x) = \{p_1^*(x), q_1^*(x), p_2^*(x), q_2^*(x)\}$$

where E(x) satisfies:

(13) $p_1^D(q_1^*(x)) = R_1(q_1^*(x) + q_2^*(x))$

(14) $p_2^S(q_2^*(x), x) = R_2(q_1^*(x) + q_2^*(x))$

(15) $MR_1(q_1^*(x)) = MC_1(q_1^*(x))$

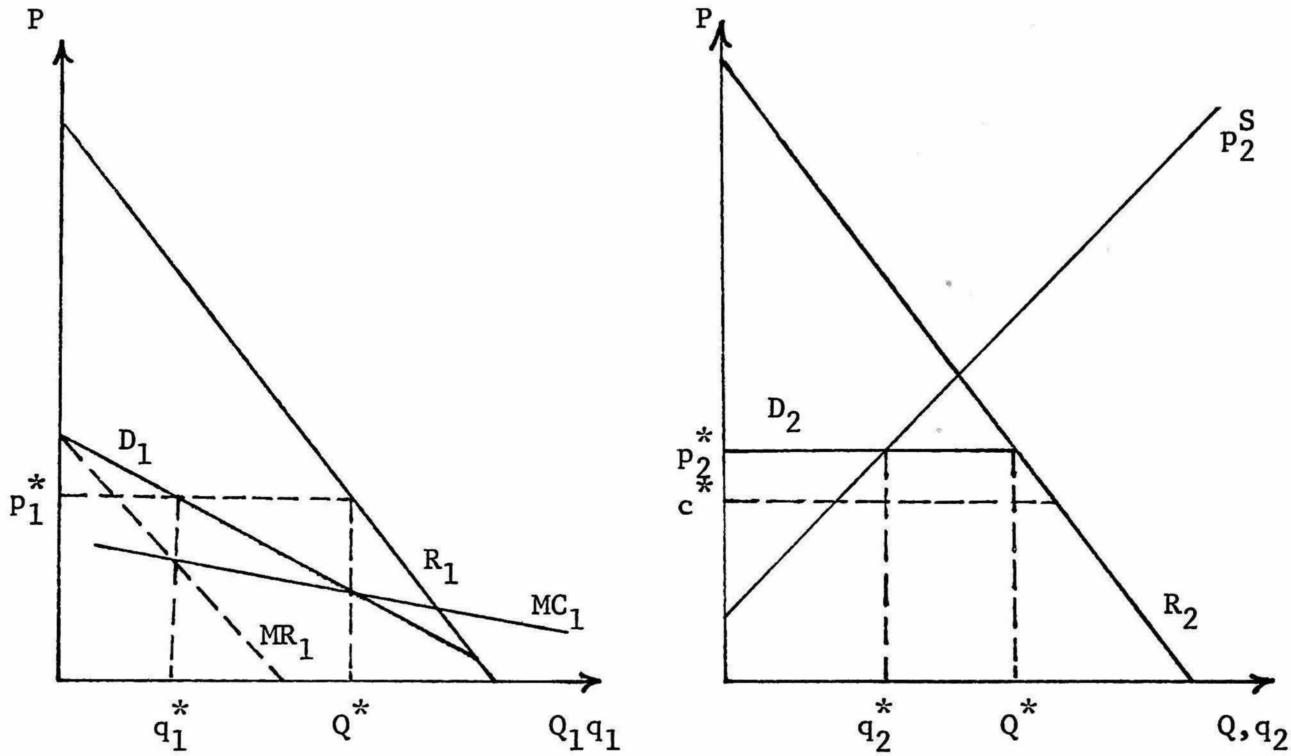
where $p_1^*(x) = p_1^D(q_1^*(x))$ and $p_2^*(x) = p_2^S(q_2^*(x),x)$. This equilibrium is illustrated in Figure 6. Equations (13) and (14) are the demand curve (mode 1) equations while (15) is the first order condition for mode one profit maximization. In other words, in the short run the market is in equilibrium when the dominant firm (mode one) is setting price optimally such that the market for the competitive mode is cleared and such that the goods markets are cleared. Equations (13)-(15) embody this.

Note, however, that the short run equilibrium price in mode two (p_2^*) may not equal the minimum long run average cost (c^*). Here we will assume that this minimum is the same for all firms offering mode two service. If $p_2^* \neq c^*$ then there is an incentive for entry or exit, which means that the number of firms will change. Let x_t denote the number of competitive firms in period t. We will describe a simple entry-exit process such that if $p_2^* > c^*$ firms enter and if $p_2^* < c^*$ firms leave. It is:

$$(16) \quad x_{t+1} = f(x_t) \equiv x_t + h(p_2^*(x_t) - c^*)$$

where $h(0) = 0$ and $h^* > 0$.

Thus if we start with x_0 firms then the short run equilibrium is $E(x_0)$ with prices $p_1^*(x_0)$, $p_2^*(x_0)$. The number of firms in mode 2 now adjusts (if $p_2^*(x_0) \neq c^*$) and we get $x_1 = f(x_0)$ firms. This determines a new short run equilibrium $E(x_1)$ and so on. A sequence of short run equilibria $\{E(x_t)\}_{t=0,1,...}$ is generated. The natural



SHORT RUN UNREGULATED EQUILIBRIUM

Figure 6

question concerns whether (and when) this sequence settles down to a long run equilibrium. A long run equilibrium is attained when there is some number of firms x^* that doesn't change from period to period (no net entry or exit). Thus we are interested in when $x^* = f(x^*)$. This occurs only when $p_2^*(x^*) = c^*$. Since in general we can assume that there is a value of c^* such that for some value of x^* the equality is true, then we can generally assume that there is at least one long run equilibrium in which both modes are represented.

3.3 Stability of Intermodal Splitting

In this section we will explore a (sufficient) condition for stability. This condition is derived in ([4]). It can be shown that a sufficient condition for stability is $|f'(x^*)|<1$ ([13]). This implies that a sufficient condition for stability is

$$(17) \quad -\frac{2}{h'(0)p^S_{2,x}} \cdot \frac{(p^S_{2,q}-R_2')}{R_2'} < \frac{P_1^{D'}}{MR_1'-MC_1'} - 1 < 0$$

In general, if $h'(0)p^S_{2,x}$ is small then we would expect the left hand inequality to be satisfied. $h'(0)$ is small if entry and exit takes time and is not instantaneous. If small changes in the number of mode two firms does not cause radical shifts in supply, then $|p^S_{2,x}|$ will be small. Thus, one would expect the left hand inequality to be met.

The right hand inequality holds if and only if $MR_1' - MC_1' - p_1^{D'} < 0$. This is true since second order conditions for firm one profit maximization require $MR_1' - MC_1' < 0$. This condition is, unfortunately, far from straight-forward. In order to examine it for policy conclusions, we will assume that the demand function is linear. This is not quite as restrictive as it might first appear for at least two reasons:

1. Linearity provides the starkest of cases. Highly non-linear functions in fact tend to increase the number of stable solutions rather than reduce them.
2. Linearity of demand is often used in econometric studies and our results will provide a caution concerning the results of those studies.

If p_1^D is linear then $MR_1' = 2p_1^{D'}$, and the condition reduces to $p_1^{D'} - MC_1' < 0$. Clearly when marginal costs are increasing, this condition is always met, i.e. stable intermodal competition obtains. Traditional theory however, indicates that when marginal costs are decreasing one should not expect stability. The reason that this is not necessarily correct is that it ignores service differentiation. It can be shown (see [4]) that even with decreasing marginal costs service differentiation acts as a countering force to what would otherwise be a destabilizing influence.

To be slightly more specific, it can be shown that if marginal costs are only somewhat declining then in fact the competitive mode must not provide service that is significantly superior to the monopolized mode if stability is to be maintained. As marginal costs decline more rapidly the maximal separation in service is reduced until with rapidly declining marginal costs stability is only consistent with significantly superior service on the monopolized mode.

3.4 Policy and Research Implications

What does all this mean with respect to transport policy and research? A couple of implications are the following:

1) Increasing returns-to-scale, in and of themselves, do not necessarily mean that the only way to provide for stable multi-modal service is to regulate the monopolized mode's price. Service differentiation can act as a strong brake on market power. In fact, if the monopolized rate is set so that the competitive rate is above long run average cost for the competitive mode, competitive mode entry will be encouraged. As long as this situation continues the residual demand for the monopolized mode will fall, eroding its market share (see [4]). Thus regulation of the rate may, in fact, lead to eliminating an alternative mode (in this case the regulatee) from the shippers choice set.
2) Econometric studies of returns-to-scale are not typically sufficient to determine whether real market power exists.

Studies must be extended to examine the relationships between service parameters. The _extent_ of returns-to-scale and the _degree_ of service differentiation must be estimated together.

A caution is in order. There is nothing in the above analysis that addresses the welfare implications of the long run stable intermodal relationship. In the long run price will equal marginal cost in the competitive mode but price may be above marginal cost in the monopolized mode. The question of which course to follow, i.e. whether or not to control rates, revolves around the potential welfare losses arising from the inequality of price and marginal cost versus the cost of achieving and administering their equality.

4. Estimation of Freight Demand: Transportation of Grain

4.1 Introduction

In this section we shall propose a model of grain elevator shipping decisions and shall discuss alternatives ways to estimate the model. The major premise of this work is that the demand for transportation is derived from the needs of the firm to sell its product in spatially separated markets. The focal point is the individual elevator, and hence the model is disaggregate.

We find that the model allows for two types of estimation methods; regression and choice models. The regression approach, while conceptually preferable to the choice model approach, is impractical because of data requirements and certain theoretical problems concerning costs. Thus use of choice models in estimating transport demand seems more practical in the light of data requirements. Choice models have been justified on theoretical grounds and are shown to be applicable in our model in certain circumstances.

4.2 The Model of a Grain Elevator's Shipping Decisions

The production process for a grain elevator consists of two basic functions. The first function is to purchase grain from various suppliers such as farmers and other elevators and make it available for sale and distribution. This function involves such activities as the procurement of grain on the local market, processing, handling, storing and hedging. It is assumed that a total cost can be associated with each level of saleable product.

The second function is the sale and distribution of total output to distant markets and/or to the local market (which includes storing the grain). Two types of costs must be reckoned within this shipping function. The first consists of the direct transport charges that are levied by the carriers. The second type of costs are only indirectly attributable to transportation. These indirect costs are discussed below.

<u>Equipment delay costs</u>: A shipment must be placed in a holding position ready for loading onto the transport equipment at some planned equipment arrival date. If there is uncertainty as to when the equipment will be supplied, this date is typically earlier than the actual arrival date. Thus there is an expected equipment delay time which is assumed to vary by mode. Grain that is kept in a holding position incurs a warehouse carrying cost over the length of the delay time.

<u>Transport loading and unloading costs</u>: It will be assumed for simplicity that common costs in loading and unloading can be allocated.

<u>Moving inventory interest costs</u>: During transit an interest cost is incurred on the value of the shipment. The interest cost per unit of shipment varies by mode and destination because transit times vary.

<u>Risk related transit costs</u>: Grain in transit is subject to loss, damage and spoilage. Thus insurance costs are incurred on each shipment. Furthermore, variance in scheduled transit

times increase the risk of incurring penalties due to late arrival of goods and loss of future customers.

For a typical firm at location i (we suppress the superscript n from before for clarity) let:

H^L_{ijm}: per unit loading and unloading costs from i to j by mode m.

d_m: expected equipment delay time for mode m.

s_i: warehouse carrying cost (¢ /bushel).

$rT_{ijm}P_j$: per bushel moving inventory interest cost from i to j by mode m.

$s_i d_m$: per bushel equipment delay cost.

The general model of the elevator's decision problem is to find total output q_i and shipments q_{ijm} for the planning period such that profits are maximized:

$$\max \sum_{j \neq i} \sum_{m} \{(P_j - rT_{ijm}P_j - t_{ijm} - H^L_{ijm} - s_i d_m - P_i) q_{ijm} - H^R_{ijm}(q_{ijm})\}$$

$$(18) \qquad + P_i q_{ii} - C(q_i)$$

$$\text{subject to: } q_i - q_{i++} \geq 0,\ q_i \geq 0,\ q_{ijm} \geq 0$$

Here $q_{i++} \equiv \sum_j \sum_m q_{ijm}$ and $q_{ii} = q_i - q_{i++}$, the amount distributed locally.

Assuming $q_i > 0$ (the non-trivial case), it can be shown that the necessary conditions for profit maximization are as follows:

$$P_i - C' + \lambda = 0$$

$$P_j - rT_{ijm}P_j - t_{ijm} - H^L_{ijm} - s_i d_m - H^{R'}_{ijm} \leq P_i + \lambda$$

(19) with equality when $q_{ijm} > 0$

$$q_i - q_{i++} \geq 0 \text{ and } \lambda(q_i - q_{i++}) = 0$$

$$q_{ijm} \geq 0.$$

Here $\lambda \geq 0$ is the Lagrangian multiplier for the constraint $q_i - q_{i++} \geq 0$. Therefore, it is the value of an additional unit distributed to the local market.

By solving (19) for q_{ijm} in terms of the P_i, P_j's, t_{ijm}'s, H^L_{ijm}'s, T_{ijm}'s, s_i, d_m's and r yields the optimal shipments

(20) $q^*_{ijm} = \eta_{ijm}(P_i, P, t, H^L, T, s_i, d, r)$,

optimal output of saleable grain

(21) $q^*_i = \eta_i(P_i, P, t, H^L, T, s_i, d, r)$,

and optimal allocation to the local market

(22) $q^*_{ii} = q^*_i - q^*_{i++} = \eta_{ii}(P_i, P, t, H^L, T, s_i, d, r)$.

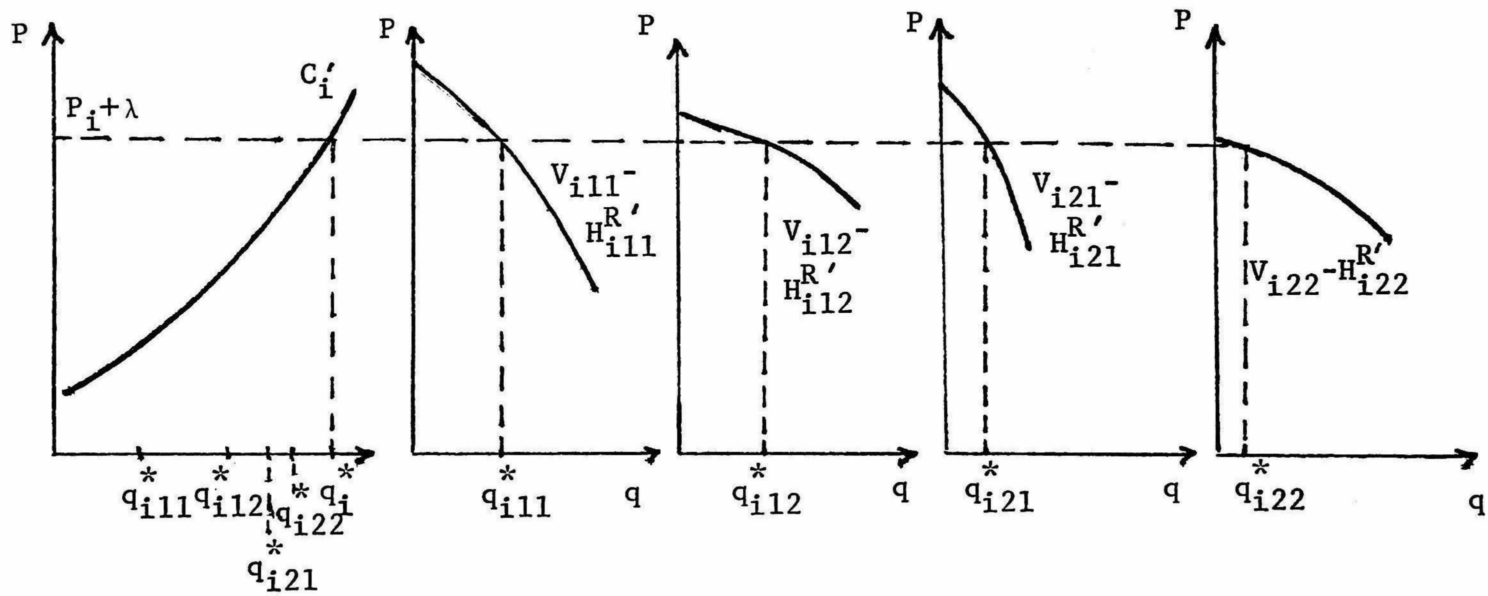
where symbols without subscripts represent vectors of the associated subscripted parameters.

The optimal solution is illustrated (Figure 7) for the three market, two mode case. Here we let

$$V_{ijm} = P_j - rT_{ijm}P_j - t_{ijm} - H^L_{ijm} - s_i d_m.$$

In the example shown a positive quantity is shipped to every mode-market pair. However, since

$$q^*_{ii} = q^*_i - q^*_{i11} - q^*_{i12} - q^*_{i21} - q^*_{i22} > 0$$



OPTIMAL DISTRIBUTION PATTERN

Figure 7

then the shadow price, $\lambda = 0$. Notice that if the local price P_i were too low relative to the net prices in the distant markets, i.e. if the solution q_i°, q_{ijm}° to the conditions

$$P_i - C' = 0$$

$$V_{ijm} - H_{ijm}^{R'} - C' \leq 0, \quad q_{ijm}^\circ (V_{ijm} - H_{ijm}^{R'} - C') = 0$$

$$q_{ijm}^\circ \geq 0.$$

satisfied $q_i^\circ - q_{i++}^\circ < 0$, then the shadow price λ of the last unit of grain allocated to the local market would be positive.

4.3 Econometric Methods of Estimating the Transport Demand Functions for the Movement of Grain

4.3.1 Regression Models

It has been shown that the solution of the elevator's profit maximization problem (18) yields optimal shipments

$$(23) \quad q_{ijm}^* = \eta_{ijm}(P_i, P, t, H^L, T, s_i, d, r).$$

These are the elevator's transport demand functions for the shipments of grain from location i to market j by mode m. Analysis of the theoretical model indicates that these demand functions can in principal be derived if the functions H_{ijm}^R and C are known.

As discussed in the previous section, H_{ijm}^R are the supply curves of perfectly competitive insurance industries. Thus using a regression approach implies that I·J·M supply curves would have to be estimated. Since the H_{ijm}^R's are expected to be convex, the form of the supply curves might be specified as quadratic.

On the other hand very little is known about the form of the cost function for the nonshipping aspects of an elevator's production process. Consequently, misspecification errors are potential problems in a cost study of grain elevators. Furthermore, since an elevator typically provides many other services besides selling grain, there are sure to be problems in appropriately measuring output. Finally, since there are no standard procedures for keeping and collecting cost records, obtaining data for a representative sample of elevators will be extremely difficult and expensive.

An alternative regression method of obtaining shippers' transport demand functions is to try to estimate the function (23), directly. However, a functional form must be specified. This form will depend on the form of the cost function $C(q_i)$ and the $H^R_{ijm}(q_{ijm})$ functions. Thus this approach will encounter many of the same difficulties as the first approach.

The key elements in regression based models for estimating transport demand functions are the shipper's production-related and shipment-related cost functions. These models will be severely limited because cost studies are difficult to conduct. The major obstacles appear to be a paucity of accurate cost data and incomplete knowledge about the grain elevators' production process.

4.3.2. Choice Models

A basic premise of choice models as applied to the demand for transportation of grain elevators is that in a typical planning period (covering the purchase of raw grain to the sale of the finished product) grain is either shipped to one market by one mode <u>or</u> is allocated locally. This initially appears to invalidate the use of such models since the optimal shipping pattern calls for splitting. However, since in the case of grain we are considering low value goods and low-to-medium size shipments (expecially in relation to the total stock of grain) then under these conditions one would expect significantly less splitting then under conditions wherein the value of the good was high or volumes were large. Thus we assume that the importance of splitting to ameliorate risk rises with the value of the good and the volume of transaction.

This premise can be incorporated into the general model (18) of the grain elevator's decision problem by assuming that the functions $H^R_{ijm}(q_{ijm})$ are linear. In this case H^R_{ijm} can be absorbed into the coefficient terms for the q_{ijm}'s in (19). Then the optimality conditions can be written

$$P_i + \lambda - C' = 0$$
$$V_{ijm} - P_i - \lambda \leq 0 \quad \text{with equality when } q_{ijm} > 0$$
$$q_i - q_{i++} \geq 0, \ \lambda(q_i - q_{i++}) = 0$$
$$q_{ijm} \geq 0, \ q_i > 0.$$

These conditions imply that the elevator chooses the alternative with highest price V_i^* from among the V_{ijm} and P_i. The amount available for distribution is then determined by $C'(q_i^*) = V_i^*$ and the whole amount q_i^* is distributed to the alternative corresponding to V_i^*. The shadow price of grain locally distributed is then determined by

$$\lambda = V_i^* - P_i.$$

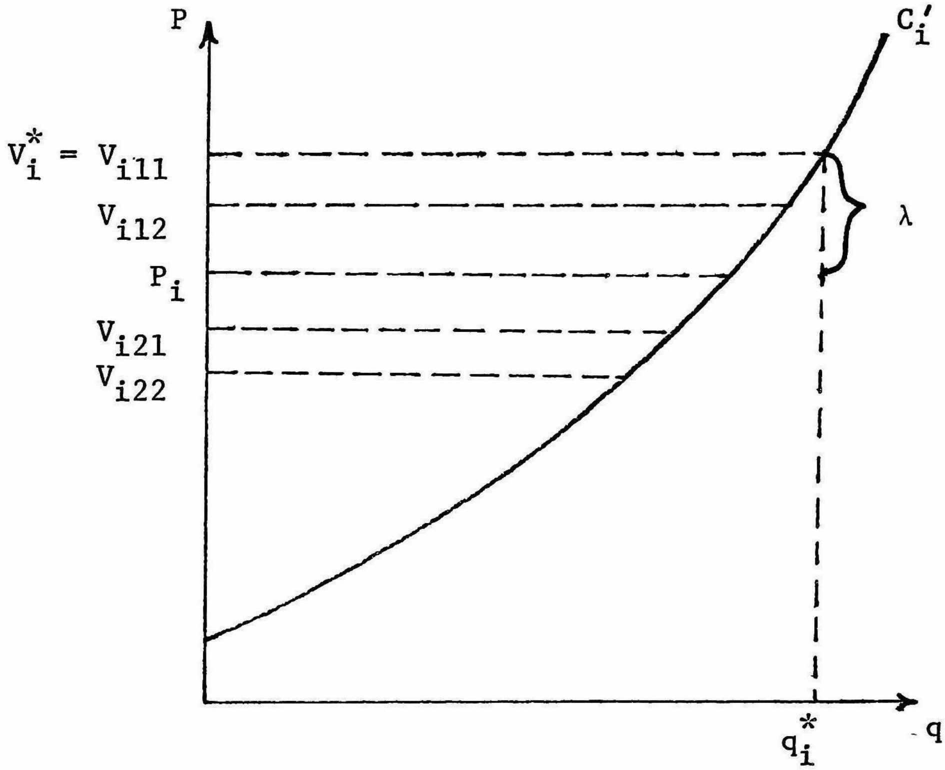
The solution is illustrated for the three market, two-mode problem in Figure 8.

Thus when the indirect transport costs $H_{ijm}^R(q_{ijm})$ are linear in q_{ijm} the decision problem of the elevator is sequential. First, an alternative is chosen which yields the highest net price. The alternatives are the market-mode pairs and the local market. Then the quantity to be allocated to the chosen alternative is determined in accordance with marginal cost pricing.

As in the regression models, the second step of the choice model requires information about the elevator's cost function, $C(q_i)$. However, in empirical applications of the choice model, surrogates can be used in place of marginal cost pricing. For example, if one is willing to assume that elevators approximately price at marginal cost, then historical patterns of market-mode shares can be used to allocate sales to the alternative with the best price. The flexibility to substitute marginal cost pricing with surrogates is an important advantage of the choice models over the regression model.

Logit models are presently used in modelling urban travel demand choice behavior [5, 9, 10]. We shall give a brief description of this class of models and shall relate how they can be applied to the market-mode decisions of grain elevators.

Define the choice variable y that takes the value $y=\eta$ if the individual chooses the $\underline{\eta th}$ alternative, $\eta \in N$. Let x_η



DECISION BASED ON THE CHOICE MODEL

Figure 8

be a vector of observable attributes of the ηth alternative. Let z be a vector of observable characteristics of the individual and let w be a vector of unobservable variables. We assume that the individual's decision depends on the x_η's, z and w. Thus the probability distribution of y is determined by the vector $X = (x_\eta)_{\eta \in N}$, z and the unknown parameters that characterize the distribution of the unobservable w. The most general choice model (see [1]) can be mathematically represented as

$$\text{Prob}\{y=\eta\} = \frac{\exp F_\eta(X,z,w)}{\sum_{\eta \in N} \exp F_\eta(X,z,w)} .$$

McFadden [8] has given the following argument that behaviorally justifies a special case of the general choice model. Suppose that the individual's choice index (e.g. utility, profit, net price, etc.) associated with the ηth alternative is the sum of a nonstochastic part and zero-mean random variable, i.e.,

$$V_\eta + \varepsilon_\eta$$

Here only the non-stochastic part depends on the x_η and z i.e., $V_\eta = V_\eta(x_\eta, z)$.

If we assume that the individual behaves so as to maximize his choice index, then we have

$$\text{Prob}\{y=\eta\} = \text{Prob}\{V_\nu + \varepsilon_\nu < V_\eta + \varepsilon_\eta, \ \forall \nu \neq \eta\}.$$

McFadden has shown that if the ε_η's are independent and with the distribution $\exp[-\exp(-\varepsilon_\eta - \alpha_\eta)]$ where α_η is a parameter, then

$$\text{Prob}\{y=\eta\} = \frac{\exp[V_\eta(x_\eta,z)-\alpha_\eta]}{\sum_{\nu\in N}\exp[V_\nu(x_\nu,z)-\alpha_\nu]} .$$

We now formulate the choice behavior of the elevator as a logit model. First, we replace the market-mode combinations (j,m) by the index η. The location i of the elevator is one of its characteristics and so we can replace the location index by a vector z of the elevator's characteristics. We might want to include in z such characteristics as capacity, ownership structure, as well as its location. Now define V_η as the inner product

$$V_\eta \equiv \langle \beta_\eta, x^*_\eta\rangle \equiv \sum_\ell \beta_{\eta\ell}x^*_{\eta\ell}$$

where β_η is a vector of coefficients and x^*_η is a vector function of x_η and z with components that include $P_j, rT_{ijm}P_j, t_{ijm}, H^L_{ijm}, s_i d_m$. Thus the logit model for the elevator choice problem is

$$\text{Prob}\{y=\eta\} = \frac{\exp V_\eta(x_\eta,z)}{\sum_\nu \exp V_\nu(x_\nu,z)} = \frac{\exp\langle\beta_\eta,x^*_\eta\rangle}{\sum_\nu \exp\langle\beta_\nu,x^*_\nu\rangle} .$$

Since the alternative η's represent market-mode pairs, the above model is a multivariate logit rather than multinomial.

In this choice framework the shipper selects the alternative with the highest choice index and then uses marginal cost pricing to decide how much to ship. Predictions of the quantity shipped on the chosen alternative thus requires information about the firm's marginal cost curve. However, as stated above, surrogates for marginal cost pricing can be developed when information about costs is not available.

Let Q denote the quantity to be shipped. Then given the choice $y = \eta$, Q is determined by $MC(Q) = V_\eta + \epsilon_\eta$. Hence its conditional distribution is

$$Pr[Q \leq q | y = \eta] = Pr[V_\eta + \epsilon_\eta \leq MC(q)]$$
$$= P_\eta(MC(q) - V_\eta)$$

where P_η denotes the probability distribution of the error term ϵ_η.

Now let Q_η denote the quantity distributed by alternative η. Its probability distribution is given by

$$Pr[Q_\eta \leq q] = Pr[Q \leq q | y = \eta] \; Pr[y = \eta]$$
$$= P_\eta(MC(q) - V_\eta) \cdot p_\eta \; .$$

Therefore, the expected quantity shipped by alternative η is

$$E(Q_\eta) = p_\eta \int q \; dP_\eta(MC(q) - V_\eta).$$

Hence

$$E(Q_\eta) = p_\eta \; E(Q | y = \eta). \tag{24}$$

Since p_η is estimated by the choice model, then an estimate of $E(Q | y = \eta)$ used in (24) will yield an estimate of $E(Q_\eta)$. Since, in general, most elevators seek to maximize turnover of stock then the real limit on shipment size (besides indivisibilities present in the system) is the quantity of grain available for distribution. Thus one can estimate $E(Q | y = \eta)$ by estimating the quantity available for distribution.

We intend to estimate this model using data which are being collected in a survey of grain elevator shipping decisions. Since the choices essentially involve simultaneous selection of market and mode, the model calibration is more complicated than those usually applied to urban travel studies. However, several recent studies have suggested ways to estimate multivariate logit models ([1, 2, 11]). In future research we shall try to apply these methods to our model. We shall also try to develop new ways of handling simultaneous (multivariate) choice models.

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