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"I think it's fun, and sort of challenging, but this is just me–I like challenges”: Exploring Early Elementary School Students’ Emerging Mathematics Identities

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Abstract

This dissertation explores the mathematics identities of early elementary school students. Throughout school, many students come to believe that mathematics is not for them or is not a part of who they are or want to be. Previous research has revealed mechanisms that lead to negative relationships with mathematics for middle grade students and beyond, but despite the importance of early childhood for later development, there is little related research with elementary school children. This dissertation addresses this gap in the literature by investigating young children's perceptions of and relationships with mathematics. Specifically, it asked three questions: Which facets of early elementary school students' relationships with mathematics differentiate among their emerging mathematics identities? What features of classroom environments contribute to the development of positive mathematics identities for early elementary school students? And, what can we learn about young children's emerging mathematics identities from the mathematics activities that they initiate?

Data collection involved three rounds of semi-structured interviews with 30 young children in the Chicagoland area, who at the beginning of the study were enrolled in Kindergarten, first, or second grade. The first round of interviews took place in the spring of 2020, soon after school buildings had first closed due to the COVID-19 pandemic. The second round took place during the summer, and the third round took place the following fall when students were in the proceeding grade level. The interviews prompted the participants to tell stories about their experiences with and feelings towards learning and doing math both within and outside of school, in the present and the future.

Qualitative analysis involved both inductive and deductive methods, and resulted in three primary findings that aligned with each of the research questions. First, whether or not students liked math and whether or not they opted to engage in mathematics outside of school seemed to be more meaningful facets of their relationships with mathematics than whether they saw mathematics as a part of their futures or whether they perceived themselves as successful. Second, having conceptual agency, opportunities for self-expression, the ability to be with peers, access to support and learning, being physically comfortable, having control over pace and schedule, and having fun were all features of the learning environment that appeared supportive of children's positive experiences learning mathematics. Third, when children seized mathematical agency and orchestrated mathematics experiences for themselves outside of their classrooms, they did so in ways that were more expansive and creative than in school.

Across these findings, agency emerged as an important through line. Specifically, children in this study expressed having, appreciating, or seeking five distinct types of agency as parts of their mathematics learning experiences: conceptual, creative, physical, temporal, and interactive. These Five Dimensions of Agency extend the literature's previous descriptions of agency to better encapsulate the intellectual, emotional, physical, and social experiences of a student in a mathematics classroom. The relationship between agency and mathematics identity can be described as mutually reinforcing, and in turn attending to young students' opportunities for agency is key in supporting their positive mathematics identity development. This takeaway has implications both for future math identity research and for classroom educators looking to ensure that more of their young students form positive relationships with mathematics.

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Preface

As an elementary school teacher in two different urban contexts, I saw first hand the polarizing reception my young students gave to mathematics; they seemed to love or hate math more so than they loved or hated other parts of the day. Ariah was in my first 2nd grade class. She always arrived smiling, and was curious, kind, and determined to learn. However, Ariah scored poorly on standardized assessments. At the time, the school required teachers to align grades with test scores, and so, despite my personal inclination against doing so, I assigned Ariah a 'C' in mathematics on her report card. The next morning, she came to school and said, "I was kinda disappointed that you gave me a C, but my mom said it was ok, I don't have to be a math person." I have wondered ever since if Ariah's sense of self in mathematics has been able to rebound.

Several years later, Sophie was in my 3rd grade class. She was creative, thoughtful, and uniquely insightful. By all standard measures, Sophie was extremely successful in mathematics. Yet, she shared with me early on that she believed she was not good at math and she did not like it. Over time, I began to see that what Sophie did not like was not the mathematics itself, but rather her classroom experiences with it. She was a steady and deliberate problem-solver, and she felt diminished when students rushed or shouted out answers. She was meticulous, and she felt frustrated when working with others who were less thorough. I tried to shape my classroom to support Sophie in developing a positive mathematics identity, but I struggled to do so. Sophie, Ariah, and many other students motivate my dissertation and my broader desire to conduct research that investigates the development of math identities in young children and that begins to shed light on how to help students develop positive relationships with mathematics.

For my former students and colleagues at Harvard School of Excellence, The University of Houston Charter School, and Mary Gage Peterson Elementary School, who showed me what it means to learn and teach mathematics.

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Chapter 1
"It's hard and you don't really get a bunch of fun": Introduction and Literature Review on Identity, Learning, and Mathematics Education

What is a ‘math person?’ Is it the person who gets straight A’s in math in school? The person who scores in the top percentile on standardized mathematics tests? The person who memorizes the most digits of pi? The person who can, with great facility, make calculations mentally? The person who can reason through a tricky situation and come to a logical conclusion? The person who sees patterns all around them? There is a pervasive perception that whoever that person is, there might only be one in a crowd. But what if there is no such thing as a ‘math person’ at all? Imagine that everyone in that crowd–the straight A student and the student getting C’s, the quick calculator and the person who takes their time, the person who considers herself a ‘math person’ and the person who fervently does not–is completely capable of engaging in mathematical thinking and using mathematics in a multitude of ways. Despite common beliefs, research supports this perspective and is emphatic that there is nothing inherently exclusive about learning or doing mathematics (e.g. Boaler, 2022; Lakoff & Nuñez, 2000). Yet, many people feel and are excluded from the discipline. This begs two questions: first, why is the societal narrative that only a few particular people can be successful in mathematics so resilient? And second, what can we do to change that?

Schooling is a significant piece of the answer to the first question. In a variety of structural and pedagogical ways, schooling perpetuates the discourse that some people are inherently good at math while others are not (Black et al., 2011). Despite reform efforts, many math classrooms remain narrow in their pedagogies, thereby encouraging forms of knowing and ways of learning that are incongruous both with the discipline of mathematics as seen by mathematicians and with many capable students who are seeking opportunities for agency, collaboration, and creative thought (Boaler & Greeno, 2000). Further, even more so than other school subjects, math has been used to sort, rank, and label students, often along lines of gender, race, home language, and dis/ability status (e.g. Martin, 2012; Barajas-Lopez, 2014; Langer-Osuna, 2011; Tan et al., 2019). People who are not successful in mathematics early on often assume that they are not capable enough to pursue it later (Gutiérrez, 2013), and even students who do experience success in mathematics often believe that their achievement is fragile (Solomon et al., 2011). When students reject mathematics, their reasons span beyond their sense of their own abilities or even their liking of the subject; rather, students often reject math because they feel that the subject is not intended for them, that it is not a part of who they are as students now, or that it is not a part of the people they are hoping to become (Gutiérrez, 2013; Boaler & Greeno, 2000).

The ways in which mathematics education excludes and marginalizes students becomes even more problematic when paired with the fact that mathematics achievement is highly valued within and beyond schools. The high regard for achievement in mathematics means that grades and scores specifically in mathematics are often used as gate-keepers for other academic opportunities in high school and in college (Esmonde, 2009). These opportunities are important for access to a wide range of careers and positions of power and societal influence (Esmonde, 2009). The right to access quality mathematics education also has arguments beyond economics; mathematics is a tool for understanding, seeing, navigating, and flourishing in the world (Su, 2020). It allows people to problem-solve, think creatively, and contribute to the betterment of their livelihoods and the livelihoods of others. Therefore, pushing back against the narrative that only some people are ‘math people’ and shifting mathematics education to be more supportive and inclusive is part and parcel with the fight for educational justice and equity.

This dissertation takes the perspective that every child is capable of thinking mathematically and contributing mathematical ideas, and further that every child has the right to mathematics experiences in schools that support them in developing a positive relationship with the discipline. Through three rounds of interviews with 30 early elementary school students during which the children told stories about their experiences with and feelings towards learning and doing math, this study explored young children’s emerging mathematics identities. The findings elevate young children’s perspectives on what math learning could and should become.

Theoretical Perspectives on Learning and Identity

Around the turn of the millennium, researchers in mathematics education began addressing the second part of my aforementioned question–how we can change course and challenge the exclusive nature of mathematics and mathematics education. They did so by shifting the research discourse away from solving problems within learners and instead investigating how individuals become learners and doers of mathematics in relation to their contexts. These researchers saw learning mathematics as a fundamentally social and cultural process and believed that in classrooms, students do not just learn content, but they also learn how to be math learners and what it means to be someone who does mathematics (Langer-Osuna & Shah, 2021).

This perspective drew on a sociocultural theory of learning; emphasizing that learning is a social process and that social interaction plays a fundamental role in how and what people learn (Vygotsky, 1978). It also relied on the theory of situated learning, which was developed from a sociocultural perspective and elevates the importance of context. Brown et al. (1989) argued that the situations in which learning occurs are key, active players in understanding what is learned and how it is learned, and that it is not possible to isolate these pieces from one another. As an example, learning without recognition or valuing of the context is akin to memorizing definitions of vocabulary words from a dictionary as opposed to learning through conversation with others in daily life; the former is far more likely to lead to future errors and limited knowledge flexibility than the latter (Miller & Gildea, 1987). Accordingly, in order to understand learning, and to design for robust learning, one must attend to the participants in a physical space, their interactions with each other, the activities the participants are engaging in, and the materials being used (Greeno, 1997). Situated learning suggests that when context changes, or when any of the components that make up context change, the potential learning also changes.

Sociocultural and situated learning theories also articulate the relationship between learning and the learner's identity. According to Wenger (1998), people learn as they engage in and contribute to the practices of their communities, and "What they learn is not a static subject matter but the very process of being engaged in, and participating in developing, an ongoing practice," (p. 95). In other words, learning is the process of becoming a more full participant in a particular community, or a process of becoming a particular type of person. Learning is, in part, a change in a person's identity. As a learner becomes more fluent in the community's practices, their participation becomes more central and they have greater potential for contributing to the community (Wenger, 1998). When a person is a full member of a community of practice, they feel and are recognized as competent by other community members, and they view their own participation in the practice as an integral part of who they are (Nasir & Cooks, 2009).

Conversely, a person who is not, or not yet a full member of a community of practice may harbor feelings of unfamiliarity and incompetence. In some situations, a person on the periphery may be on an inbound trajectory in which their participation and recognition by the community is increasing. However in other situations, a person may remain marginalized on the periphery or may in fact be on an outbound trajectory in which their participation and identification with the community of practice lessens over time (Wenger, 1998).

Additionally, these theories posit that learning and identity development are fluid with changing membership in communities that are also themselves changing (Wenger, 1998). People are members of multiple communities, and prioritize different memberships at different times. Further, communities as collectives learn as they refine their practices and as new generations of members participate and contribute. They act as repositories of shared histories of learning that collectively define what current competent membership looks like and involves, thus shaping how newcomers see themselves and are seen by others. New members need to see a potential future of increased participation within the community in order to learn and eventually identify with the practice. In other words, learning as becoming only exists for an individual when the community of practice makes identity trajectories available (Wenger, 1998).

In bringing sociocultural learning theory and math education research together, Sfard and Prusak (2005) sought to further clarify the concept of identity. They agreed with Wenger and other sociocultural theorists that identities are some combination of created in the moment, influenced by context, and accumulated over time. People neither remake themselves in every interaction nor do they reach an imagined stasis, but rather who we are gets renegotiated over time and across contexts (Wenger, 1998). However, Sfard and Prusak were concerned that regardless of theorists' intentions, definitions of identity focused on being or becoming a particular kind of person risked being understood as a search for a person's immutable essence, and therefore negated the very heart of identity as collectively developed and shaped through engagement and interactions. They wanted to emphasize that people have agency in the creation of their identities.

In response to this concern, Sfard and Prusak (2005) focused on how people tend to describe themselves and others as a way to understand and explain actions, experiences, and situations. These descriptions are often reifying, meaning they suggest a known reality. For example, after finishing a map-making school assignment with confidence, a student might declare, "I am a good map-maker!" In that moment of declaration, the student responded to her particular context–the expectations put on her by the teacher, the characteristics of the task in front of her, and her relationship to them–and she identified herself in a specific way. Put otherwise, the way that the student described herself formed her identity, showing that identity can be understood as the stories people tell about themselves. Sfard and Prusak (2005) put forth the theory that stories are not just windows into an identity that remains potentially unchanged, but rather that narratives *are* identities, and are influential themselves, shaping actions and beliefs in the moment of their utterance.

Like in the example above, sometimes stories are told by the identified person themself, thus capturing the storyteller's first person identity (Sfard & Prusak, 2005). Other times, stories are told to the identified person by someone else, thus forming a second person identity (Sfard & Prusak, 2005). An example of this is in the case of a parent telling their child that they are a certain kind of person or a teacher telling a student about their strengths. It is also frequent that people talk about other people who are not in their presence. Third person identities are stories told about an identified person between others (Sfard & Prusak, 2005). In the realm of schools, this often happens when parents and teachers discuss children. These stories may not all be the same, and while some may reflect an objective state of affairs, more often stories reflect what Sfard and Prusak referred to as a designated identity, or a state of affairs that is expected to be, now or in the future. Stories reflect reality as the storytellers see it. Still, all of these stories, or all of these identities, are relevant to a person's ongoing engagement with the world around them.

Sociocultural, situated, and narrative theories all converge on the belief that learning and identity do not develop in isolation, but rather are processes of interaction with places, people, and materials. Sociocultural and situated theories highlight that when contexts change, the potential for learning and identity development also changes. A narrative theory of identity does not disagree, but it shifts the focus to the stories we tell about ourselves. As contexts shift, we are introduced to new possibilities for what our stories might include, and in turn we revise and develop our stories, and therefore our identities (Sfard & Prusak, 2005; McAdams, 1993; Langer-Osuna & Esmonde, 2017). Importantly, across these theories, identity is malleable and can be constructed differently in particular moments and framed differently across time. Its development is a combination of individual agency and collective influence. With this balance in mind, math education researchers began thinking about how learning experiences in schools and classrooms offer varying opportunities for students to develop identities in relationship with mathematics.

Foundational Studies of Identity in Mathematics Education

Though their specific focuses and methodologies varied, math education researchers who began to investigate the role of identity in learning mathematics shared the goal of supporting more students in forming positive, empowered relationships with mathematics while challenging beliefs that certain students or groups of students were inherently less capable. In their seminal work, Boaler and Greeno (2000) investigated features of learning environments to understand how particular approaches to instruction shaped students' experiences. From interviews with high school calculus students across six schools, they found that students were more likely to enjoy mathematics and to intend to continue learning mathematics in their futures if they perceived their classrooms as collaborative spaces where they were able to work with others and participate actively in the learning process. In contrast, students who perceived their classrooms as individualistic and felt they were expected to sit, listen, and absorb content were more likely to disassociate from mathematics. Their findings emphasized that the classroom context shaped not only the opportunities to learn mathematics content, but also the relationships that students formed with the discipline.

Martin (2000) also looked at how learners form relationships with the discipline of mathematics. Specifically, through an ethnographic and observational study of African American students along with their parents and teachers, he focused on the ways in which mathematics experiences in schools are racialized and how influences across intrapersonal, school, community, and sociohistorical levels shape the mathematics opportunities of African American students. In describing his findings, Martin (2000) defined a student's *mathematics identity* as their beliefs about their abilities to perform mathematics, participate in mathematical environments, and use mathematics to shape and change their lives. This definition, which emphasizes how an individual perceives themselves as opposed to how they are perceived by others and which relies on the stories students tell, became important for future work in the field.

Like Martin, Nasir (2002) also focused on the experiences of African American students. She brought together the literature on situated learning with research on how learning happens outside of school (e.g. Saxe, 1988) to understand how cultural, mathematical practices offered youth opportunities for identity development. Her study of these practices, including dominoes and basketball, revealed that becoming certain kinds of learners in general, and mathematics learners in particular, was related to the characteristics of the contexts in which learning was happening. Nasir argued that a practice's organization, norms, and structures all influence the relationships participants can form with the practice, or their *practice-linked identities*. Defined as "the identities that people come to take on, construct, and embrace that are linked to participation in particular social and cultural practices," (Nasir & Hand, 2008, p. 147), practice-linked identities reflect to what extent a person sees a particular practice as a part of who they are. Nasir's work also highlighted that the constructs of engagement, participation, learning, and identity are cyclically related. When one feels closer to a practice, they are more likely to be more engaged, participate more, learn more, and thereby further strengthen their practice-linked identity.

As the field of mathematics identity research continued to develop, Cobb et al. (2009) crafted a framework intended to further clarify the construct of mathematics identity or mathematics-linked identities with the intention of moving the field towards a shared definition that could be applied across contexts. Their framework delineates two forms of identity in mathematics classrooms: normative and personal identities. Normative identities reflect what it takes to be an effective student in a particular classroom, whereas personal identities are the extent to which individual students identify with, comply with, or resist the collective normative identity. In other words, personal identities describe individual students' relationships with mathematics as they are experiencing it in their classrooms. When a students' personal identity is strong and they are affiliating or beginning to affiliate with the discipline of mathematics, the classroom obligations that were initially obligations-to-others become obligations-to-self. Their framework also breaks down the features of classroom environments that impact identity. Cobb et al. (2009) describe normative identity as influenced by the ways in which students can express agency in a classroom, how authority is distributed, and what students are expected to be able to do and know in order to be considered as competent. Personal identity reflects students' perceptions of these same features. As an example of this framework's potential application, they compared the experiences of two groups of eighth grade students, one in a traditional algebra class and one in a design experiment class focused on data literacy. They found that students who felt that authority was shared in the classroom between teacher and students and who experienced conceptual agency were more likely to feel successful and to begin to affiliate with the mathematical activity happening in their class compared with students who perceived authority as distributed solely to the teacher and who felt limited to reproducing established procedures.

This foundational research on identity in mathematics education shared several threads. It drew on theories that defined identity as malleable and in relationship to context. It assumed that the problems of students struggling with and dissociating from mathematics were not rooted in the students, but rather were a result of the interactions with and between features of their learning environments that limited their opportunities for competence, positive relationships with mathematics, and the types of mathematics learners they could become. These researchers held equity front and center, striving to support more students, and specifically historically marginalized students, in having their mathematical selves recognized and encouraged.

Ongoing Mathematics Identity Research

Researchers have carried on these threads and continue to tackle the challenge of uncovering the factors that support positive mathematics identity development. Ongoing research approaches this from a variety of angles. Some researchers are continuing work specifically with communities of students that have been historically and are still frequently marginalized in mathematics classrooms. For example, Berry et al. (2011) emphasized the intersection of multiple identities–including racial, gender, and academic–in their work that focuses on mathematics learning for social justice and that seeks to uplift the stories of Black boys in particular. Relatedly, Gholson and Martin (2019) investigated the intersecting influences and negotiations of race, gender, and class in the mathematics learning experiences of Black girls. These researchers intentionally elevated stories of Black students' success in mathematics as counter-narratives to the frequent documentation of their struggles.

Barajas-Lopez (2014) similarly elevated stories of Mexican immigrant students' success in mathematics. He wrote about both engagement and disengagement in mathematics classrooms, highlighting the importance of context and teacher-student relationships. Turner et al. (2015) also emphasized the important role of teacher-student interactions. They described that math tracking practices in schools especially marginalize students who speak languages other than English and discussed the importance of positioning students who are learning English to have opportunities for active and meaningful participation in math discussions so that they can contribute to the collective activity and learning in the classroom. Oppland-Cordell and Martin (2014) showed that when students had the opportunity to engage and feel a sense of belonging, they began to construct more positive images of themselves and their mathematical worlds. They did so by telling the story of a group of Latinx students who experienced increasing success, participation, and identification with mathematics through a culturally diverse calculus workshop. With themes of resistance and resilience, all of these researchers, along with others, continue to push the field to decenter the experiences of white, middle and upper class students and change systems to better support, reveal, and celebrate the mathematical brilliance of students from diverse racial, cultural, and linguistic backgrounds.

Other researchers have put the role of teacher at the front and center, seeking to understand how teachers can support the development of their students' positive mathematics identities. Battey and Franke (2008) presented case studies of how teachers' mathematics identities related to and influenced the ways in which they taught mathematics in their classrooms and what they expected of their students, thus influencing the opportunities students had to relate to the discipline. For the teachers in their study, shifts in identity, and therefore in learning and teaching, were slow and difficult; becoming a new kind of mathematics teacher or a new kind of mathematics learner is much more complex than learning the steps of addition. This highlighted the importance of teacher professional development and reflective identity work, which is the subject of Aguirre et al.'s (2013) book written for teachers and teacher educators. Their thesis is that when teachers reflect on their own and their students' multiple identities, they open up more possibilities for students to learn and grow as students who are seen as and feel like competent mathematical thinkers. Louie (2018) also elevated the idea that teachers can take steps to widen definitions of and opportunities for competence in their classrooms. Specifically, she built on the work of teacher noticing (Sherin et al., 2010) and showed that mathematics teachers can attend to, interpret, and respond to students' mathematical ideas in ways that position students from non-dominant communities as mathematical sense-makers, thus expanding who is seen as contributing to the community.

A third way in which researchers have approached ongoing work on mathematics identity is through centering specific pedagogical practices or constructs. This research has looked at classrooms and other learning environments to understand the features that are supportive of students' positive mathematics identity development. For example, Kazemi and Hintz (2014) focused on mathematical discussions and the ways in which they foster both mathematical sense-making and the development of community in classrooms. They argue that fostering intentional mathematical discussions is one way to expand opportunities for competence and shared authority in mathematics classrooms. Langer-Osuna (2011, 2017) has investigated collaboration in mathematics classrooms and its relationship to authority and mathematics identity development. She has highlighted the ways in which students' bids for authority can be differently interpreted by their peers in relation to gendered expectations and emphasized that it is critical for teachers of collaborative mathematics to take social identities, power dynamics, and relationships into account when organizing opportunities for collaboration because how these factors interact with each other influences students' opportunities for learning and for forming mathematics identities. Relatedly, Ruef (2021) looked at how mathematical argumentation can be a supportive space for expanding the understanding of what it means to be 'good at math.' She followed a single teacher and class over the course of the year to see how the participants together co-constructed an expansive conception of mathematical competence.

Spurred in particular by the COVID-19 pandemic and the preponderance of virtual learning, recently there have been studies that looked at how online learning can support students in developing positive relationships with mathematics outside of physical classrooms. Hodge-Zickerman et al. (2021) encouraged educators to keep learning theories at the forefront of design of online learning spaces and to use technology to leverage instructional priorities including fostering meaningful teacher-student, student-student, and student-content interactions. Jessup et al. (2020) argued that though there were challenges with the rapid change to online instruction during the pandemic, teachers can, within the space of online learning, build authentic relationships with students, allow multiple modalities for engagement, and provide flexibility for participation that together support students in building community and seeing themselves as growing, contributing members of that community.

Like the foundational research in mathematics identity, this ongoing work focuses on equity, inclusion, and expanding opportunities for students to develop positive relationships with mathematics. The researchers who do work with specific, marginalized communities, those who focus on the role of teachers, and those who investigate particular pedagogical practices all see educators and designers of learning environments as having significant influence on students' mathematics identities and therefore the responsibility to make intentional decisions that support more students in seeing themselves as competent mathematics learners with the potential to contribute to the learning of their classroom communities. They also maintain that part of this responsibility is ensuring that students have agency in their mathematics learning experiences.

The Role of Agency

Agency, which can be defined as the extent to which students are able to decide what to do and how to do it, is a critical construct in understanding mathematics identity. Research has shown that when students are positioned as agentive, and are able to take an active role in their classrooms, they are more likely to enjoy mathematics and see it as a part of their futures (Boaler & Greeno, 2000). Further, when mathematics learning environments value students' whole selves and allow them to make meaningful decisions about their learning, students' identification with the discipline is strengthened (Nasir & Hand, 2008). As students exercise agency in math classrooms they find their place in and form a relationship with mathematics. Put otherwise, students' experiences of agency are important both to how students build and enact their mathematics identities. In turn, for researchers who are committed to understanding students' relationships with mathematics, it is meaningful and productive to explore students' experiences of agency in math classrooms.

Agency is often understood as a continuum, meaning that students are seen as being able to exercise somewhere between very little and a lot of agency (Ruef, 2021; Cobb et al., 2009). However, instead of focusing on quantity, some researchers have delineated specific types of agency that students may or may not experience in classrooms. Cobb et al. (2009) described two forms of agency: disciplinary and conceptual. This delineation built on work from Pickering (1995) who studied professional mathematicians and found that sometimes the mathematicians developed their own, new concepts while other times they were constrained by existing structures of the discipline. In applying this research to mathematics classrooms, Cobb et al. (2009) defined disciplinary agency as the requirement for students to use established solution methods; in other words, in these cases it is the discipline of mathematics that has the agency, more so than the students. This form of agency actually limits opportunities for students' success and constrains possibilities for students to bring themselves into the work. In contrast, conceptual agency involves students having the opportunity to choose and develop problem-solving methods that make sense to them. Students with conceptual agency build relationships between concepts and establish understandings of big ideas (Cobb et al., 2009). This allows for multiple pathways to success and opens up possibilities for students to bring themselves into mathematics, thus supporting students in developing positive mathematics identities.

Beyond conceptual agency, there are other ways that a student might experience agency in their math classroom. Ruef (2021) explained that one component of student agency is the extent to which they are able to "compose their bodies" (p. 15) through decisions such as where and how to sit or when to stand up to access a supply or sharpen a pencil. Students might also have the agency to decide what materials to use, who to work with, or even what activity to participate in. Further, students might exercise agency over their own mathematical contributions, deciding what ideas to share with their community and how to share them. Given that mathematics does not only live within the walls of classrooms, students' agency in mathematics could also be carried into their mathematical endeavors outside of school. At home and in their communities, students can make more decisions about how and when they do or use mathematics. As they do so, they continue to strengthen their relationships with mathematics while simultaneously revealing their current mathematics identities through the ways they choose to engage with the discipline.

Agency is related but distinct from several other constructs. Authority, for example, or who is in charge of making mathematical contributions and deciding if solutions are legitimate (Cobb et al., 2009) is less about making decisions for oneself and more about influence over others within the community. Ruef (2021) defined authority as “the arbiter of accuracy and correctness,” which can be a role that is shared between teacher and students but is often left in the hands of teachers alone. Authority is more a description of social power than a description of how an individual decides what is best for them. Still, agency and authority are tightly related. If authority in the classroom is solely distributed to the teacher, then students are less likely to have agency, and if authority is shared between teachers and students, then students are more likely to experience agency over multiple dimensions of their learning (Cobb et al., 2009).

Engagement is another related construct, but instead of focusing on decision-making, engagement describes a global sense of a student’s involvement or participation. Agency and engagement are mutually supportive; a student may be more likely to be engaged in a context that they perceive to value who they are as a participant. As their engagement increases, students become more likely to exercise agency that is granted by that supportive context (Nasir & Hand, 2008). And in the reverse, as students exercise agency, they may experience an increase in their engagement. Further, greater engagement and experiences of agency in activities both support students in developing stronger affiliation or identification with those activities (Nasir & Hand, 2008). Ongoing research on mathematics identity involves continuing to disentangle these related constructs so as to better understand the roles that each of them play.

Identity Development in Young Children

The vast majority of mathematics identity research thus far has focused on older students, especially at the high school level (e.g. Boaler & Greeno, 2000; Nasir & Hand, 2008), with some studies reaching into late elementary school (e.g. Langer-Osuna, 2011) and middle school (e.g. Cobb et al., 2009). Research is scant on the development of mathematics identities in young children. However, studies from other fields offer reason to believe that mathematics identities are in active formation in elementary school, if not even earlier. For example, work in developmental science has revealed that young children are constantly making sense of the worlds around them. According to developmental intergroup theory (Bigler & Liben, 2006), very young children begin to notice different social groups based on their salience in society. Environmental conditions, developmental patterns, and individual differences all contribute to the ways in which children understand stereotypes and develop prejudice. Relatedly, social identity theory (Tajfel & Turner, 1985) highlights that as members of communities, children develop social selves through categorization and comparison. As young children develop a sense of belonging, often accompanied by pride and self-esteem, they separate people as like them (in-group) or different from them (out-group). As an illustration of this, when Rogers et al. (2012) sampled a diverse group of elementary school students, they found that the students had impressive sophistication in their awareness of the social meanings attached to ethnicity, both in relation to themselves and their peers.

In addition to interpreting their contexts, young children also make sense of who they are as individuals. As children enter elementary school, they navigate new social structures, form ideas about their own talents and aspirations, and develop attitudes towards school (Coll & Szalacha, 2004). These attitudes and self-perceptions strengthen over time and become more deeply intertwined with their specific schooling experiences (Coll & Szalacha, 2004). In fact, young children's perceptions about who they were, are, and may become in relation to the contexts in which they find themselves are most certainly influential on their later life trajectories both within and beyond their time in school (Coll & Szalacha, 2004; McAdams, 2013). Further, not only can young children perceive complexity in their environments and in their own development, they can also articulate their perceptions. Young children can be valuable and reliable reporters about themselves and their experiences and can offer unique information not otherwise accessible to adults (Sabol et al., 2020).

Together, this work suggests that if we aim to understand students' mathematics identities, it would be fruitful to investigate young children's experiences with and relationships to mathematics, at an earlier age than has been represented by research thus far. Previous studies have shown that children in late elementary school, middle school, and high school already have complex mathematics identities. Sometimes these mathematics identities are quite negative in nature, with students distancing themselves from the discipline of mathematics. In order to support students in forming positive relationships with mathematics, it would behoove us to look closely at the mathematics identities of younger children so as to understand how mathematics identities emerge, develop, and change over time as students get older and move through school.

Bringing the Pieces Together: A Study of Young Children's Emerging Mathematics Identities

This dissertation builds on the body of literature that has investigated the intersection of mathematics learning and identity. Here, I define mathematics identity as the stories that children tell about their relationships with mathematics. I see these relationships as reflecting a combination of children's feelings about their mathematics experiences and how they perceive themselves as learners and doers of mathematics both within and outside of the classroom, now and in the future. This definition builds on sociocultural, situated, and narrative theories, believing that in mathematics classrooms, the interactions of people, materials, and ideas shape how students learn mathematics content alongside what it means to be learners and doers of mathematics and that identities are developed as people reflect and tell their stories (Sfard & Prusak, 2005). It pulls from the foundational work of Martin (2000) who defined mathematics identity as a collection of personal beliefs, the work of Cobb et al. (2009) who delineated personal mathematics identity as reflective of an individual's perceptions, and the work of Boaler and Greeno (2000) and Nasir and Hand (2008) in valuing the influence of features of learning environments on students' experiences within them, both in and outside of classrooms.

The challenges cited by researchers who began investigating the role of identity in mathematics education thirty years ago still exist; mathematics continues to be a field that marginalizes and excludes students, and the societal belief that only some people are 'math people,' persists. Continuing to respond to this challenge matters for equity in mathematics education–all students have the right to be successful in mathematics and to see themselves as mathematically competent. It also matters for students' ongoing academic and career opportunities. And, it matters because mathematics is a powerful way to navigate, problem-solve, and flourish in the world. This project places importance on "attentiveness to the hearts of children" (Davis & Shaeffer, 2019, p. 386) and takes the stance that the field of mathematics education's understanding of mathematics learning and identity can be enriched by looking closely at and taking seriously the mathematics experiences of young children, including their opportunities to exercise agency in mathematics learning spaces, at the beginning of their academic journeys. The roots matter for the growth of the tree.

Specifically, this dissertation asks three questions: Which facets of early elementary school students' relationships with mathematics differentiate among their emerging mathematics identities? What features of classroom environments contribute to the development of positive mathematics identities in this same group of students? And, what can we learn about young children's emerging mathematics identities from the mathematics activities that they initiate?

To answer these questions, I engaged in an interview-based study with 30 early elementary school children from across the Chicagoland area. Over the course of seven months, between May and November 2020, I interviewed each student three times, spanning two academic school years with the summer in between. This was a period of cultural and educational upheaval due to the COVID-19 pandemic. For students, it may be that the significant contextual changes they experienced throughout the duration of this study heightened their feelings and insights or made particular moments and experiences salient. I took this unique moment in time as a rich opportunity to learn from young students about their mathematics learning experiences in general and about the shifts they experienced because of the pandemic in particular. At each interview, I asked students to reflect on their experiences learning and doing mathematics within and outside of their classrooms. By describing their feelings about their mathematics experiences, how they perceived themselves as learners and doers of mathematics in and out of school, and the role they saw math playing in their lives now and in their futures, children's responses to the interview questions showed how they understood their own mathematical agency and revealed their relationships with the mathematics–their emerging mathematics identities. Using both inductive and deductive methods (Miles et al., 2020), I transcribed, coded, and analyzed the interviews, looking for patterns and trends within and across individual students, grade level groups, and the study participants as a whole.

In the remainder of this dissertation, I delve into the details of this study and what it means for the field of mathematics education. In Chapter 2, I describe the study design and methods including recruitment, data collection, and data analysis. In Chapter 3, I apply frameworks from previous mathematics identity research to investigate their applicability with young children. In doing so, I show that certain metrics, including whether students felt successful and whether students saw mathematics as a part of their futures, did not sufficiently differentiate amongst this study population, while others, such as whether students enjoyed mathematics and whether they initiated mathematics activities in their own time, seemed more fruitful for understanding students' developing relationships with the discipline. I also highlight that opportunities for self-expression and conceptual agency, two features of learning environments discussed in previous literature, appear important to young children's emerging mathematics identities. In Chapter 4, I investigate students' perceptions of their mathematics learning experiences in their physical classrooms and when they were learning remotely at home, due to COVID-19. I show that students found certain priorities better met at home, while others were better met at school, and I reveal the ways in which students emphasized the importance of varying forms of agency in shaping their feelings about each context specifically and learning mathematics in general. In Chapter 5, I dive into the mathematical activities that children described initiating outside of school, specifically looking at what those activities were, students' reasons for engaging in them, and how they compare to the ways students describe the mathematics activities in their schools. In doing so, I show that when students seize agency and engage in mathematics on their own terms, they are often driven by pleasure and curiosity, and they do so in ways that are expansive and creative. Finally, in the discussion I explain the ways in which agency appeared as a through line across my analysis, and I describe how agency and mathematics identity are cyclically related, reinforcing each other within early elementary school students' mathematics learning experiences. I end with a description of limitations to this study, implications for mathematics education researchers and practitioners, and directions for future work.

Chapter 2
Study Design and Methods

On Monday, March 9, 2020, I met with a principal at a Chicago-area elementary school and concretized plans for beginning research in three specific classrooms. In order to investigate the emerging mathematics identities of young children, I had planned a study that centered on observations of students during math class. Three days later, on Thursday, March 12, that particular school district announced temporary closure. On Friday, March 13, the governor of Illinois ordered the closure of all public and private K-12 schools across the state for two weeks. At that point in time, 46 people in the state had tested positive for coronavirus, and none of us could have imagined how the next months into years would progress. That is the context in which I embarked on this dissertation.

About three weeks after local schools closed, I decided that waiting for schools to reopen was not going to be my answer, and that I would instead need to redesign my study in a way that worked with the changing times and all of the associated unknowns. So, I shifted gears. Though the conditions of the pandemic were in many ways constraints–I could not, for example, visit schools and actually see children engaging in math learning–I found that they also opened up unique opportunities for talking with and learning from young children. In the end, I designed an interview-based study. First and foremost, interviews could easily be conducted online, allowing for consistency in data collection despite social restrictions and school closings during the pandemic. Further, semi-structured interviews (Spradley, 1979) were a meaningful methodology for my dissertation because in interviews, participants could reflect and make sense of their experiences and predict their own futures; they could choose their words and therefore how they represent themselves (Braathe & Solomon, 2015). In this way, interviews are actually a site for the emergence of identity. As interviewees speak and respond to questions, they literally craft who they are (Braathe & Solomon, 2015). Interviews also allowed me as the researcher to access and probe for information about people that is often unobservable–their perceptions, ideals, and emotions (Lamont & Swidler, 2014)–all of which are deeply relevant to understanding identities.

Over the course of seven months, I interviewed 30 early elementary school children from the Chicagoland area three times each. These interviews spanned two academic school years with one summer in between. They were designed to collect the data that would help me answer my three research questions: Which facets of early elementary school students' relationships with mathematics differentiate among their emerging mathematics identities? What features of classroom environments contribute to the development of positive mathematics identities for this same group of students? And, what can we learn about young children's emerging mathematics identities from the mathematics activities that they initiate?

In the remainder of this chapter, I describe in detail the design and execution of this study. First, I explain how I recruited participants, and who those participants ended up being. Then, I detail my data collection process and explain each phase of my analysis. Finally, I close with sharing how my own positionality as a previous elementary school teacher and a mother of two young children during the pandemic impacted my perspective on this work. Together, this chapter paints a picture of how this study came to be and what it looked like as it was happening.

Recruitment

I aimed to enroll 30 participants in this study, evenly spread across Kindergarten, first, and second grade. The Kindergarten through second grade age span was selected due to the lack of representation of early elementary school children's voices in the literature on math identity development and because young students have just recently embarked on their formal schooling, thus opening up more opportunities for developing disciplinary affinities and mathematics identities. I also decided to limit participation to participants within the Chicagoland area. Though all participation in the study was virtual, I limited recruitment geographically so that all participating families were living with similar ebbs and flows of virus transmission and with shared public policy guidelines.

With limited options during stay-at-home orders, I recruited participants for this study through snowball sampling (Coleman, 1958; Biernacki & Waldorf, 1981). First, in May of 2020, I informally reached out to parents of kindergarten through second grade students within my personal networks, in particular from the ethnically and linguistically diverse neighborhood school on the northwest side of Chicago in which I most recently taught. I described the project and posed the question of whether or not their child may be interested in participating. If a family expressed interest, I shared the formal recruitment email with them, which detailed steps for participation in the study, followed by consent forms. At that point, I also asked families to suggest others in their networks who they thought might also be willing and available to participate. In this way, the sampling snowballed from my initial outreach emails; in the end, approximately half of the participants (16/30) were recommended by other families. Throughout the recruitment process, I maintained a careful balance of the number of students in each of the three grade levels. Also, as I reached out to families I did so with a broad desire and intention to end with a group of participants who reflected diversity across gender, race, and home language. I completed recruitment with the enrollment of the 30th participant in early June.

Participants

In total, 30 children, along with one caregiver per child, enrolled in the study. The families lived across 15 zip codes, though approximately half of them lived in two zip codes on the northwest side of Chicago. Twenty-five of the families lived within the city's boundaries, while five of the families lived in four different suburbs or nearby cities to the north. At the start of the study, 26 of the students were enrolled in public schools. Twenty-one of those schools were neighborhood schools, with attendance determined by home address. Five of the students attended selective enrollment public schools, which required applications for entry. Four participants were enrolled in private schools, three of which were religious. One participant transitioned from public school to homeschooling during the course of the study. Nine participants began the study in Kindergarten, eleven in first grade, and ten in second grade. During the study, students progressed to the following grade level, meaning the Kindergarteners became first graders, the first graders became second graders, and the second graders became third graders. However, for consistency, throughout this dissertation I refer to the students in grade level groups by the grade in which they began the study. Information about participants' school enrollment is displayed in Table 2.1.

Table 2.1

Participants' School Enrollment Data

Data category		Number of students
School location	Urban	25
	Suburban	5
School type	Public	26
	Neighborhood	21
	Selective Enrollment	5
	Private	4
	Religious	3
	Independent	1
Grade Level	Kindergarten	9
	1st grade	11
	2nd grade	10

Upon enrollment in the study, I asked the caregivers to share several other pieces of demographic information about their children including their gender, race, and home language(s). As such, this data is not self-identified by the children, but it is family-identified. Of the 30 participants, 18 were identified as female and 12 as male. Caregivers identified five child participants as African American or Black, five as Asian, two as Hispanic or Latinx, 15 as white, two as multiracial, and one caregiver opted to self-describe as Middle Eastern. Twenty-one of the students spoke only English at home. In addition to English, two students spoke Gujarati, one spoke French, two spoke Spanish, one spoke Hebrew, one spoke Mandarin, one spoke both Mandarin and Spanish, and one spoke both Arabic and French. This demographic data is displayed in Table 2.2.

Table 2.2

Participants' Demographic Information

Data category		Number of students
Gender	Female	18
	Male	12
Race	African American or Black	2
	+ White	2
	+ Asian	1
	Asian	5
	Hispanic or Latinx	2
	White	15
	Multiracial	2
	Prefer to self-describe: Middle Eastern	1
Language	English	21
	+ Gujarati	2
	+ French	1
	+ Spanish	2
	+ Hebrew	1
	+ Mandarin	1
	+ Mandarin + Spanish	1
	+ Arabic + French	1

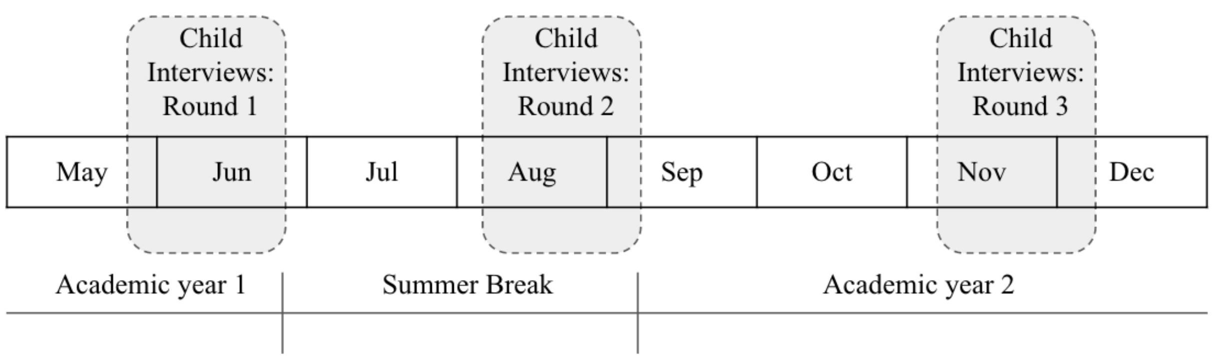
Data Collection

My ultimate goal in data collection was to hear participants' stories about their experiences learning mathematics as a way of understanding their emerging mathematics identities. To achieve this goal, I engaged in three rounds of semi-structured interviews

(Spradley, 1979) with each child participant. I scheduled interviews at times that were convenient for the participants, and because all interviews were conducted online, participants could engage from the comfort and safety of their homes. Further, the online format made it easier to reschedule at the last minute when a family's needs changed, and I was even able to conduct a few interviews while children held their parents' phones, buckled into booster seats in the back of moving cars. I conducted all 90 child participant interviews over Zoom, for a total of 29 hrs and 26 seconds of video data. These interviews lasted an average of 19 minutes and 37 seconds each, with a minimum length of 12 min and 20 seconds and a maximum length of 29 minutes and 56 seconds. The range in interview lengths reflects differences in individual participants. Some children tended to offer longer responses, while others were more brief in their stories and descriptions. Figure 2.1 illustrates the timing of the three rounds of child interviews, which took place between May and December of 2020.

Figure 2.1

Timeline of Child Interviews



The first round of child interviews began on May 28 and ended on June 20, right at the end of the school year. I began each of these interviews with an informal description of the study and offered the participants a chance to ask any questions they had. Then, we transitioned into a mathematics activity. I showed students an image of four pieces of produce and asked them to talk about which one they thought did not belong with the others. This activity served primarily as an entrypoint for our conversations about mathematics; by posing a question with no correct answer, I hoped that it would help the children gain comfort with sharing their thinking. In the remainder of the first round interviews, I asked students to talk about three main topics: their conceptions of what math is, their experiences in their math classes at school prior to school closures, and their recent experiences learning math remotely. I ended the first round of interviews by asking students about their plans for the summer and whether or not those plans might include math as a way into understanding the role they saw mathematics playing in their lives outside of school. (Complete interview protocols for all three rounds of interviews can be found in Appendix A.)

The second round of child interviews took place between August 11 and September 3, at the end of the summer and immediately before the beginning of a new school year. This round of interviews again focused on three main topics: students' engagement with mathematics over the summer, their conceptions of what math is, and their expectations for the upcoming school year. The questions about the summer specifically asked students if they had done any math over the summer, and if so, who had initiated those activities. The second set of questions–about their conceptions of what mathematics is–was repeated from the first round of interviews. I finished the interview by having students describe what they thought might be different about learning math in the upcoming school year both compared to what math was like in the previous school year and to what doing math was like over the summer.

The third and final round of child interviews took place between November 8 and December 8, once students had solidly settled into their new school year. Yet again the interview questions were divided into three topics. This time, they focused on students' conceptions of what mathematics is, their experiences learning math during school, and their perceptions of the role that mathematics plays in their lives beyond school. The first set of questions repeated what I had asked in the previous two interviews. The second set of questions began by asking students to describe what math was like for them in school this year and involved discussions of the activities they had been doing, the content they had been learning, parts they liked or disliked, and their teachers' expectations. I also asked students to compare their math class this year with last year, as another way to uncover specific features of their learning environments and experiences that they found to be salient. In the third set of questions, I asked students about whether and in what ways they saw themselves doing and learning mathematics with their family or with other people in their communities outside of school, if they ever initiated mathematics activities on their own, and whether and in what ways they expected to continue learning and doing math as they got older.

Together these three sets of interviews gave the participants the opportunity to reflect on their experiences learning and doing mathematics within and outside of their classrooms. In doing so, the participants told stories about who they were as mathematics learners. By describing their feelings, their perceptions of themselves, and the role they saw math playing in their lives now and in their futures, they revealed their relationships with mathematics and shared their emerging mathematics identities.

In addition to the interviews with children, I also collected data from caregivers. Specifically, before each child interview, I sent the caregivers a survey. The surveys were completed in Qualtrics (http://www.qualtrics.com) and were designed to take approximately ten minutes to complete. Their primary purpose was to provide information to contextualize students' stories. The caregiver surveys addressed four topics. The first was how the caregiver related to math themselves and how they felt about supporting their child in math. The second was the caregiver's perspective on how their child seemed to be engaging with math in and outside of school. The third topic was the schooling structures at play for their child. For example, were they learning in person, remotely, or in a hybrid model? If remotely, did they meet with their teacher live or watch pre-recorded videos? Finally, the surveys with caregivers also served to collect demographic data including the caregiver and participating child's race, gender, primary language(s), zip code, and school. (See Appendix B for complete surveys.)

In early April 2021, I reached out to the caregivers one final time to offer participation in an optional caregiver interview. Twenty five caregivers opted to participate. The parent interviews took place between April 26 and May 21. The interviews lasted an average of 20 minutes and 3 seconds each, with the minimum length being 12 minutes and 52 seconds and the maximum length being 31 minutes and 32 seconds. In total, I recorded 8 hrs and 21 minutes of caregiver interviews. Like the child interviews, the caregiver interviews took place virtually on Zoom at times that were convenient for the participants. Their purpose was to learn more about the children's experiences with math from the caregivers' perspectives. Given the intensity of parental support during remote learning in particular, I expected that the caregivers would have significant and relevant input. During the interviews, I asked parents about five primary topics:

their own relationship with mathematics, their perceptions of their child's experiences in mathematics in school that year, their child's previous school math experiences, their child's engagement with mathematics outside of school, and finally, their hopes for their child's future trajectory with mathematics. (See Appendix C for caregiver interview protocol.)

Table 2.3 summarizes all of the data that I collected.

Table 2.3

Project Data

Category	Quantity	Duration
Child interviews	**90**	**Total: 29 hrs 26 sec** **Avg: 19 min 37 sec** **Max: 29 min 56 sec** **Min: 12 min 20 sec**
Caregiver Surveys	90	-
Caregiver interviews	25	Total: 8 hrs 21 min Avg: 20 min 3 sec Max: 31 min 32 sec Min: 12 min 52 sec

Note. Only bolded data was analyzed for this dissertation.

Analysis

In my analysis, I focused on the child interviews, and I kept my three research questions front and center: Which facets of early elementary school students' relationships with mathematics differentiate among their emerging mathematics identities? What features of classroom environments contribute to the development of positive mathematics identities in this same group of students? And, what can we learn about young children's emerging mathematics identities from the mathematics activities that they initiate? These questions directed three primary phases of analysis. The first phase addressed both the first and second questions. The second phase further addressed the second question. And the third phase addressed the third question.

However, before beginning these phases of formal analysis, I spent time getting to know the children through a process of data condensation (Miles et al., 2020). I transcribed the interviews, and as I did so I took brief notes on what stood out to me with regards to the participants' relationships with mathematics. After completing the initial round of transcriptions, I went back and reread the interviews, this time pulling specific quotes that reflected how students described themselves and their mathematics experiences. I compiled my notes and quotes into paragraphs about each child, and I attempted to capture what seemed to make them unique. This stage of pre-analysis was vital in making sure that as I moved forward I continued to see each student as a whole person, and more than a set of data points or a composite of codes.

Phase 1: Applying Patterns from the Literature

In my first phase of analysis, I looked at three patterns from previous studies of mathematics identity. Each of the studies had foregrounded participants' perspectives in understanding students' perceptions of their mathematics classroom experiences and of themselves as mathematics learners. The findings from this phase of analysis can be found in Chapter 3.

Phase 1a. I began with the work of Jo Boaler and James G. Greeno (2000) who studied the mathematics learning experiences of high school calculus students across six schools. They found that compared to students who perceived their classrooms as individualistic–centering individual work and achievement–, students in collaborative environments were more likely to enjoy mathematics and to intend to take more mathematics courses in their futures. In other words, their findings suggested that a particular feature of the classroom environment, that being the collaborative nature of the classroom, had an impact on two facets of students' relationships with mathematics, their affinity for mathematics and their intent to continue learning mathematics in their futures.

I sought to determine if this same pattern held for my data. In order to do so, I engaged in a process of deductive coding (Miles et al., 2020). This involved first establishing a set of a priori codes to reflect the pieces of what Boaler and Greeno (2000) described: students' perceptions of their classroom classroom environments as individualistic or collaborative, students' affinity for mathematics or lack thereof, and students' intentions or lack of intentions to continue learning mathematics in their futures. I began with the classroom features. Based on the work of Boaler and Greeno (2000), I defined the code *collaborative* as a student mentioning student discussion, working together in pairs or groups, helping or being helped by classmates, or getting ideas from their classmates. In contrast, I defined the code *individualistic* as a student describing practicing and repeating procedures on their own after being shown the procedure by a teacher, working through a workbook on their own, or being personally responsible for doing what the teacher told them to do. Table 2.4 lists these particular codes, their definitions, and examples for each from student interviews.

Table 2.4

Coding for Environment (Adapted from Boaler & Greeno, 2000)

Code	Sub-code	Definition	Example
Environment	Collaborative	The student mentions: ● Frequent or meaningful student discussion ● Working together in pairs or in groups ● Helping or being helped by classmates ● Getting ideas from peers	"[When I got stuck] I either asked a friend or my teacher." - Suriyah "I liked working with my group and learning math together." -Torrin
	Individualistic	The student mentions: ● Practicing or repeating procedures on their own after being taught by the teacher ● Working through a textbook/ workbook on their own ● A lack of discussion amongst classmates ● Doing what the teacher tells you to do alone	"In school, first we do math, like she shows us how, my teacher shows us how to do it. And then we write it." -Alice "...They won't look at my work and I won't look at theirs...because we aren't allowed." -Sigrid

Next, I turned to establishing codes for the ways in which Boaler and Greeno (2000) described students' relationships with mathematics. I defined the code *liking math* as a student expressing positive feelings towards math, and I defined the code *disliking math* as a student expressing negative feelings towards math. Finally, I defined the code *ongoing math* as a student saying that they expected to continue learning or doing mathematics both in school and beyond as they got older, and I defined the code *ending math* as a student saying that they expected to only learn or do mathematics within the confines of when it is mandatory in school. Table 2.5 lists these particular codes, their definitions, and examples for each from student interviews.

Table 2.5

Coding for Affinity and Future (Adapted from Boaler & Greeno, 2000)

Code	Sub-code	Definition	Example
Affinity	Liking math	The student says: ● That they like math or that they sometimes like math. ● That they feel generally positive feelings (e.g. happy, excited) when they do math.	"I just like math. I don't know why. I just like it." -Benji “[When doing math] I feel good. I feel great." -Ayisha
	Disliking math	The student says: ● (More often than not) that they dislike math or usually dislike math. ● That they feel generally negative feelings (e.g. sad, mad, frustrated) when they do math.	"It's not fun. I have to write a lot...I feel mad...because I get confused and I get really mad." -Valentina
Future	Ongoing math	The student expects to continue learning and/or doing math, both in school *and* beyond.	"I do think I will keep learning math because math can be anywhere and when it's everywhere you can learn anything anywhere." -Suriyah
	Ending math	The student expects to only learn or do math within the confines of when it is mandatory in school.	"Math, for me, it's not, I'll do it, I like it. But for me, it's not fun. So if I don't have to in school, I don't really want to do it." -Alice

With these codes, I returned to the interview transcripts and highlighted sections of the transcripts that reflected each code. I did this in rounds, first looking across the transcripts for the environment codes, then for the affinity codes, and finally for the future codes. Due to the nature of the interview protocols, coding for individualistic and collaborative environments was relevant to the first and third interviews for each student, when school was in session. I categorized a student as having experienced a *collaborative environment* (or rather, as having perceived their environment as collaborative), if at any point in either of those two interviews they described experiencing any of the defining conditions of the code. I did this even if at other points the student described their environment as individualistic. I made this decision because I wanted to understand the impact that experiencing a collaborative environment had while also acknowledging that in many cases students might experience their classroom environments as a mix of collaborative and individualistic. Coding for the affinity codes was relevant across all three sets of transcripts. In turn, I categorized a student as *liking math* if they in two or three of the three total interviews shared predominantly positive feelings towards mathematics, and I categorized a student as *disliking math* if they more often than not shared predominantly negative feelings towards mathematics. Finally, coding for the future codes was only relevant to the third round of interviews. So, I categorized a student as *ongoing math* if in their third interview they said that they would continue learning or doing math as they got older, both in school and beyond. I categorized a student as *ending math* if they responded that they expected to stop learning or doing math at some point in their schooling.

Once I had coded all of the relevant interviews and had categorized all 30 students as perceiving their environments as collaborative or individualistic, as liking or disliking math, and as seeing math as ongoing or ending, I looked across the data for patterns (Miles et al., 2020). First, I calculated the total number of students that fell into each category. Then, I looked for the relationships described by Boaler and Greeno (2000). Of the students who were categorized as perceiving their classrooms to be *collaborative*, I calculated the percentage who were also categorized as *liking math* and expecting math to be *ongoing*. Along the same lines, I calculated the percentage of students who perceived their classrooms as *individualistic* who also *disliked math* and saw math as *ending*. I also looked at the relationships in reverse, calculating of the students who liked math, how many saw their classrooms as collaborative and how many expected to continue learning or doing math. And, of the students who expected to continue learning or doing math, how many saw their classrooms as collaborative and how many liked math. Additionally, I repeated these same calculations for each grade level group, looking at the patterns amongst the Kindergarten, first, and second grade students. Finally, I noted how many students overall, and how many within each grade level group, seemed to fit within the pattern described by Boaler and Greeno (2000). I looked at the groups of students who did and did not seem to fit within the pattern and noted connections between them.

Phase 1b. The second pattern from the literature that I examined came from the work of Paul Cobb, Melissa Gresalfi, and Lynn Liao Hodge (2009). They found that compared to eighth grade students who perceived authority in their math class as distributed solely to the teacher and who felt limited to disciplinary agency by reproducing established procedures, students who felt that authority was shared in the classroom between teacher and students and who experienced conceptual agency were more likely to feel successful and to begin to affiliate with mathematics. Cobb et al. (2009) defined affiliating as seeing mathematics as a part of oneself and as the transformation from classroom activity being experienced as an obligation to someone else (presumably the teacher), to being experienced as an obligation to oneself. This required a certain level of desire and eagerness. They distinguished affiliating from two other possible mathematics identities: cooperating, which they described as participating in mathematics activities as required, and resisting, or actively pushing against mathematics activities and separating oneself from the discipline. In summary, this pattern described by Cobb et al. (2009) described two features of the classroom environment, those being whether or not students perceived authority as solely in the hands of the teacher or shared between teacher and students and whether students were limited to disciplinary agency or experienced conceptual agency. They looked at how these features of the classroom environment impacted students' perceptions of success and students' overall mathematics identities described as affiliating, cooperating, or resisting.

Again, I sought to determine if this pattern held for my data, and in order to do so I followed a similar process as described above for Phase 1a. I engaged in deductive coding (Miles et al., 2020), and first established a set of a priori codes to reflect the pieces of what Cobb et al. (2009) described: students' perceptions of teacher or shared authority, students' perceptions of disciplinary or conceptual agency, students' sense of their own success, and students' mathematics identities as affiliating, cooperating, or resisting. I began with the classroom features. Based on the work of Cobb et al. (2009), I defined students' experiences of *teacher authority* as a student describing their classroom responsibilities as practicing or repeating procedures on their own after being taught by the teacher, doing what the teacher says, or following the teachers' instructions. In contrast, I defined students' experiences of *shared authority* as a student describing frequent or meaningful student discussion, working together in pairs or in groups, helping or being helped by classmates, and getting ideas from peers. Next, I defined *disciplinary agency* as a student describing following particular procedures to get a single right answer, and I defined *conceptual agency* as a student describing choosing their own strategies to solve problems and having the opportunity to compare those strategies with those of their peers. Table 2.6 lists these codes, their definitions, and examples from student interviews.

Table 2.6

Coding for Authority and Agency (Adapted from Cobb et al., 2009)

Code	Sub-code	Definition	Example
Authority	Shared authority	Student mentions: • Frequent or meaningful student discussion • Working together in pairs or in groups • Helping or being helped by classmates • Getting ideas from peers	“[When I got stuck] I either asked a friend or my teacher." - Suriyah “I liked working with my group and learning math together.” -Torrin
	Teacher authority	Student describes: • Practicing or repeating procedures on their own after being taught by the teacher • Doing what the teacher says on their own • Following the teacher’s instructions	"In school, first we do math, like she shows us how, my teacher shows us how to do it. And then we write it." -Alice "...They won't look at my work and I won't look at theirs...because we aren't allowed." -Sigrid
Agency	Disciplinary agency	Student describes following particular procedures to get the right answers (usually or always)	"I learned the strategy and then I'd do it." -Asad
	Conceptual agency	Student describes choosing their own strategies to solve problems and/or comparing strategies with those of their peers (usually or always)	"Yes, I always do [pick my own strategy]. Because, if we didn't, I don't think math would be as fun as it already is." -Meera

After establishing codes for these classroom features, I shifted to the facets of students’ relationships with mathematics–students’ perceptions of their own success and whether they were affiliating with, cooperating with, or resisting mathematics as they were experiencing it in their classrooms. Based on the work of Cobb et al. (2009), I defined a perception of *self-success* as a student describing themselves as sometimes or often successful in math class, and I defined a perception of *lack of self-success* as a student describing themselves as rarely or never successful in math class. In an attempt to capture the spirit of how Cobb et al. (2009) defined affiliating as exhibiting a personal eagerness or desire to learn and a transformation of obligations-to-others to obligations-to-self, I defined *affiliating* as a student describing excitement about participating in math class and learning and doing mathematics within *and* outside of school. I then defined *cooperating* as a student describing a willingness to learn math and participate in math class but limited excitement about doing so on their own. Finally, I defined *resisting* as a student describing a lack of desire to learn math and participate in math class. Table 2.7 lists these codes, their definitions, and examples from student interviews.

Table 2.7

Coding for Success and Mathematics Identity: Affiliating, Cooperating, and Resisting (Adapted from Cobb et al., 2009)

Code	Sub-code	Definition	Example
Success	Self-success	Student describes themselves as sometimes or often successful in math class	"I get ten points out of ten." -Asad
	Lack of self-success	Student describes themselves as rarely or never successful in math class	"I'm bad at math." -Owen
Mathematics identity	Affiliating	The student describes excitement about participating in math class and learning mathematics within *and* outside of school. The student likes math *and* self initiates math activity outside of school.	"I really focus, and not every kid in my class does that… and I think it's fun. And sort of challenging, but this is just me. I like challenges." -Benji
	Cooperating	The student describes willingness to learn math and participate in math class but limited excitement about doing so on their own. The student: ● Likes math *and* does not self-initiate math activity outside of school *or* ● Sometimes dislikes math *and* self-initiates math activity outside of school *or* ● Sometimes dislikes math *and* willingly does math outside of school with a caregiver	"Sometimes I don't like [math] because, like, sometimes I have a lot of math to do. And I don't like it when I have a lot of math to do because I just wanna play, but sometimes I like doing it because, like, I don't have really anything to do." -Alice
	Resisting	The student describes a lack of desire to learn math and participate in math class. The student: ● Dislikes math *and* does not self-initiate math activity outside of school *and* does not willingly do math outside of school with a caregiver.	"I feel happy when I finish [my math work], because that means that…I have lots of hours with no pain and misery." -Asher

With these codes, I returned to the interview transcripts and highlighted sections of the transcripts that reflected each code. The first and third interviews, when school was in session, were relevant. I did this in rounds, first looking across the transcripts for the authority and agency codes, and then returning for the success and mathematics identity codes. When highlighting transcript sections relevant to the *authority* codes, I noticed that they overlapped entirely with the sections previously identified as relevant to the *environment* codes. Students' experiences of a classroom as having shared authority between teacher and students mirrored students' experiences of a collaborative classroom, and students' experiences of a classroom as having only teacher authority mirrored students' experiences of an individualistic classroom. Though there are nuanced differences in how the original researchers defined these terms, with collaboration being more focused on how classroom activities are conducted and authority being more focused on how mathematical decisions are made, these distinctions were not clear in the stories of the young students who participated in this study. In turn, I opted to combine these codes. Students who were categorized in Phase 1a as perceiving their classrooms as collaborative were here categorized as experiencing shared authority, and students who were categorized in Phase 1a as perceiving their classrooms as individualistic were here categorized as experiencing teacher authority.

As I continued coding the transcripts, I categorized students as having experienced *conceptual agency*, or as having perceived themselves as having conceptual agency, if at any point in either of those two interviews they described experiencing any of the defining conditions of the code. I did this even if at other points the student described feeling limited to disciplinary agency. I made this decision because I wanted to understand the impact that experiencing conceptual agency had while recognizing that students were likely to experience a mix of both. I applied a similar structure to categorizing students as perceiving *self-success* or *lack of self-success*; if in either interview the student described themselves as sometimes, often, or always successful, I categorized the student as *self-success*. A student was only categorized as *lack of self-success* if in both interviews they described themselves as rarely or never successful.

Finally, I coded the transcripts in order to then categorize students as affiliating with, cooperating with, or resisting mathematics as they were experiencing it in their classrooms. I operationalized the code of *affiliating* as students who both liked math (i.e. were previously categorized as *liking math* in Phase 1a) and who initiated mathematics activity outside of the confines of their math classrooms. This showed both their affinity for the discipline and their engagement with it on their own. I categorized students as *cooperating* if they had one but not both of the characteristics for *affiliating*. So, a cooperating student might like math but not self-initiate math activities outside of school, or they might dislike math but sometimes self-initiate mathematics activities outside of school, or they might dislike math and sometimes willingly engage in mathematics outside of school with prompting by a caregiver. A student was categorized as *resisting* if they both disliked math and did not self-initiate math activity outside of school, nor did they describe willingly engaging in mathematics outside of school with a caregiver.

Once I had coded all of the relevant transcripts and had used those transcripts to categorize all 30 students as perceiving their classrooms as spaces of teacher or shared authority, as experiencing disciplinary or conceptual agency, as perceiving themselves as successful or not, and as affiliating with, cooperating with, or resisting mathematics, I looked across the data for patterns (Miles et al., 2020). First, I calculated the number of students in each category overall and within each grade level group. Then, I looked at connections across the features. I compared the students who experienced shared authority with those who experienced conceptual agency, to determine the extent of the overlap of those groups. Similarly, I compared the students who experienced teacher authority with those who felt limited to disciplinary agency. Next, I looked at the relationships between each of those conditions and students' perceptions of their success, first separately and then together. Finally, I looked at the relationship between those conditions and whether students were categorized as affiliating, cooperating, or resisting. I also made the calculations in reverse, noting of the students who were categorized as affiliating, cooperating, and resisting, how many experienced shared authority, conceptual agency, or both. This set me up to be able to see how many students overall, and how many within each grade level group, seemed to fit within the pattern described by Cobb et al. (2009). Finally, I looked at the groups of students who did and did not seem to fit within the pattern and noted connections between them.

Phase 1c. The third pattern from the literature that I considered is from the work of Na'ilah Suad Nasir and Victoria Hand (2008). Their observations of and interviews with high school varsity basketball players who were also geometry students revealed that having meaningful access to the activity's domain, the availability of integral roles, and opportunities for self-expression, together created the conditions that supported the students in developing practiced-linked identities with basketball, and the lack of the same features left students disconnected from geometry. When Nasir and Hand (2008) defined access to the domain, they described students understanding the context of a broader practice and being able to link what they were doing within a given moment with other opportunities for learning. They described integral roles as opportunities for students to participate in meaningful and productive ways in the activity. And, they explained that when students had opportunities for self-expression they were able to bring their whole selves into the space, sharing their personalities and having their unique contributions appreciated by those around them. Finally, Nasir and Hand (2008) defined *practice-linked identities* as "the identities that people come to take on, construct, and embrace that are linked to participation in social and cultural practices," (p. 147). This concept can be applied to the primary focus of this dissertation, mathematics identities, as mathematics identities can be understood as the relationship between people and the practice of mathematics. In summary, Nasir and Hand (2008) saw that three features of the learning environment–access to the domain, integral roles, and opportunities for self-expression–impacted to what extent students saw the practices of basketball and of geometry as a part of who they were.

Yet again, I engaged in a process of deductive coding (Miles et al., 2020) to determine if the pattern identified by Nasir and Hand (2008) could be applied to my data. I began by establishing a set of codes based on the definitions in Nasir and Hand's (2008) work that fit with the stories young children told. For the young students in my study, the idea of *access to the domain* emerged when students talked about developing skills along a meaningful trajectory such that they felt like they were making progress toward a greater goal, or when students described believing that what they were learning in their classroom had a purpose that extended beyond the school's walls. I defined *integral roles* as students describing helping others or sharing their ideas with the class–these were the primary ways in the data that students expressed making important contributions to their class' collective learning. The idea of *self-expression* emerged when students talked about making things their own, arranging things in ways that were comfortable to them, making choices about the activities they wanted to do, and having fun while doing so. Table 2.8 lists these features as codes, their definitions, and corresponding examples from interviews with students.

Table 2.8

Coding for Access to the Domain, Integral Roles, and Opportunities for Self-Expression (Adapted from Nasir & Hand, 2008)

Code	Definition	Example
Access to the domain	The student describes: ● Learning connected skills *or* ● Developing skills or knowledge along a meaningful trajectory *or* ● Feeling ongoing purpose or application of math skills outside of the classroom	"I think it's just a way to strengthen the mind to know how to do coding or if you want to be a coder, you're going to need to know what all the one zeros come out in...also math both strengthens your mind to do anything because everything has some sort of math diagram or something." -Asher
Integral roles	Student describes: ● Helping others ● Being helped by others ● Sharing their ideas with the class	"[When I got stuck] I either asked a friend or my teacher." - Suriyah "I liked working with my group and learning math together." -Torrin
Opportunities for self-expression	Student describes: ● Doing things their own way (besides strategy) ● Making significant choices ● Being themselves and having fun while doing so	"We just did what we wanted to do in math class...We had to choose from the three math shelves in our classroom." -Ayisha "I liked [math] because when I did the worksheets, I had these cool pens at school, and I got to write with them." -Alice

With these codes, I returned to the transcripts, and highlighted moments of conversation that reflected each code. These codes specifically related to students' experiences in math classrooms, so they were again applicable to the first and third interviews, which took place when school was in session. In this phase of analysis, I only looked for evidence of the presence of each of these codes, and did not specifically look for evidence of their absence. For example, I highlighted when a student discussed the ways he expected math to help him figure out all sorts of problems in the world as that student experiencing *access to the domain*. However, I did not highlight the opposite, such as a student expressing that math had limited purpose.

When highlighting transcript sections relevant to *integral roles*, I noticed that they mirrored the sections previously coded as *collaborative environment* and *shared authority*. Students' perceptions of meaningful participation through contributing ideas aligned with their understanding of a classroom space in which they could work together with their peers and influence their peers' learning. Put otherwise, for the young children in my study, the integral roles they described taking on revolved around learning mathematics as a classroom community. In turn, I combined these codes. Students who were categorized in Phase 1a as perceiving their classrooms as collaborative and in Phase 1b as experiencing shared authority were here categorized as having the opportunity to play integral roles.

Once I had coded all of the relevant transcripts and had used those transcripts to categorize all 30 students as having or not having access to the domain, integral roles, and opportunities for self expression, I sought to understand how to translate Nasir and Hand's (2008) practice-linked identities in this study data. To do so, I reflected back to the work of Cobb et al. (2009) as described in section Phase 1b. Nasir and Hand's (2008) definition of a practice-linked identity as reflecting how people embrace a practice as a part of who they are resonates with Cobb et al.'s (2009) definition of a student who is affiliating with mathematics. In both of these cases, the emphasis is on an intimate connection between a person and a discipline. A student who is affiliating with mathematics embraces mathematics as a part of who they are and therefore participates in mathematics activities beyond what is required of them by others. So, I leaned on the previous codes of affiliating, cooperating, and resisting as a way to operationalize practice-linked identities.

The next step was to unpack the ways in which access to the domain, integral roles, and opportunities for self-expression related to each other and to whether students were previously categorized as affiliating with, cooperating with, or resisting mathematics. First, I noted how many students experienced each classroom feature in total and within each grade level group. I noted how frequently students experienced one, two, or three of the classroom features and paid close attention to which were emerging more frequently and which seemed to appear together. Then, for students who did not describe themselves as having experienced any of the three features, I looked at how many of them were affiliating with, cooperating with, or resisting mathematics. I did the same for the groups of students who had described themselves as having experienced one of the features, two of the features, and all three of the features. After those calculations, I went back to investigate the possible impact of each individual feature. Together, these steps helped me to uncover to what extent the pattern previously identified by Nasir and Hand (2008) held true in my data–that experiencing access to the domain, integral roles, and opportunities for self-expression opened up opportunities for identity formation. Finally, I looked closely at and hypothesized connections between the students who did not seem to align with this pattern because either they did experience multiple of these supportive features but were not previously categorized as affiliating with mathematics or the opposite, that they did not experience these supportive features but they were previously categorized as affiliating.

Phase 1d. After examining the applicability of these three patterns from the literature, I brought the analyses from Phase 1a, 1b, and 1c together. I compared which students' stories seemed to fit within and across multiple patterns, and which did not. I also compared which grade level groups seemed to reflect these patterns, and which less so. At this point, I looked at the facets of identity side-by-side: students' affinity for mathematics, their desire to continue learning mathematics in the future, whether they saw themselves as successful, and whether they engaged in mathematics on their own outside of school. I noted which of these seemed to be spread across my data set, as opposed to being clustered amongst the older or younger students, omnipresent, or generally absent. This helped me answer my first research question about which facets of young students' relationships with mathematics we can use to understand and differentiate between their emerging mathematics identities. Similarly, I looked across the classroom features: the collaborative nature of classrooms (which aligned with students' experiences of classrooms as spaces of shared authority and where they played integral roles), opportunities for conceptual agency, access to the domain of mathematics, and opportunities for self-expression. Again, I noted which of these seemed to be spread across my data set, and which seemed impactful on the various facets of students' mathematics identities. This helped me to begin answering my second research question, about which features of classroom environments are supportive of positive mathematics identity development for young children. I moved forward into the next phase of analysis to continue responding to this question.

Phase 2: Analyzing Students' Perceptions of Features of Mathematics Learning Environments

In the second phase of analysis, I shifted from the existing literature and began with the words of the participants themselves to further understand what features of learning environments were supportive of the development of positive mathematics identities. Specifically, I drew on the first round of interviews with participants, which took place in May and June of 2020, just after school buildings first closed. At that time, the students had recently experienced a dramatic change in their learning environments. Dramatic changes or violations of expectations can lead people to have particularly strong emotional and cognitive responses (Burgoon, 1993). Therefore, it may be that the surprising shifts students experienced from learning in their classrooms to learning remotely set them up to be particularly reflective and to notice features of their learning environments that they may not have previously given much thought to. During the first round of interviews, I asked students to compare their experiences learning mathematics in school buildings and at home. What did they like better about each setting? What about each setting helped them learn? In this phase of analysis, I took an inductive approach (Miles et al., 2020) and began with students' responses to these comparative questions. The findings from this analysis can be found in Chapter 4.

After gathering student responses to interview questions comparing their experiences learning mathematics in physical classrooms and from their homes, I reread each quote and identified a central theme–what was the quote really about? What was at its heart (Miles et al., 2020)? I focused on understanding what it was about the learning environment that led to a student expressing a positive or negative feeling towards their experience. Next, I grouped related themes together. Those groups became codes. I cycled back to the students' quotes using the new codes, and iterated between coding the students' quotes and clarifying the code definitions until I had assigned a specific code to each relevant student quote. This process involved developing sub-codes in addition to the primary codes (Miles et al., 2020).

In total there were six primary codes: *being with peers, access to support, access to learning, physical comfort, pace/schedule*, and *having fun*. These codes are not specific to whether students were learning mathematics in school or at home, but rather emerged as salient features across both contexts with varying frequencies. *Being with peers* reflected student quotes that suggested interactions with their classmates was important to their experience learning mathematics. This was sometimes about physical co-presence, and other times was more specifically about collaboration. *Access to support* described student quotes that focused on how a student got the help they felt they needed. This support could come from their teacher, peers, family, or specific materials. Related but distinct from access to support, I defined *access to learning* as when a student described that in a particular learning environment they were able to learn more content, or they felt that they were able to be more successful. I applied the code *physical comfort* to student quotes that focused on how their bodies interacted with the space around them–the ways they could move around and their options for seating. The code *pace/schedule* was applied when a student emphasized conditions of their workflow, including when they worked, how fast they worked, and whether their progress aligned with that of their classmates. Finally, *having fun* described student quotes that focused on students' pleasure and enjoyment. Sometimes, student quotes mentioned having fun in more general terms, and other times they described specific activities. The final codes for what emerged from student reflections as salient features of their learning environments are found in Table 2.9 along with definitions and examples of student quotes for each code.

Table 2.9

Coding for Salient Features of Learning Environments

Code	Sub-code	Definition: The student describes…	Example
Being with peers	General	Being co-present with peers, being able to see peers, or being near peers.	"It's nice because you could see all your friends." -Maulik
	Collaboration	Doing mathematics together with peers.	"I actually like being right there in person and doing it together." -Torrin
	Away from	Being away from their peers, rather than with their peers, as a benefit of the environment	"Sometimes when I do work, I like doing it alone." -Ling
Access to support	Teacher	Receiving support or help from the teacher, especially when they are stuck.	""She guides, [our teacher], guides us through it." -Sigrid
	Peers	Receiving support or help from their peers, especially when they are stuck.	"I like doing math at school 'cause my friends kind of helped me with my work when I needed help." -Louise
	Family	Receiving support or help from family members, including parents and siblings, especially when they are stuck.	"I get helping from my mom and dad and my sister." -Claire
	Materials	Using materials such as manipulatives, posters, presentation slides, and other supplies to help them, especially when they are stuck.	"I wish I was still in the classroom because like, there's a lot more things that can help you." -Kira
Access to learning	Quantity	Feeling that there is *more* opportunity to learn	"It teaches me more." -Darius
	Success	Feeling that they are able to be more successful either based on their own evaluation or based on the evaluation of their teacher (e.g. they can get better grades).	"[In the app] if you got a problem wrong, it gives you a second try at it…" -Asher
Physical comfort	Seating	Being comfortable because of the types of seating available.	"In the classroom...we have really fun bouncy chairs, like really fun donut chairs you sit on...and these others chairs that...rotate." -Kira
	Movement	Being comfortable because of the freedom, or lack thereof, to move	"I can like move around." -Suriyah

		around.	
Pace/schedule	Time commitment	The amount of time they spend on math work.	“We have to do it longer.” -Essey
	No rush	Being able to work at their own pace, take breaks, or complete work without time limits.	“I'm at home and I could rest and do my work…I could take breaks." -Asad
	Acceleration	Being able to work ahead of their peers at their own choosing via a workbook or assigned app.	“You don’t have to go at the same pace, you can just go as fast as you want to…” -Sebastian
Having fun	General	Finding a particular context to be more fun without further explanation as to why.	“...it’s funner.” -Alice
	Activity	A particular context to be more fun because they find a specific named activity that they get or do not get to do in that context to be fun.	“Our coloring...That's my favorite part of math." -Claire

Once I finished coding, I returned to the quotes and separated them this time by whether they were describing positive features of being physically in school, challenges of being physically in school, positive features of learning from home, or challenges of learning from home. For each of these categories, I counted how many times each of the codes was applied. This allowed me to compare the ways in which students were describing each learning environment and the ways in which the environments impacted how they learned mathematics. This also offered a lens into what students expressed was important to them overall, across both contexts. In addition to making these calculations for the study population at large, I also narrowed in and compared the frequency with which each of the codes was applied to the three grade level groups, which allowed me to see if there were differences in what students found salient between those groups. Taken together, these processes helped me to further answer my second research question because they revealed, from the students’ perspectives, which features of the classroom environment were supportive of positive experiences learning mathematics. I moved into the next phase of analysis to approach the third research question.

Phase 3: Exploring Child-Initiated Mathematics Activities

In the third phase of analysis, I explored what it meant for children to engage in mathematics on their own terms, in their own ways. In other words, I investigated what children did when they authored their own mathematical experiences. This analysis foregrounded students' mathematical agency, which is a critical construct in developing and understanding students' mathematics identities. I looked at whether or not each child reported initiating mathematics activities, and if so, what were the nature of those activities. I saw this as an important area for study because in describing instances when they initiated mathematics activities–or instances in which they exercised agency–the children showed both what they understood mathematics to be and the role they saw it playing in their current lives. They also highlighted what was important to them when engaging in mathematics. These are important pieces of children's relationships with the discipline. Findings from this phase of analysis can be found in Chapter 5.

To begin, I looked back through all three sets of interviews for student responses about the math that they did "just because they wanted to, not because anyone was telling them to." I created a list of all of the students who, in any of their interviews, responded that they did in fact sometimes do math "just because." Then, for all of the students on this list, I collected their responses about the mathematics that they did on their own. This included descriptions about math that students opted to do over the summer when school was not in session and math that they did with their families, neighbors, or friends that they initiated. I excluded descriptions of mathematical activities that students engaged in outside of school if those activities were explicitly initiated by someone other than the student. For comparison, I also gathered responses from these same students about the types of math activities that they participated in in school. Specifically, during both the fall and spring interviews, I asked students, “Can you tell me about what math time is like in your classroom?” and “What activities do you do during math time?” This data would eventually allow me to understand the differences between child-initiated mathematics activities and children’s mathematics experiences in schools.

Similar to my process in Phase 2, in this phase of analysis, I took an inductive approach (Miles et al., 2020). After gathering the relevant data, I looked across all of the activities that students described doing, both on their own and in their classrooms. For each student quote, I identified the activity being described, and then I clustered similar activities into groups. These groups became codes (Miles et al., 2020). I returned to the students’ quotes using the new codes, and iterated between coding the students’ quotes and clarifying the code definitions until I had assigned a specific code to each relevant student quote. This process resulted in ten codes, representing ten types of mathematics activities that students engaged in on their own, at school, or across both contexts, including: *solving equations, counting, creating equations, asking questions, measuring, drawing, discussing/explaining, building, telling time,* and *estimating.*

Solving equations involved problem-solving or engaging in computation with multiple numbers and a specific operation, resulting in an answer. *Counting* described determining a quantity of something or reciting numbers in a specified order. *Creating equations* was when students chose the numbers for, wrote, and/or declared to someone else number sentences to be solved. I applied the code *asking questions* when students shared that they posed a question or sought out specific information about any mathematical topic. *Measuring* involved determining a quantity through measurement, often with the use of specific tools. *Drawing* described illustrating a mathematical idea, scenario, or problem. *Discussing/explaining* was when students engaged in conversation with at least one other person with the purpose of conveying or sharing mathematical ideas. I applied the code *building* when students shared that they composed structures or shapes. *Telling time* involved using a clock. And finally, *estimating* described hypothesizing about the quantity of a given set of objects. The final codes for the mathematics activity types that emerged from student reflections are listed in Table 2.10 along with definitions and examples of student quotes for each code.

Table 2.10

Coding for Students' Mathematics Activities

Code	Definition	Example
Solving equations	Student describes problem-solving with multiple numbers and a specific operation that requires computation and results in an answer	"We've been doing plusses and minuses, yeah, addition and subtraction." -Reese
Counting	Student describes determining a quantity of objects or marks or reciting numbers in a specified order	"I counted how many dinosaurs I have." -Mia
Creating equations	Student describes choosing the numbers for, writing, or announcing a situation that requires problem-solving with multiple numbers and a specific operation and could result in an answer	"So usually, I could just get a piece of paper from my desk, I'd write like, equations, hard equations, and sometimes answer 'em, sometimes not." -Louise
Asking questions	Student describes posing a question about any mathematical content	"I remember how I learned multiplication. I learned just like asking my mom at home." -Sebastian
Measuring	Student describes determining a quantity through measurement, often with the use of specific tools	"I've also done some measuring with my mom while I was making challah, and that was probably it…I picked helping out with measuring with the challah." -Benji
Drawing	Student describes illustrating a mathematical idea, scenario, or problem	"I would say I draw shapes a lot like I draw shapes to make dresses like I would draw a heart, now I draw a triangle to make a dress." -Meera
Discussing/Explaining	Student describes engaging in conversation with at least one other person with the purpose of conveying or sharing mathematical ideas	"Sometimes we did…group things…where [our teacher] showed us on a whiteboard and we did small talks, math chats about what we're doing." -Torrin
Building	Student describes composing shapes or structures	"I do legos." -Valentina
Telling time	Student describes using a clock to identify the time	"And I see and tell the time." -Asad
Estimating	Student describes hypothesizing about quantity of a given set of objects	"For the estimating there was like a jar of apples shown on the TV and then we're supposed to estimate how many were in there." -Suriyah

Next, I returned to the original student quotes about the mathematical activities that they initiated on their own to gather information on why students engaged in these activities. Sometimes students offered this information organically, and other times I asked this as a follow-up question. This part of the analysis only applied to the students' descriptions of the mathematical activities that they opted to do; I did not ask students to describe why they were engaging in specific mathematical activities in their classrooms because in most cases those activities were being directed by a teacher. Like with the activities, I clustered similar purposes into codes and iterated between the student quotes and codes to clarify definitions until I had assigned a specific code to each relevant student quote (Miles et al., 2020). This process resulted in eight purpose codes: *for fun, because it's embedded in something else, to teach or challenge others, to practice or improve, to cure boredom, to learn something new, for a challenge,* and *to prepare for the next grade.* The final codes for students' purposes for engaging in mathematical activities on their own are found in Table 2.11 along with definitions and examples of student quotes from each code.

Table 2.11

Coding for Purposes of Students' Self-Initiated Mathematics Activities

Code	Definition	Example
For fun	Student describes doing mathematics because they found it fun, pleasurable, or joyful	"I'll take any problems…I like to do them… cause it's fun." -Claire
Because it's embedded in something else	Student recognizes that mathematics was integral to or involved in an activity that they opted to do	"Like earlier today when we were about to set, when we were setting up the golf course, I had to choose a certain amount of clubs and golf balls." -Torrin
To teach or challenge others	Student describes doing mathematics because they want to explain a concept to someone else or offer someone else a mathematical challenge	"I was trying to teach…my younger sister…what one plus one equals. Two." -Ayisha
To practice or improve	Student describes doing mathematics because they want to get better at a specific skill	"Sometimes we'll practice math games that I need to work on a lot, so that next time we play those games I know more about it." -Gal
To cure boredom	Student specifically mentions boredom as a reason for engaging in mathematics	"When I get really bored I probably do math." -Louise
To learn something new	Student seeks out information about mathematics that they did not previously know	"I was learning some math on Youtube…I just don't know my time tables, so I found [the video] myself… and then I just learned." -Darius
For a challenge	Student does mathematics because they want to feel challenged	"Sometimes I go ahead of my assignment pages…because the math we're doing in school is too easy for me." -Mia
To prepare for the next grade	Student does mathematics because they believe that doing so will help them in the next grade in school	"My dad gave me math problems cause I asked him to [because] well, I'm about to be in first grade." -Lucas

Once I had coded each relevant student quote for purpose, I further clustered these student-named codes into three larger, umbrella categories. *Because it's embedded in something* *else* remained its own category. I paired *to practice or improve* and *to prepare for the next grade* into a broader purpose category of *meeting school-based expectations.* The remaining student-named codes–*for fun, to teach or challenge others, to cure boredom, to learn something new,* and *for a challenge* all reflected instances where students were *driven by pleasure or curiosity*. In all of these cases, students opted to engage in mathematics because they found it satisfying; it made them feel good. Combining the original student-named codes into fewer categories offered a more clear lens into students' motivations for initiating mathematics activities.

Then, once again, I returned again to the original set of student responses and gathered information about with whom students were engaging in mathematical activity. This information was only sometimes available, but existed for some examples of child-initiated mathematics activities and for some descriptions of mathematics activities that took place in classrooms.

At this point in the analysis, I had two lists. One list named each student who at some point described initiating mathematics activities outside of school. Next to their names were the codes for each activity type that they initiated at home, followed by the codes for why they initiated each activity, and with whom they participated in the activity (when available). The second list began with the same set of student names, followed by each activity that they described doing at school, and with whom they did those activities (again, when available). These two lists allowed me to make comparisons between the activities that students engaged in across these contexts. The first comparison I made was to calculate the number of students who engaged in each type of mathematics activity at home and at school. I also looked at the participation in activities in each context by grade level group to see if there were meaningful differences by grade. I found that *solving equations* emerged as the most frequent activity both in students' descriptions of their math classes and in the child-initiated mathematics activities. So, I returned one final time to the original student quotes to more deeply understand what that specific activity involved for students. What were students doing as they solved equations? Did solving equations look the same for students when they were in school and when they were opting to solve equations on their own? Narrowing in on solving equations offered another way to understand the unique qualities of the mathematics that participants chose to engage in. Taken together, these processes helped me respond to my third research question and learn from child-initiated mathematics activities.

After completing all three phases of analysis, I reviewed their findings side-by-side at the scale of the whole data set, grade level groups, and individual students. This allowed me to note patterns and themes that emerged across multiple findings. I discuss these themes in Chapter 6.

The Role and Impact of the Researcher

Who I am as a researcher impacted how I designed and implemented this study, as well as how I analyzed the data and interpreted the results. First, I am a former elementary school teacher. My experiences as a teacher inspired this work, and I deeply believe that not only can all children do math, but that all children deserve the opportunity to feel success, curiosity, and joy in math. I approached all of my conversations with the children and their caregivers through this lens. Many of the participants knew that I was a former elementary school teacher. On the one hand, this gave us shared footing to begin conversations. The child participants were often excited to hear that I was in some ways like their teachers who they for the most part explicitly adored. My experiences as a teacher also helped me understand the children when they shared thoughts that were not entirely coherent or when they used lingo that is common in elementary classrooms but not in other settings. On the other hand, it is possible that participants left out certain details under the assumption that they were things I already knew or that they saw as taken for granted. It is also possible that my own experiences shaped my interpretations of students' stories in ways that were not entirely accurate. To combat this, I did my best to stay close to the data in my interpretations by returning repeatedly to the children's exact words.

Second, I had personal connections to approximately half of the participating families. I knew some families from my career as an elementary school teacher because the parents had been involved at the school in which I most recently taught, or because I had taught the participating child's older sibling. Some of the caregivers were actually former teacher colleagues of mine. Other times I knew the families from living in the neighborhood, and had been at backyard barbecues where their children had played with my children. Even for the half of participants with whom I did not have a prior relationship, I was still only one degree removed. I knew someone who they knew, which offered a starting point of familiarity. This familiarity created a level of comfort in my conversations with participants over Zoom that likely would not have been otherwise as easily achieved. In other circumstances it may have been more difficult to move through as many questions as I did, and the children or their caregivers may have been less forthright or responsive than they were. However, knowing that I had relationships with other families and teachers at their schools, it is also possible that children or their parents could have withheld sensitive information that they may have actually been more willing to tell a disconnected stranger.

Third, I am a white woman. According to Federal data from 2020, 79.3% of public school teachers in the United States were white, and 76.5% were female (U.S. Department of Education, 2020). In other words, I reflect the demographic that is most heavily overrepresented in the teaching profession and can easily be associated with the institution of schooling. The institution of schooling has caused and continues to cause great amounts of trauma for many students, especially students of color. In turn, it is possible that my position as a white woman created a certain amount of distance from or distrust with some of the families who participated in this study. This may have impacted the data that I was able to gather. Relatedly, I conducted all interviews in English, and some of the participants and their caregivers may have been more comfortable in other languages. This too may have impacted my data.

Finally, I am a mother of two young children. While I did my best to schedule interviews at the convenience of the participants, I was also scheduling interviews between nap times and snack times. My older child was at the end of his last year of preschool when the pandemic began. That spring, he had some live Zoom meetings with his teachers and classmates, but for the most part, the school year just ended early for him. He began Kindergarten the next fall "at" our neighborhood public school, entirely online. Because of him, I had a very intimate experience with elementary school in general, and with math learning in particular during the pandemic. I have no doubt that what I saw as strengths and challenges of that year in school for my child and for our family in some ways colored my expectations for what others might have experienced as well. On the other hand, my experiences as a mother gave me an enormous amount of empathy for each of the participating families in this study, and I deeply appreciated their time, willingness to talk with me, and thoughtfulness in their responses to my questions.

Chapter 3
"I'm not 18. I'm only six and a half":
Investigating Mathematics Identity Development Patterns with Young Children

When Reese and I had our first conversation over Zoom together, she was just a few days away from finishing Kindergarten. She shared with me that it had been a bumpy year; there was a lengthy teachers' strike early on, then her teacher had a baby and was on maternity leave for a few months, and then the pandemic hit, sending Reese and her classmates home to log-in to school from tablets. She hoped that her future years in school would be a little less interrupted, and she knew she had a lot of learning ahead of her. When I asked her if she expected to keep learning mathematics as she got older she said, "I'm still learning math. And I'm only six." In our following conversation later that summer, Reese again reminded me, "I'm not 18. I'm only six and a half."

Though maybe unintentional, Reese's reminders of her age were apropos to this study because much of the literature exploring mathematics identity is in fact with participants who are much closer to eighteen (Radovic et al., 2018). The children in this study were recruited to participate because they were at the beginning of their formal schooling story. Like themselves, their relationships with mathematics, especially school-based mathematics, were still young. In this chapter, I look at established patterns in the literature that illustrate how various classroom features impact mathematics identities of middle school and high school students, and I explore the applicability of these patterns to children in the early years of elementary school.

Specifically, the findings here respond to the research questions: Which facets of early elementary school students' relationships with mathematics differentiate among their emerging mathematics identities? And, what features of mathematics learning environments contribute to the development of positive mathematics identities for this age group? I sought to uncover whether what we know about older students is also relevant for young children, and further, whether there are differences in what matters even amongst the early elementary grades. Through my analysis, I argue that though general patterns documented in previous literature appear to hold when looking at my study data as a whole, a closer look at individual students and grade level groups paints a more nuanced picture. Some classroom features and descriptions of students' relationships with mathematics appear to have developmental trajectories, meaning they shift in their relevance as children get older. This makes those features less relevant for understanding this study population overall. However, students' experiences of conceptual agency and opportunities for self-expression do seem meaningful to students' emerging relationships with mathematics across the Kindergarten through second grade study participants. Further, a combination of whether students enjoy mathematics and whether or not they bring mathematics outside of school into their own lives is more helpful for differentiating between these students' relationships with mathematics than looking at whether students perceive themselves as successful or whether they see mathematics as a part of their futures.

To reach these claims, I examined three patterns from the literature that each emerged from studies that took seriously students' perceptions of themselves and their mathematics classroom experiences. These studies all recognized that students' emotions matter, and that students' affective responses to their classrooms and to learning play a meaningful role in their current and future participation in mathematics. None of these studies looked at more standardized measures, such as test scores, as outcomes, but instead they focused on elements of mathematics identity, or students' relationships with the discipline of mathematics. Further, like this dissertation, each of these studies employed interviews in order to hear directly from their participants, and took an interpretive stance (Erickson, 1985) in foregrounding participants' perspectives in their analysis. The analysis in this chapter learns from and extends the work of these past studies by respecting and prioritizing the voices of students, by focusing on students' feelings and perceptions, and by investigating the applicability of their findings for students in kindergarten, first, and second grade.

First, I explored the work of Jo Boaler and James G. Greeno (2000) from their seminal essay, "Identity, Agency, and Knowing in Mathematics Worlds." Responding to statistics that many seemingly capable students opt out of advanced mathematics coursework and mathematics-related careers, they unpacked the relationships between students' perceptions of their classroom environments, their affective reactions to their learning experiences, and their intentions for their future participation in mathematics. The researchers interviewed high school calculus students across six schools and found that students who perceived their environments as collaborative were more likely to enjoy mathematics and to intend to continue learning mathematics after high school in comparison to students who perceived their classrooms as individualistic, or centering individual work and achievement. In examining this work within the context of my project, I found that though this same general pattern held across my data, both students' perceptions of the environment and the presence of mathematics in their futures seemed to shift across the early elementary grade level groups, making the links between these features less clearly relevant to students' emerging mathematics identities.

Then, I investigated the framework created by Paul Cobb, Melissa Gresalfi, and Lynn Liao Hodge (2009) in their article, "An Interpretive Scheme for Analyzing the Identities that

Students Develop in Mathematics Classrooms.” They first observed classrooms to determine norms and expectations, and then engaged in interviews to understand students’ personal identities in response to and relationship with those collectively established practices. In doing so, they found that students who felt that authority was shared in their classroom between teacher and students and who experienced conceptual agency were more likely to feel successful and to begin to affiliate with the mathematical activity happening in their class. This was in comparison to students who perceived authority in their classroom as solely in the hands of the teacher and who felt limited to disciplinary agency, or reproducing established procedures. When I explored this framework within the data of the current study, analysis revealed that whether or not children perceived themselves as successful had limited usefulness in differentiating across this study population. However, experiences of conceptual agency seemed impactful and relevant across the grade level groups for young children’s relationships with mathematics.

Finally, I considered the work of Na'ilah Suad Nasir and Victoria Hand (2008) in their article, “From the Court to the Classroom: Opportunities for Engagement, Learning, and Identity in Basketball and Classroom Mathematics.” Their goal was to understand the factors that lead to increased engagement and the development of *practice-linked identities*–meaningful connections between oneself and the activity one is participating in–when looking at extra-curricular activities in comparison to classroom experiences. They observed and interviewed high school varsity basketball players who were also geometry students, and they found that on the basketball court, students had meaningful access to the activity’s domain, the availability of integral roles, and opportunities for self-expression, which together supported the students in developing practiced-linked identities with basketball. This was not the case for geometry. To bring this work to the current study, I analyzed the relationship between the participants' perceptions of these features and their relationships with mathematics and found that while the combination of these features does appear potentially impactful, only opportunities for self-expression seemed to maintain similar relevance across the grade level groups.

Together, the work of Boaler and Greeno (2000), Cobb et al. (2009), and Nasir and Hand (2008) highlighted a range of important and overlapping classroom features that have been found to impact students' mathematics identities. Bringing their findings to my data allowed me to investigate the possible impacts of students' perceptions of their positions, roles, freedoms, and learning on their emerging relationships with mathematics. In the sections that follow, I describe in detail my findings from the analysis of each pattern from the literature. Then, I discuss key takeaways from looking across all three analyses, and I end with questions of what else matters to young children and their relationships with mathematics that is not integrated into these three previous studies.

The Relationship Between Environment, Feelings, and the Future

Suriyah was nearing the end of first grade when I first met her. She was calm and thoughtful in our interview, and she exuded a certain confidence in her responses that seemed to suggest she had already given quite a bit of thought to her experiences learning and doing mathematics. Suriyah described her math class as *collaborative*; she explained that students often worked in groups, and they supported each other within those groups. When she got stuck trying to solve a problem in math, she was just as likely to ask a peer as her teacher. In reflecting on what she missed about school while learning at home, she mentioned the chance to help others. She shared, "I miss helping some of my friends during math… I'm the youngest at home so I don't really have anyone to help." Suriyah *liked math*, "because it can be fun and also challenging at the same time," and she anticipated that she *will continue learning math* well into her future, "because math can be anywhere and when it's everywhere, you can learn anything anywhere."

Suriyah's reflections aligned with Boaler & Greeno's (2000) findings that students who perceived their learning environments to be collaborative were more likely to enjoy mathematics and to intend to take other mathematics courses in their futures. In their study, 94% of the high school AP calculus students who described their classes as centering meaningful student discussion, peer support, questions amongst and directed at both classmates and teachers, and an openness to consider multiple approaches and solutions to the problems said they enjoyed mathematics, and 80% of those same students had intentions of continuing in the discipline. Boaler and Greeno sought to understand why, given no apparent struggles or lack of ability, many students opted to stop learning mathematics after high school. Their analysis revealed that part of the answer to their question was in how students understood their mathematics learning environments and relatedly, their role within them. They wrote that, "The mathematics classroom may be thought of as a particular social setting…in which children and teachers take on certain roles that help define who they are," (p. 173). In other words, when students saw themselves as part of a community, or part of a team in their classrooms, this contributed to students' positive feelings towards learning mathematics.

In both mine and Boaler and Greeno's (2000) data, the opposite relationships also emerged. In their study, students who described their classrooms as *individualistic* were less likely to enjoy mathematics and less likely to intend to take additional mathematics courses.

Such students' perceptions of their mathematics classrooms emphasized the importance of practicing and repeating procedures after demonstration by their teachers. They described minimal discussion and the importance of obedience for success. Owen, a kindergarten student in my study, had perceptions of and feelings towards mathematics that aligned with this pattern and that were consistent across all three of our interviews. He said, "[Math in my classroom] was the worksheets for like half of the day…it's very quiet. There was no music." He elaborated that though students sat at tables together, they could not help each other. He found math to be tiresome, difficult, and frustrating. Even when he got an answer right, he still felt sad that he was doing math. To him, the only good part of math in school was that lunch came afterwards. Owen said that there was nothing he wanted to learn in math, and when I asked him if he might learn or do math down the line in his future he declared, "No! Never! Never! Never!"

As previously described in Chapter 2, in my analysis of interviews, students were categorized as perceiving their classrooms as *collaborative* if they mentioned student discussion, working together in pairs or groups, helping or being helped by classmates, or getting ideas from their classmates in any of our interviews. In contrast students were categorized as perceiving their classroom experiences as *individualistic* if they described practicing and repeating procedures on their own after being shown the procedure by a teacher, working through a workbook on their own, or being personally responsible for doing what the teacher told them to do. Then, I coded for the relevant facets of students' relationships with mathematics. In order to be labeled as *liking* math, a student had to express more positive than negative feelings towards math, while the opposite was true for students who I labeled as *disliking* math. Finally, students were categorized as *ongoing math* if they said that they expected to continue learning or doing math both in school and beyond as they got older. They were categorized as *ending math* if they expected to only learn or do math within school requirements. Suriyah, described at the beginning of this section, is an example of a student who was categorized as perceiving a *collaborative* classroom, *liking* math, and expecting *ongoing math* in her future. In contrast, Owen was categorized as perceiving an *individualistic* classroom, *disliking* math, and imagining *ending math* when school ended. These codes are listed with definitions in Chapter 2, Tables 2.4 and 2.5.

In a way, my data for this study appeared to reflect the relationships and patterns that Boaler and Greeno (2000) described. 89% of the students who I interviewed who described their math classrooms as collaborative said that they enjoyed mathematics, and 100% expected to continue learning and doing mathematics in their futures both within and beyond school. Those percentages are higher than for the students who described their learning environments as individualistic; in that group, 67% of students said they liked math, and 75% expected to continue learning and doing mathematics in their futures. Put simply, the kindergarten through second grade students in this study were more likely to like math and imagine themselves continuing doing math if they perceived their classrooms as collaborative spaces. However, the students who perceived their classroom environments as individualistic were still more likely to like math than to dislike math, and to see math in their futures than not. Table 3.1 illustrates this data.

Table 3.1

Percentages of Students by Perceived Environment Type

Perceived environment	Liked math	Disliked math	Ongoing math	Ending math
Collaborative *n=18 (60%)*	16 (89%)	2 (11%)	18 (100%)	0 (0%)
Individualistic *n=12 (40%)*	8 (67%)	4 (33%)	9 (75%)	3 (25%)
Total *n=30*	24 (80%)	6 (20%)	27 (90%)	3 (10%)

A closer look at grade level groups and individual students added more questions about the relevance of the links between environment, affinity, and sense of the future for the participants in my study. 40% of the students (n=12) did not fit cleanly along the lines of the patterns described above, and an even higher percentage of the kindergarteners (67% or 6/9 students) did not align with the documented trends. Amongst the nine kindergarteners, seven of them described their environments as individualistic, and yet, many students still liked math and saw it as a part of their futures. Table 3.2 breaks down the data by grade level. The bolded text highlights that a high percentage of Kindergarteners perceived their math classroom environments as individualistic compared to the older age groups. In the older age groups, significantly more students perceived their classroom environments as collaborative.

Table 3.2

Percentages of Students by Perceived Environment Type, By Grade Level

Grade level	Perceived environment	Liked math	Disliked math	Ongoing math	Ending math
Kindergarten	**Collaborative** ***n=2 (22%)***	1 (50%)	1 (50%)	2 (100%)	0 (0%)
	Individualistic ***n=7 (78%)***	4 (57%)	3 (43%)	4 (57%)	3 (43%)
	Total *n=9*	5 (56%)	4 (44%)	6 (67%)	3 (33%)
1st grade	Collaborative *n=8 (73%)*	8 (100%)	0 (0%)	8 (100%)	0 (0%)
	Individualistic *n=3 (27%)*	3 (100%)	0 (0%)	3 (100%)	0 (0%)
	Total *n=11*	11 (100%)	0 (0%)	11 (100%)	0 (0%)
2nd grade	Collaborative *n=8 (80%)*	7 (88%)	1 (12%)	8 (100%)	0 (0%)
	Individualistic *n=2 (20%)*	1 (50%)	1 (50%)	2 (100%)	0 (0%)
	Total *n=10*	8 (80%)	2 (20%)	10 (100%)	0 (0%)

Claire was one of the kindergarten students who both saw her classroom as individualistic and liked math. When Claire described what her teacher did during math class, she said, “[She] comes and looks around us to make sure we’re doing our stuff…she’s checking on us because she doesn’t want us to be playing..she expects us to be doing our work, not playing around.” The following fall, Claire explained that in order to learn math, you had to follow the teacher’s instructions. There was no ambiguity that Claire perceived her classrooms, both in Kindergarten and the following fall in first grade, as individualistic environments. Still, she was excited to learn, and she loved making patterns when those patterns involved her favorite foods. She was eager to be able to do more and more of what she saw her older sister being able to do, and she had aspirations of being a fourth grade teacher someday, so she definitely expected to continue doing math. For Claire, the individualistic nature of her classroom did not, at least at this point in her development, overshadow the pieces of learning math that she saw as fun and exciting. It seems feasible that for a kindergartener like Claire, adjusting to the rules of school puts a heavy emphasis on following the teacher's instructions and doing what you are supposed to do; in this way, the newness of school could be a significant factor in kindergarteners' experiences and one that limits the significance of differentiating between collaborative and individualistic environments.

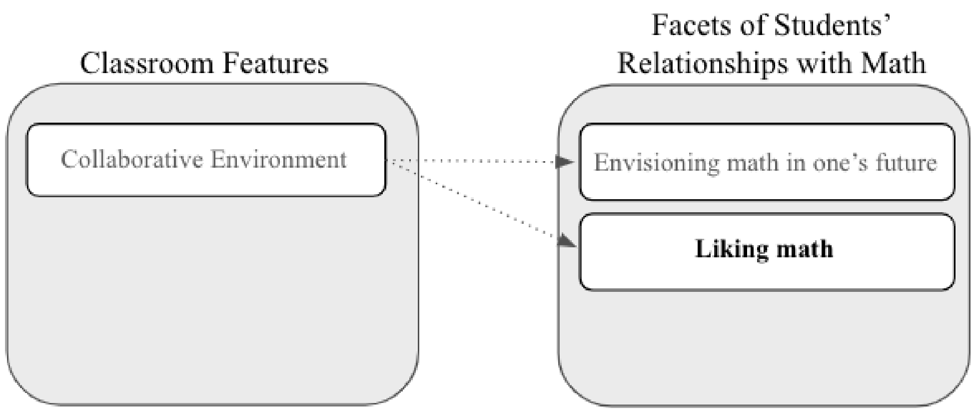
Another trend across the three grade levels was that all of the students who began the study in first or second grade imagined continuing to learn and do math in their futures. This may be because these young students have so much schooling still ahead of them in comparison to Boaler and Greeno's (2000) AP Calculus students who were on the cusp of having the opportunity to decide what they want to learn. In turn, this particular facet of children's relationships with mathematics could not be used to differentiate between any of the students in these two groups. This also had the ramification that anyone in those grade level groups who either saw their environment as individualistic or disliked math did not fit within the expected pattern.

Asher, a second grader, was one of the students who perceived his classroom as a collaborative learning space and saw himself as continuing to learn and do mathematics well into his future, but he disliked math. In our first interview, Asher begrudgingly acknowledged that math would likely be a part of his future because he saw his mother, along with people like me, doing math, but for now, math was at the bottom of his list of things he wanted to do. In his words, "I feel happy when I finish [my math work], because that means that…I have lots of hours with no pain and misery." And yet, Asher shared that he and his classmates often worked in partnerships during math and that students were expected to help each other solve problems and complete assignments. Why then was Asher's experience learning mathematics so negative? It turns out that for Asher, collaboration was not a positive experience. He described often feeling frustrated by assigned partner work because, "your partner could be way ahead of you, and then you'll have no idea what your partner's doing because they're so far ahead…" Asher is an example of a student for whom having collaborative structures in the classroom was not enough; it seems that he needed agency and other support to make those collaborative structures positive and productive for him. As he was experiencing it, collaboration was actually isolating. Effective collaboration requires learning skills like sharing, listening, and joint decision making. These skills take time and practice to develop. If the nature of Asher's collaborative experiences shifted, would his feelings towards mathematics also shift? Asher's case raises the question of whether or not perceptions of a collaborative environment are what matter, or if it is actually positive experiences with collaboration that the data in this project and in Boaler and Greeno's (2000) study are generally reflecting. This question warrants further future investigation. Certainly, the AP Calculus students in Boaler and Greeno's study had been in school much longer than my study participants, and in turn had more time to develop effective collaboration skills, possibly making their collaborative experiences more likely to be positive.

In summary, in this phase of analysis I looked at one classroom feature, that being the collaborative nature of classrooms, and two facets of students' relationships with mathematics–students' affinity for mathematics and whether or not they see mathematics as a part of their futures. Boaler and Greeno (2000) found these variables to be linked. These links can be seen as arrows in Figure 3.1.

Figure 3.1

The Relationship Between Environment, Affinity, and Future



Note. The arrows are dotted rather than solid to suggest that the link between a collaborative environment, students liking math, and students envisioning math as a part of their future was not clear for this study population. "Liking math" is bolded to show that it emerged here as the more relevant facet of students' relationships with mathematics.

However, my analysis revealed that though perceiving an environment as collaborative may support students in developing positive feelings towards mathematics, on its own it is not enough of a measure to make clear distinctions between young children's emerging mathematics identities. For this reason, the arrows in Figure 3.1 are dotted instead of solid. This is particularly true for kindergarten students who may be prone to perceiving their classroom environments as individualistic. Further, though it was the case that two of the three students who expected to stop learning and doing math entirely after reaching a certain point in school also did not like math, the vast majority of students in my study expected math to be an ongoing part of their futures, making it impossible to decipher any clear relationships between that outcome and students' perceptions of their classrooms. Whether or not students liked mathematics was a more meaningful way to begin understanding students' experiences and relationships with mathematics, which is represented by it being bolded in Figure 3.1. Still, other dimensions of students' perceptions are needed in order to better understand the nuanced differences between childrens' feelings and experiences. To look at other dimensions, I turned next to the work of Cobb et al. (2009).

The Relationship Between Authority, Agency, Success, and Affiliation

When I sat down to talk with Torrin for the first time, he carefully adjusted his headset and then eagerly detailed to me exactly how a math class in his second grade classroom was structured. He explained that the class was divided into groups, and he loved working with his group to solve problems together. That cooperative spirit also came through as Torrin described whole class discussions. He said that if he made a mistake, he could talk about it to the class and a classmate could help him fix it, and that often, he learned new strategies by listening to his peers. It was clear that Torrin perceived his teacher as *sharing authority* over what counts as mathematically correct and legitimate with his classmates. Further, Torrin felt like he had the *conceptual agency* to develop strategies of his own and solve problems in ways that made sense to him. He said that when he wants to learn something new, instead of jumping to ask someone right away, "Sometimes… I can bring another thing that I already know with this new thing, and bring them together and so I can find something out." Torrin saw himself as *successful* in math; he noted that his math group was a "higher level" than the others in the class. And, not only did Torrin like math, he also opted to do math in his own time. He played games that involved math, and enjoyed finding math throughout his day-to-day life. He explained that this was because, "I know if we didn't have math we wouldn't be able to do a lot of stuff." Torrin was already taking mathematics on as a part of who he was; he was *affiliating* with mathematics as he understood it.

Torrin's reflections aligned with Cobb et al.'s (2009) findings that students who saw authority as shared between teacher and students and who perceived themselves as able to exercise conceptual agency were more likely to feel successful and to see mathematics as it was happening in their classroom as a part of who they were. Their study took place across two eighth grade mathematics classes in a single school. One of the classes was algebra, taught by a teacher at the school, and the other was a design experiment class focused on data analysis, taught by a member of the research team. Cobb et al. emphasized in their article that their intent was not to evaluate the school's algebra class or teacher, to suggest better ways of teaching mathematics, or even to illustrate particular relationships between specific outcomes. Instead, their goal was to illustrate the use of an analytic tool for describing students' relationships with mathematics. However, their tool incorporated particular features of the learning experience–authority and agency–, making it useful for my analysis.

Cobb et al. (2009) found that students' mathematics identities fell in three categories: affiliating, cooperating, and resisting. Students who affiliated with mathematics were students who felt like their classroom obligations were obligations to themselves. These students took on their classroom mathematics activities as important and meaningful. Differently, students who cooperated with mathematics followed through on classroom expectations, but did so out of obligation to their teacher. Finally, students who resisted mathematics actively pushed against their classroom obligations, feeling as though they wanted nothing to do with the mathematics that was happening in their classes. The students in the design experiment class who experienced shared authority and conceptual agency nearly entirely saw themselves as successful and affiliated with the mathematics happening in their class. In contrast, in the algebra class where students experienced teacher authority and were limited to disciplinary agency, fewer students felt successful, and they cooperated with the mathematics happening in their class, but did not affiliate with it.

As I began to investigate the applicability of these relationships to my study data, I coded the student interviews along the classroom features that Cobb et al. (2009) described. First, they discuss whether students perceived mathematical authority in the classroom as solely in the hands of the teacher or as shared between teacher and students. For young children, this translated to whether or not they felt like they could work with, help, and be helped by other students in the classroom. This mirrored the description of a collaborative classroom in the previous round of coding, so here I used the collaborative and individualistic environment codes as proxies for *shared authority* and *teacher authority.* Next, I coded along the lines of *disciplinary* and *conceptual agency*. When students are limited to exercising disciplinary agency, they consistently follow particular procedures to get singular right answers. In contrast, when students exercise *conceptual agency*, they are able to problem-solve, make connections, and investigate a variety of solutions in ways that make sense to them (Cobb et al., 2009). For the young children in this study, this looked like having the freedom to choose their own strategies and thoughtfully compare those strategies with those of their peers. Definitions and examples of these codes can be found in Chapter 2, Table 2.6.

After coding for students' perceptions of these classroom features, I turned toward students' mathematics identities. Using Cobb et al.'s (2009) categories, I labeled Torrin, introduced at the beginning of this section, as *affiliating* with mathematics because he both liked mathematics *and* opted to do mathematics on his own, outside of the confines of his classroom, which was a sign that he was taking on mathematics as a part of his personal identity. He actively carried mathematics across the walls of home and school. I categorized other students who liked math but did not self-initiate mathematical activity outside of school or who sometimes disliked math but willingly did math outside of school with a parent as *cooperating* with mathematics. Lastly, I categorized students who disliked math, did not self-initiate mathematical activity outside of school, and did not willingly engage in mathematics with a parent or other relevant person in their life as *resisting* the mathematics that they were experiencing. In total, I categorized 15 students as affiliating, 12 as cooperating, and three as resisting. Additionally, I noted whether or not students perceived themselves as *successful* in order to explore how that perception related to the students' perceptions of their classroom experiences as well. These identity codes are further elaborated in Chapter 2 in Table 2.7.

As I analyzed the coded interviews, it appeared that in some ways, the links that Cobb et al. (2009) described between authority, agency, success, and mathematics identity seemed to resonate with my data. Of the students who perceived themselves as experiencing shared authority and conceptual agency (n=13), 100% felt successful and 69% were coded as affiliating. In contrast, of students who experienced entirely teacher authority and disciplinary agency (n=5), 80% were coded as cooperating with mathematics. These relationships are bolded in Table 3.3.

Table 3.3

Students' Mathematics Identities by Perceptions of Authority and Agency

	Affiliating	Cooperating	Resisting	Perceived themselves as successful
Shared authority *and* conceptual agency *n=13*	9 **(69%)**	4 (31%)	0 (0%)	13 (100%)
Teacher authority *and* disciplinary agency *n=5*	1 (20%)	4 **(80%)**	0 (0%)	5 (100%)
Shared authority *but* disciplinary agency *n=5*	2 (40%)	2 (40%)	1 (20%)	5 (100%)
Teacher authority *but* conceptual agency *n=7*	3 (43%)	2 (29%)	2 (29%)	5 (71%)
n=30	15 (50%)	12 (40%)	3 (10%)	28 (93%)

Note. The percentages in bold highlight the difference in mathematics identities of students who perceived themselves as experiencing both shared authority and conceptual agency and students who perceived themselves as experiencing neither. The percentages that are underlined show that both of these groups of students perceived themselves as successful.

However, in the same table you can see that 100% of both of these groups of students perceived themselves as successful. These percentages are underlined. In fact, all but two students in the study described themselves as at least sometimes experiencing success. This made the category of perceived success less useful in differentiating between students' experiences. In other words, given that these young students who felt successful still had significant variation in their relationships with mathematics, it seems that feeling successful is not on its own a clear marker of affiliation with this population.

Lucas was an example of a student who perceived himself as successful, but his relationship with mathematics was still limited to cooperating with classroom obligations. This may be because besides the fact that he perceived himself as successful, his perceptions of his math learning experiences aligned with Cobb et al.'s (2009) description of students in the more traditional algebra class. A first grader at the beginning of the study, he saw authority as limited to his teacher and his participation as limited to disciplinary agency. He explained that in math class he just did what he was supposed to do: "I just do the answer." He found math in kindergarten to be boring, and expected that first grade would be "way more boring. Like 10 million times more boring." Still, across all three of our interviews, Lucas saw himself as one of the most successful students in his class and noted that his classmates "have a hard time." Despite disliking math class, Lucas opted to do some math on his own outside of school to help ensure that he would stay ahead of his peers, and he was disappointed once when another student scored higher than him. Though Lucas did not affiliate with mathematics as he was experiencing it in his classroom, he also did not resist it. He continued going through the motions and plowing ahead. It is possible that the fact that Lucas perceived himself as successful offered some positive buffer against his otherwise negative classroom experiences even if his success was not enough for Lucas to feel more positively towards mathematics overall.

I only categorized three students as developing mathematics identities that were resisting mathematics as they were experiencing it in their classrooms, but of those three students, none of them fell perfectly along the predicted patterns. One of the three felt successful, and two of the three perceived themselves as experiencing conceptual agency. Kira was one of these students. Though Kira saw authority as limited to the teacher, described herself as generally unsuccessful, did not like math, and did not opt to do math outside of when it was mandatory, Kira did describe herself as having conceptual agency in math class. She shared that she got to use whatever strategies she wanted when solving problems, and for her that typically meant drawing out the problem “Cuz I loooove drawing little pictures. I like doodling." This freedom was not enough on its own to support a positive relationship with math for Kira. In her words, “I don’t like math…It’s hard, and…you don’t really get a bunch of fun. Everyone sits at the desk being quiet, doing their own work, they don’t get to talk to other people. It’s like, sad.” Further, she shared that everyone else in her family–her mom, dad, brother, and sister– are “all good at math” and do a lot of math. She perceived herself as standing out and only being “good at reading.” It seems that Kira would have needed other supportive features beyond experiencing conceptual agency in order to change the trajectory of her emerging mathematics identity.

Despite these examples, when I looked separately at the students who experienced shared authority and the students who experienced conceptual agency, my analysis continued to suggest that experiencing these classroom features may be supportive of positive relationships with mathematics. As seen in Table 3.4, of the 15 students who were coded as affiliating with mathematics, 73% of them perceived their classroom as having shared mathematical authority and 80% of them perceived themselves as experiencing conceptual agency. This contrasts with 50% of the students who were categorized as cooperating with mathematics who perceived conceptual agency or shared authority, separately.

Table 3.4

Percentages of Students Perceiving Shared Authority and Conceptual Agency, By Identity

	Experienced shared authority	Experienced conceptual agency	Experienced neither
Affiliating *n=15*	11 **(73%)**	12 **(80%)**	1 (7%)
Cooperating *n=12*	6 **(50%)**	6 **(50%)**	4 (33%)
Resisting *n=3*	1 (33%)	2 (67%)	0 (0%)

Note. The percentages in bold highlight that a higher percentage of students who were coded as affiliating with mathematics perceived themselves as experiencing shared authority and conceptual agency in comparison with students who were coded as cooperating.

However, these percentages were not evenly distributed across the grade level groups. As I noted in the previous section when describing classrooms as collaborative or individualistic, perceptions of shared authority seemed to shift as students got older. Very few Kindergarteners perceived themselves as experiencing shared authority. This makes looking at perceptions of authority less helpful in understanding students' experiences when looking across this study population as a whole. This was not the case with students' perceptions of conceptual agency, which were relatively more similar across the three grade level groups. This can be seen in Table 3.5, which divides the shared authority and conceptual agency data by grade level.

Table 3.5

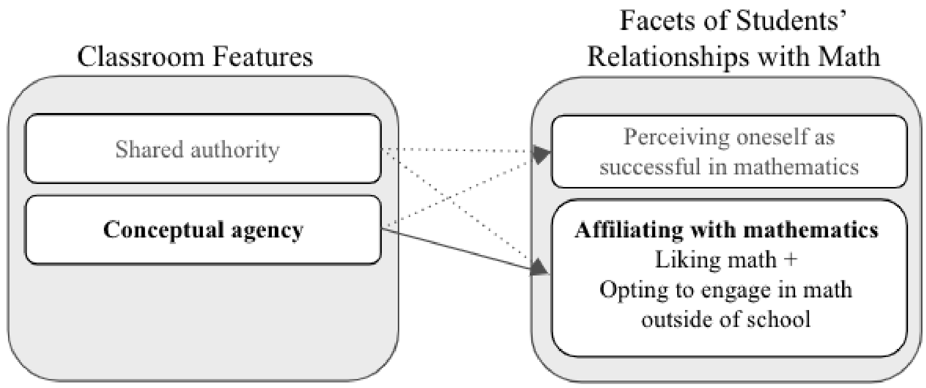
Percentages of Students Experiencing Shared Authority and Conceptual Agency, By Grade Level

	Experienced shared authority	Experienced conceptual agency
Kindergarteners *n=9*	2 (22%)	7 (78%)
1st graders *n=11*	8 (73%)	6 (55%)
2nd graders *n=10*	8 (80%)	7 (70%)
Total *n=30*	18 (60%)	20 (67%)

In summary, in this phase of analysis I looked at two classroom features–authority (teacher or shared) and agency (conceptual or disciplinary). I also looked at whether students perceived themselves as successful, and whether they seemed to be affiliating with, cooperating with, or resisting mathematics as they were experiencing it in their classrooms. Cobb et al. (2009) found these variables to be linked. These links can be seen as arrows in Figure 3.2.

Figure 3.2

The Relationship Between Authority, Agency, Success, and Affiliation



Note. The arrows coming from *shared authority* and pointing to *perceiving oneself as successful in mathematics* are dotted instead of solid to show that those links are unclear with this study population. *Conceptual agency* and *affiliating with mathematics* are bolded to highlight their relevance to my data. In addition, the link between them is more clear and therefore that arrow is solid.

However, my analysis revealed that several of these links were less clear with the early elementary school students who participated in my study. For one, students' sense of authority in the classroom appeared to shift as students got older; the Kindergarteners were very unlikely to perceive their teachers as sharing authority with their peers. Still, many of these students liked and felt successful in math. For this reason, the arrows in Figure 3.2 connecting shared authority to the facets of mathematics identity are dotted instead of solid. Second, the vast majority of students in the study perceived themselves as successful, making that characteristic unhelpful in differentiating between students' emerging relationships with mathematics. In turn, the arrows leading to that facet of mathematics identity are also dotted instead of solid. The one link that did resonate with my data was between conceptual agency and affiliating with mathematics. Students experienced conceptual agency across the grade level groups, and when they did so, they were more likely to be taking mathematics on as a part of who they were. These variables are bolded in Figure 3.2. This is a key takeaway. For young children, whether or not they feel successful does not seem to be the metric to use to evaluate students' emerging relationships with mathematics. Instead, we should look at whether or not they like mathematics and engage in mathematics on their own, outside of school. Further, when determining whether or not classrooms are supportive of positive mathematics identity development, these findings suggest that it is worthwhile to attend to students' opportunities to exercise conceptual agency. Besides conceptual agency, I was curious what other features might support positive mathematics identity development, and so I next turned to the work of Nasir and Hand (2008).

The Relationship Between Domain, Roles, Self-Expression, and Mathematics Identities

Meera, a soft-spoken second grader explained to me that "Math was invented to help you figure out things." She saw purpose in learning math, and in addition to expecting to continue learning and doing math in her future, Meera appreciated math because she understood it as meaningful to her everyday life. In her words, "In your everyday life [when] you come to a challenge or you need to do something with a lot of numbers you can just use math to find out the answer." Meera had been shown or had come to see that the *domain of mathematics* was expansive outside of the specific skills she was learning in her classroom. That said, Meera also spoke with great enthusiasm about math in school. She loved working with her assigned math group; they often helped her figure things out. And, she felt like she was good at math because she was also able to help others; she believed that she played an *integral role* in her class. To top it off, Meera shared that her math group got to pick its own name as an *opportunity for self-expression*; she loved being a member of the "Painted Ponies!"

In describing her understanding of and experiences with mathematics, Meera brought to the conversation the three features of learning environments that Nasir and Hand (2008) identified as important for supporting students in developing practice-linked identities with a given activity: providing students with access to the domain, students playing integral roles, and offering students opportunities for self-expression. Meera seemed to be developing a practice-linked identity with mathematics; in the previous round of analysis, she was categorized as affiliating with mathematics because she both liked math and frequently opted to engage in mathematics on her own outside of school. In turn, Meera was an example of a second grade student who was aligning with the pattern that Nasir and Hand put forth. Their study took place in a high school in two settings: a geometry classroom and the basketball court. The participating students were varsity basketball players and successful geometry students, but their engagement and identification with basketball was much higher because their geometry classroom offered only a very limited window into the domain of mathematics, positioned them as passive receivers of knowledge (Belenky et al., 1986), and provided them with very few opportunities to express who they were in meaningful ways.

Nasir and Hand's (2008) work offered new classroom features to unpack in relation to students' mathematics identities: access to the domain, the opportunity to play integral roles, and the opportunity for self-expression. When Nasir and Hand defined *access to the domain*, they described students seeing a purpose to the content they were learning in the moment and being able to connect their learning to a broader practice. For the young students in my study, this idea emerged when students talked about developing skills along a meaningful trajectory such that they felt like they were making progress towards a greater goal or when students, like Meera above, described believing in a purpose to what they were learning that expanded outside of their classroom walls. Students in my study discussed playing *integral roles* when they described helping others and sharing their ideas with the class. These moments mirrored the moments that were previously coded as *collaborative environment* and *shared authority*, so here I used those as a proxy code for the feature of integral roles. Finally, students described *opportunities for self-expression* when they talked about making things their own, arranging things in ways that were comfortable to them, making significant choices in their math classrooms, being themselves and having fun while doing so. A more detailed description of the coding process can be found in Chapter 2, and specific definitions and examples of these codes can be found in Table 2.8.

As I analyzed the coded interviews, I looked at how the emergence of these classroom features related to their mathematics identities. To do this, I referred back to how students were coded in the previous analysis based on the work of Cobb et al. (2009) as either affiliating with, cooperating with, or resisting mathematics as they were experiencing it in their classrooms. These categories offered a way to capture students' relationships with the discipline of mathematics, closely tied to what Nasir and Hand (2008) referred to as practice-linked identities. Overall, of the students who described experiencing all three features in their math classroom environments (n=7), all but one, or 86%, were coded as affiliating with mathematics in the previous round of analysis. This is a much higher percentage of students affiliating with mathematics than amongst the groups of students who described experiencing only one or two of the aforementioned features (44% and 42% respectively). Therefore, broadly speaking it seems like the students described experiencing access to the domain of mathematics, playing integral roles in their classrooms, and having opportunities for self-expression were in fact more likely to develop strong relationships with mathematics. Table 3.6 includes the data that illustrates this claim.

Table 3.6

Percentages of Students Categorized as Affiliating, Cooperating, or Resisting, By Their Experiences of Identity-Supportive Features

	Affiliating	Cooperating	Resisting
Experienced 3/3 identity-supportive features *n=7*	**6 (86%)**	1 (14%)	0 (0%)
Experienced 2/3 identity-supportive features *n=12*	**5 (42%)**	5 (42%)	2 (17%)
Experienced 1/3 identity-supportive features *n=9*	**4 (44%)**	6 (56%)	0 (0%)
Experienced 0/3 identity-supportive features *n=2*	**0 (0%)**	1 (50%)	1 (50%)
Total n=30	15 (50%)	12 (40%)	3 (10%)

Note. The percentages that are bolded highlight that a high percentage of students who described experiencing all three identity-supportive features were also coded as affiliating with mathematics.

In order to better understand the impact of these features, I looked at each feature separately in comparison to students' mathematics identities. It turned out that for each of the features separately, of the students who experienced that feature a higher percentage of the group was coded as affiliating with mathematics compared to cooperating with or resisting mathematics. Put otherwise, the data seems to show that any and all of these features may be supportive of a positive relationship with mathematics on their own, and in combination even more so. This data is shown in Table 3.7.

Table 3.7

Percentages of Students Categorized as Affiliating, Cooperating, or Resisting, By Identity-Supportive Feature

	Affiliating	Cooperating	Resisting
Experienced access to the domain *n=17*	9 (53%)	6 (35%)	2 (12%)
Experienced integral roles *n=18*	11 (61%)	6 (33%)	1 (6%)
Experienced opportunities for self-expression *n=20*	13 (65%)	6 (30%)	1 (5%)

However, like in my findings from the previous patterns from the literature, dividing this data further by grade level made the links between the identity-supportive features and students' mathematics identities for this study population overall less clear. For the youngest students, describing access to the domain and playing integral roles were both quite rare. Expressing opportunities for self-expression was also less common amongst the kindergarten students than it was for the first and second graders, but the difference for that feature across grade level groups was smaller. The data by grade level is shown in Table 3.8.

Table 3.8

Experiences of Identity-Supportive Features, By Grade level

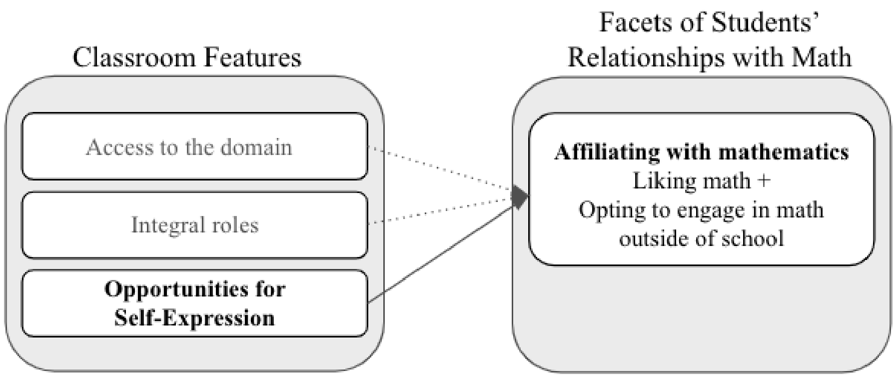
	Access to the domain	Integral roles	Opportunities for Self-Expression
Kindergarteners *n=9*	3 (33%)	2 (22%)	5 (56%)
1st graders *n=11*	7 (64%)	8 (73%)	8 (73%)
2nd graders *n=10*	7 (70%)	8 (80%)	7 (70%)
Total *n=30*	17 (57%)	18 (60%)	20 (67%)

Does this suggest that students in kindergarten are especially less likely to experience access to the domain or integral roles? This is an open question that warrants further exploration. It may be the case that Kindergarten classrooms are actually structured differently in such a way that the students experience limited roles and a more limited domain of mathematics. It is also possible that young students who are adapting to structured elementary school settings for the first time may be more prone to perceiving their roles as limited. Yet another explanation could be that these features are more difficult for children at this age to describe, thus limiting the data that I was able to collect. Regardless of the reason, what this data shows is that opportunities for self-expression could be a powerful feature to attend to when looking at how classrooms support students' mathematics identities in the early elementary grades; it both appears to support students' positive relationships with mathematics and is experienced more similarly across these grade level groups.

In summary, here I looked at three classroom features–access to the domain, playing integral roles, and opportunities for self-expression–and how those features related to students' emerging mathematics identities. Nasir and Hand (2008) found these variables to be linked for the high school students in their study. These links can be seen as arrows in Figure 3.3.

Figure 3.3

The Relationship Between Access to the Domain, Roles, Self-Expression, and Math Identities



Note. The arrows leading from *access to the domain* and *integral roles* are dotted to show that the links between these features and students' mathematics identities across this study population as a whole are unclear. In contrast, *opportunities for self-expression* is bolded to show its relevance in this data set.

Broadly speaking, it appears that these three features are in fact supportive of young children's relationships with mathematics. In particular, it seems that having experienced all three of these features made it more likely for a student to be affiliating with mathematics as they were experiencing it in their classrooms. However, access to the domain and playing integral roles were either less present or less easily described in the perceptions of kindergarteners, making the links between experiencing those features and the emerging math identities for the youngest students less clear. For this reason, the arrows leading from those features in Figure 3.3 are dotted instead of solid. In contrast, three of the four kindergarteners categorized as affiliating with mathematics did describe experiencing opportunities for self-expression in their math classroom. That in combination with the fact that a high percentage of students overall who experienced opportunities for self-expression were coded as affiliating with mathematics makes opportunities for self-expression a classroom feature worth paying attention to.

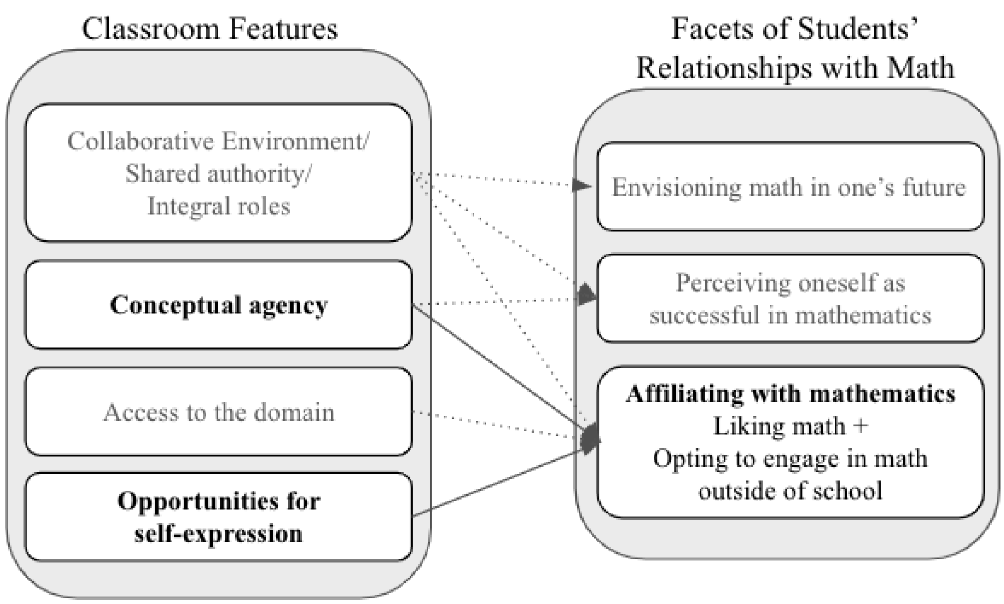
Discussion: Looking Across the Patterns And Asking Unanswered Questions

I began this chapter by describing Reese who was keenly aware that she was at the beginning of her mathematics journey. She had an older sister who she saw engaging with mathematics content that she had not yet learned, and she knew that she would get to learn more as she got older. The literature on mathematics identity has focused on students much older than Reese, and in this chapter I set out to explore the applicability of that work for early elementary school students. Having investigated the mathematics identity development patterns identified in the work of Boaler and Greeno (2000), Cobb et al. (2009), and Nasir and Hand (2008), I will conclude this chapter by discussing the findings when looking across all three phases of analysis and posing questions that are left unanswered by this work.

When looking at all 30 students in this study, an average of 68% of students (52% of the kindergarteners, 70% of the first graders, and 80% of the second graders) consistently aligned with the three general patterns from the literature that I described. First, students who perceived their classrooms as collaborative were more likely to like math and expect to continue learning and doing math within and beyond the confines of school. Second, students who perceived mathematical authority in their classroom as shared and who experienced conceptual agency were more likely to perceive themselves as successful and to begin to affiliate with mathematics–that is, take on learning mathematics as important and meaningful for themselves. And third, students who perceived themselves as having access to the domain, playing integral roles in their math classes, and having opportunities for self-expression were also more likely to affiliate with mathematics as they were experiencing it. Figure 3.4 shows all of these links. It also reflects the fact that in my analysis the features of a collaborative environment, shared authority, and integral roles were coded the same, and that I defined affiliating with mathematics as the combination of liking math and opting to engage in math outside of school.

Figure 3.4

The Relationship Between Classroom Features and Facets of Students' Relationships with Math



Note. Links between classroom features and facets of students' relationships with math that were not clear when the study data was divided by grade level are shown with dotted instead of solid lines. Variables that were less relevant or impactful for the study population are in gray. The bolded variables and solid arrows show meaningful links between classroom features and students' relationships with math for this study population.

Upon closer examination of the data by grade level group, many of these links became less clear. Some of the classroom features discussed here appeared to be perceived differently depending on grade level. The kindergarteners were more likely to perceive their classrooms as individualistic and it became more common for students to perceive their classrooms as collaborative as students got older. This aligned with their perceptions of teacher or shared authority and their sense of whether they played integral roles in their classrooms. Similarly, perceptions and descriptions of having access to the domain of mathematics were rare amongst the kindergarteners and became more common in the later grades. By pointing out these developmental shifts, I do not mean to suggest that these features are not important to students' emerging mathematics identities. Rather, they may not be the most suitable classroom features for looking at how classrooms support students' mathematics identities across this grade level band overall. For this reason, they are written in gray in Figure 3.4.

Relatedly, some of the facets of students' relationships with mathematics discussed in this chapter appear less useful for differentiating amongst students in the early elementary grades. Though a few students expected to stop learning math at a particular grade or when they were done with school, the vast majority of students in this study expected mathematics to be a continuing part of their future lives. This made it difficult to decipher if expecting to continue learning and doing mathematics was meaningfully related to any particular classroom feature. Similarly, nearly all of the students who participated in this study saw themselves as at least sometimes successful. Though feeling successful may be an important piece of young children's relationships with mathematics, for this study population, it does not offer a window into what makes students' relationships with mathematics different from each other. These two facets are also written in gray in Figure 3.4.

Experiences of conceptual agency and opportunities for self-expression were two classroom features that did appear to be varying and relevant to the students in this study across the three grade level groups. In total, there were 15 students who were coded as perceiving both conceptual agency and opportunities for self-expression. Of those students, 10 of them (67%) were also categorized as affiliating with mathematics. In this same group, four students were categorized as cooperating with mathematics and one as resisting. In contrast, amongst the students who experienced only one of these two features or neither of them, a smaller percentage of students was categorized as affiliating with mathematics and higher percentages of students were categorized as cooperating and resisting. Table 3.9 illustrates this data.

Table 3.9

Experiences of Conceptual Agency and Opportunities for Self-Expression, By Identity

	Affiliating	Cooperating	Resisting
Both conceptual agency and opportunities for self-expression *n=15*	10 (67%)	4 (27%)	1 (7%)
Either conceptual agency or opportunities for self-expression (but not both) *n=9*	4 (44%)	4 (44%)	1 (11%)
Conceptual agency but not opportunities for self-expression *n=5*	2 (40%)	2 (40%)	1 (20%)
Opportunities for self-expression but not conceptual agency *n=4*	2 (50%)	2 (50%)	0 (0%)
Neither conceptual agency nor opportunities for self-expression *n=6*	1 (17%)	4 (67%)	1 (17%)

This seems to suggest that these features are important to young childrens' relationships with mathematics, especially in combination with each other, and differing experiences with them may help us understand differences in childrens' emerging mathematics identities. In turn, these features, and the arrow linking them to affiliating with mathematics are bolded in Figure 3.4.

Undoubtedly, there is more that matters to young children and their relationships with mathematics than is captured by the classroom features explored in these particular studies. For one, the literature surrounding mathematics identity in older students is clear that mathematics learning experiences are impacted by students' gender, race, home language, and dis/ability status; Students who are male, white, and speak English as a primary language are systematically advantaged in mathematics (e.g. Martin, 2012; Barajas-Lopez, 2014; Langer-Osuna, 2011). These were not areas explicitly probed in my interviews. However, I did collect participants' demographic data around gender, race, and home language as identified by their caregivers, and in turn, I was able to calculate the ways in which students' relationships with mathematics, as categorized along the lines of affiliating, cooperating, or resisting, fell along the lines of these other components of students' identity. I chose to collapse the racial categories besides white together for these calculations. This is not intended to diminish the possible differences between racial groups, but rather to acknowledge that students of color, across racial lines, are frequently marginalized in mathematics learning environments. Further, the racial breakdown of the students of color is such that if they were not grouped together, each group would be so small that it would be unreasonable to draw any conclusions from the analysis.

Table 3.10

Study Population and Mathematics Identity, By Race, Gender, and Home Language

	Race		Gender		Home language	
	Students of color	White students	Female students	Male students	Multilingual students	English-only students
Study population *n=30*	15 (50%)	15 (50%)	18 (60%)	12 (40%)	8 (27%)	22 (73%)
Students affiliating with math *n=15*	8 (53%)	7 (46%)	9 (60%)	6 (40%)	4 (27%)	11 (73%)

As illustrated by Table 3.10, it turns out that the students who I categorized as affiliating with mathematics were nearly exactly reflective of the gender, racial, and linguistic breakdowns of the study population at large. Put otherwise, my data does not show particular identity groups as over or under-represented amongst the students with the strongest relationships with mathematics. That said, given the human development literature on identity (e.g. Rogers et al., 2012; Coll & Szalacha, 2004; Sabol et al., 2020), I fully expect that even with the youngest elementary school students, other important components of their identity matter to their relationships with mathematics. This warrants further study and exploration of how future interviews with young children may be better able to access these dimensions of their experiences.

Throughout my interviews with students, several other themes emerged repeatedly. Students talked about whether or not their teachers were “nice” and available to help them. Students talked about their relationships with their peers; sometimes these were supportive, while other times they were competitive or distracting. There were some parts of math classrooms–for example tests and homework–that were often mentioned as causing stress while others, such as games and choice activities, led to more positive feelings. It was also clear that how students engaged with mathematics with their families outside of school was relevant to their impressions of mathematics learning and mathematics at large. Are some of these features particularly impactful on young students' emerging mathematics identities? Do some follow developmental trends? What other features are yet to be uncovered by my data entirely? In the next chapter, I describe findings from my analysis of young children's reflections on the differences between learning mathematics in school buildings and at home, as they transitioned to remote school because of the onset of the COVID-19 pandemic. These conversations help to continue answering the question of what matters to young children's relationships with mathematics.

Chapter 4
"I'm not in the classroom at my table doing my math work. I'm not in the school": Learning From Children's Reflections on Their Transitions to Remote Schooling

In March 2020, teaching and learning changed dramatically as school buildings around the world closed and students found themselves engaged in schoolwork from home. Teachers and students shifted from spending hours together in classrooms to interacting through screens or work packets. The following fall, a new school year began, with some schools opening in person, many remaining entirely virtual, and some attempting a hybrid model. The COVID-19 pandemic upended life as we knew it, including schooling, for adults and children alike. In the quote used in the title for this chapter, Kindergartener Alice may have been stating the obvious; the physical location of where she was doing school in general and math in particular had changed. But the obvious was well worth stating because it was surprising, even shocking. Just a few weeks earlier, none of us, Alice included, could have predicted the shifts that she would soon be experiencing.

Because learning is fundamentally social and cultural, shifts in context, like the ones that unfolded in the spring of 2020, impact learning. People construct knowledge and meaning surrounded and influenced by other people, physical space, and the coinciding situations (Vygotsky, 1978; Wenger, 1998; Lowenthal & Muth, 2008). Further, shifts in context also impact identity development; as people engage in the particular practices of the communities in which and with which they are learning, they become particular kinds of people (Lave & Wenger, 1991; Wenger, 1998). This means that how schools and schooling are organized is of critical importance for students' opportunities to learn, and for the people they have the opportunities to become (Vygotsky, 1978; Lave & Wenger, 1991).

These intersections between context, learning, and identity are as true in mathematics as they are across other disciplines. As people learn mathematics, they form a relationship with the knowledge, skills, and practices they are learning, and that relationship becomes a part of their personhood (Wenger, 1998). People's mathematics identities–the stories they tell about their relationships with the discipline of mathematics–can develop and change over time as their interactions and contexts change (Langer-Osuna & Esmonde, 2017). This means that how mathematics classrooms and other spaces where learning mathematics happens are organized impacts not only the mathematics content that students have the opportunity to learn, but the relationships with mathematics that students have the opportunity to form and the types of mathematics identities that they have the opportunity to develop (e.g. Boaler & Greeno, 2000; Cobb et. al. 2009; Nasir & Hand, 2008).

Investigating context, and the related opportunities to be learners of mathematics (Nasir, 2021) is of utmost importance in mathematics because of the discipline's history of learning environments that marginalize and dehumanize. Sorting and tracking students based on problematically narrow evaluations, emphasizing uniform pacing guides across classes, holding singular algorithms and answers as correct, focusing on procedures and speed, and attempting to keep mathematics content "neutral" and separate from politics are all common-place practices in mathematics classrooms that limit students' opportunities to bring their whole selves into the space and in turn limit their opportunities to both learn mathematics and develop positive mathematics identities (Gutiérrez, 2018). Further, despite mathematics being frequently described as culturally universal (Shah, 2017), these dehumanizing structures are all the more present in the school mathematics experiences of historically minoritized and marginalized students including students of color (e.g. Berry, 2008; Martin, 2009, 2012; Shah, 2017), English language learners (e.g. Barjas-Lopes, 2014), and girls (e.g. Boaler, 2002; Langer-Osuna, 2011). This means that the study of context in mathematics education is deeply tied to issues of equity, and understanding features of classroom environments that support positive relationships with mathematics is one piece of opening up access to the discipline and supporting students who have historically been pushed away.

In the previous chapter, I examined patterns from the literature that explored relationships between students' perceptions of themselves and their mathematics classroom experiences. I argued that whether or not students felt successful and whether or not students saw mathematics as a part of their futures were not ideal metrics for understanding young students' emerging mathematics identities. Rather, it was more productive to look closely at the combination of whether students enjoyed mathematics and whether or not they brought mathematics into their own time outside of school as a way to understand students' developing relationships with the discipline. In addition, though experiencing classroom environments as collaborative and being granted access to mathematics as meaningful and expansive have been documented to support older students' positive mathematics identities (e.g. Boaler & Greeno, 2000; Cobb et. al, 2009; Nasir & Hand, 2008), for young learners those conditions were either less common or more difficult to perceive or articulate. In contrast, whether or not students felt that they had conceptual agency in mathematics and whether or not they had opportunities for self-expression appeared impactful across the Kindergarten through third grade participants in this study.

The analysis that led to those claims took an a priori approach, beginning with categories from existing literature (Miles et al., 2020). This made sense because my goal was explicitly deductive–to see if what the literature said was impactful for older students was also true for young children. Conversely, the analysis and resulting findings in this chapter take an inductive approach (Miles et al., 2020), beginning with the words of the participants themselves in order to continue responding to the question: What features of mathematics learning environments contribute to the development of positive mathematics identities in early elementary school children? This aligns with the broader goal of this dissertation to elevate the voices of young children themselves because they have been historically excluded from the literature. Young children are perceptive and reflective and can articulate complexity in their experiences (e.g. Bigler & Liben, 2006; Rogers et al., 2012; Sabol et al., 2020), and in turn it is important that we hear from them directly about their experiences learning mathematics. When people, young children included, have the opportunity to reflect on and make sense of their mathematics experiences, they tell stories that share who they think they are and are becoming as learners and doers of mathematics (Sfard & Prusak, 2005; Berry III, 2021). These stories offer us the opportunity to learn about what is important and meaningful to them.

Specifically, in this chapter I draw on the first round of interviews with participants, which took place in May and June of 2020. This was early in the COVID-19 pandemic, right after school buildings had closed for the first time. As I described earlier, this was a very unique moment in time; one draped in dramatic, contextual change. What the children expected school to look like that spring was not how it ended up looking. Studies in psychology have shown that when expectations are violated, people have particularly strong emotional and cognitive responses (Burgoon, 1993). This can be seen even in very young children who have been found to look longer (Perez & Geigenson, 2022) and learn more (Stahl & Feigenson, 2017) following surprising events. Applied to this study, it may be that the contextual changes that students experienced due to the onset of the COVID-19 pandemic heightened their feelings and insights into what worked and what did not, what they liked and what they did not, what was exciting, and what was a struggle in their mathematics classes. Because students were suddenly at home, they were well-positioned to remember and notice features of their learning environments, both in and now out of the classroom, that they may not have previously given much thought to. In other words, this moment of transition prompted students to articulate their insights and identify salient features of their mathematics learning experiences, and the contextual changes that students were experiencing made the stories that they told especially fruitful for understanding their perspectives.

During the first round of interviews with students, I asked students to compare their experiences learning mathematics in school buildings and at home. Were there things they liked better about each setting? What about each setting helped them learn? For this phase of analysis, I gathered student responses to these specific questions. Then, I identified the central themes of each relevant student quote, and clustered those themes into codes (Miles et al., 2020). I cycled back to the students' quotes using the new codes, and iterated between coding the students' quotes and clarifying the code definitions. The final codes for what emerged from student reflections as salient features of their learning environments included: *being with peers, access to support, access to learning, physical comfort, pace/schedule,* and *having fun.* Definitions and examples of student quotes for each code are found in Table 2.9 in Chapter 2. These codes are not specific to whether students were learning mathematics in school or at home, but rather emerged as salient features across both contexts with varying frequencies.

Through my analysis of the interview transcripts, I found that whether learning mathematics physically in school buildings or from home, the young students in this study cared about relationships, support, flexibility, and fun. They wanted to feel like they could learn and succeed. They wanted agency within their math learning experiences. For some students, these needs were better met at school. In particular, students frequently cited the importance of peer interactions as a strength of being in school buildings. However, for some students these same needs were better met at home. This was most often the case for students who expressed a desire for more control over their work pace and schedule. These findings illuminate a need to move away from the in-person versus virtual schooling binary that has dominated the education discourse in the media and instead focus on how, no matter the learning environment, educators can better meet students' needs and create positive experiences with mathematics.

In the sections that follow, I describe how I reached these claims. First, I unpack what students named as benefits and challenges of learning mathematics in school buildings. Then, I compare that to what students described as the benefits and challenges of learning mathematics from their homes while school was remote. After that, I describe differences across grade level groups. Finally, I end with unanswered questions and key takeaways for educators that emphasize how we can use insights from young students during this time of cultural upheaval to shape learning environments that support positive mathematics identity development for more students, regardless of where learning is physically happening.

Students' Reflections on Learning Mathematics at School

As a subject, math was somewhere in the middle of Agnes's list. It was not the time of day that she looked forward to most in second grade–that was reading, which she loved–but it was fine. Sometimes, it was even fun. There were some games that she played in math class that she liked, especially the card game *Sushi Go* and the computer game *Prodigy*, but she found the workbook pages hard and did not enjoy completing them. She explained to me that her teacher did not expect her math work to be perfect–she anticipated that she would make mistakes–but she did expect her to "to actually put some thought into it, to actually work on it." When I asked Agnes about the difference between learning math at school and at home she said, "I like the school a little bit better because I like working with lots of other people."

Just as Agnes described, *being with peers* was the theme that emerged most frequently in conversations with students as a benefit of learning math in school buildings. It was mentioned by three Kindergartners, four first graders, and three second graders (total n=10). Sometimes, students talked about wanting to be with or see their friends without specifying why (n=6), and other times students specifically described wanting to be able to collaborate or work together with their classmates (n=4). For the students who talked about being with peers as an important feature of learning math in classrooms, there was a sense in their reflections that they saw strength in learning mathematics in the physical presence of their classmates, and they were not experiencing the same sense of community at home.

Along with *being with peers*, *access to support* was also an important feature and strength of being physically in school buildings for several students (n=6). Students were labeled with this as a priority when they talked about wanting, needing, or getting help from someone or something in their environment. Specifically, students cited getting support from their teachers, peers, and materials in their classrooms. Sigrid, a first grader, explained that her teacher would guide them through problem-solving when she was in her classroom, which her parents were not always able to do. Similarly, Meera, a second grader, shared that even though her mom was a preschool teacher, she did not know the same things that her teacher knew. As for material support, second grader Gal described how helpful it was to have a printer at school so that each student could get the papers they needed.

Louise was a second grader who missed the help she used to get from her friends while at school. This example is different from those coded as *being with peers* because she specifically described her friends as a resource for support when she needed help. It can be pedagogically challenging for teachers to create an environment in which peer interactions are supportive of learning (Smith & Stein, 2018; Kazemi & Hintz, 2014), which makes it notable that Louise specified this as a benefit of being in her classroom. Overall, Louise had some mixed feelings about mathematics. She liked math because it was challenging, but she didn't want it to be too challenging, because then she got frustrated. She really loved math in first grade when she felt especially successful, but this year was a little different. Louise explained, "I get very annoyed sometimes when I can't do it." She clarified that she sometimes felt that way before her school building closed also–it was not as if she only started feeling frustrated after the transition to remote learning. But, she did feel like things had gotten harder. In her words, "I like doing math at school 'cause my friends kind of helped me with my work when I needed help." When Louise was at school, she had her friends' support when she got stuck, and that was not the case anymore.

Related to but distinct from *access to support* was when students talked about having *access to learning* (n=6). This emerged when students talked about being able to learn more in their classrooms (*access to learning: quantity)*, or it being easier to learn and be successful while they were in school (*access to learning: success*). In these cases, students did not elaborate on the features that came together to make learning more possible while they were physically in schools, but it was clear that in shifting to remote learning they felt like they were learning less or that learning was harder. As first grader Darius succinctly declared, “School teached me more.” Second grader Meera was a little more specific when she said, “I like it better in school because I can learn a lot more strategies.”

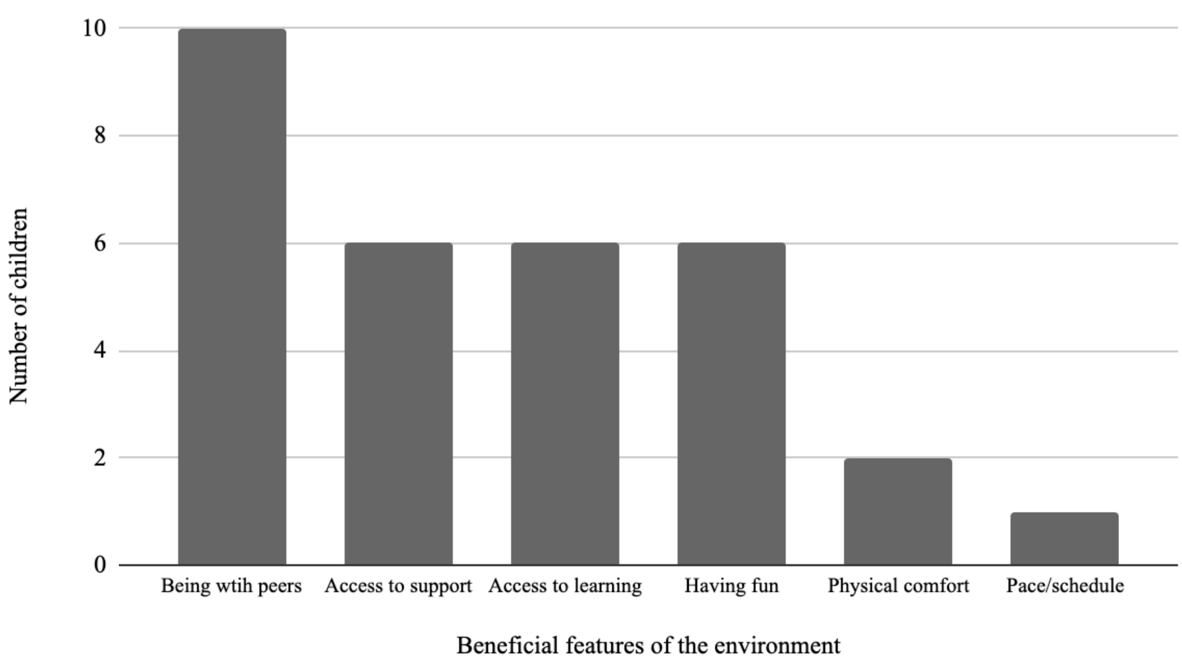
Six students noted *having fun* learning math at school. In most of these cases, students named specific math activities or class structures that they found fun that they were no longer able to do. Examples included coloring (now the work was on a screen so they didn’t use markers), playing particular games, and having math structured as rotations through stations with a different activity at each station. Given the documented impact of students’ dispositions towards mathematics on their learning and on their developing mathematics identities (e.g. Gresalfi & Cobb, 2006), students’ perceptions of fun is not to be discounted. When students enjoy mathematics and find joy in mathematics, they may feel more motivated and experience a deeper level of engagement (Cobb & Hodge, 2002; Ames & Archer, 1988; Gutiérrez, 2018).

Finally, though less common, students also discussed *physical comfort* (n=2) and *pace/schedule* (n=1) as benefits of being in the classroom. The students who talked about physical comfort explained that in their classrooms, their teachers offered them a variety of seating options. They enjoyed the chance to move around and sit in different types of chairs and in different locations; at home, they felt confined to one spot in front of their computers. The student who was labeled with *pace/schedule* as a feature he felt was a strength of learning in schools said that he felt like he had to spend too much time on his math work now. When he used to go to school, he finished nearly all of his math work while he was in math class and rarely had homework. But now, all of his math work was homework and it took him a long time to complete.

In summary, many students noted benefits of learning mathematics physically at school. Specifically, 21 students responded that there was something (or some things) that they liked better about learning math in their classrooms. Of those 21 students, some named one feature, while others named several. *Being with peers* was the most commonly named feature, followed by *access to support, access to learning,* and *having fun*. The features that students described as benefits of learning mathematics at school are displayed in Figure 4.1 from left to right in order of prevalence.

Figure 4.1

Benefits of Learning Mathematics in School, According to Study Participants



In a few instances, students named challenges with learning math at school. It is worth noting that in the interview protocol, I asked students about the benefits of each context–were there things they liked better about learning in school or learning at home? In turn, the majority of comments were framed in the positive. Still, it is interesting to look at the moments when students described a drawback, in particular because those drawbacks mirrored what other students found as benefits. For example, there were two students who talked about having limited physical comfort in school. First grader Suriyah explained, “In the classroom, we have to either stay in our seats or go in certain places that my teacher allows me to go to.” Similarly, second grader Asher shared, “You always have to sit at your own table.” And, unlike the several students who thought it was more fun to learn math in school, Kindergartener Lucas thought it was less fun because he preferred using ipads. These students saw the same features of their school learning environments in a negative light as other students saw positively, and they described these drawbacks in contrast to what they saw as benefits of learning at home.

Students’ Reflections on Learning Mathematics at Home

As a second grader, Sebastian was a serious math enthusiast. When his dad first signed him up to participate in this study, he told me in an email, “Sebastian would absolutely love to talk to you about math. There isn’t much he’d rather talk about.” Sure enough, Sebastian was extremely eager to talk about math. He described in great length the strategy he used for dividing large numbers in his head and he reiterated many times how much he liked challenges. Now that he was doing his math work at home, Sebastian felt freed to learn at a pace that was comfortable for him. He explained that at home, “You don’t have to go at the same pace, you can just go as fast as you want to. And you don’t have to finish it all at one time. Like, that’s what you usually have to do." For Sebastian, there were two pieces to this newfound flexibility. First, he could move ahead when he was ready. In school, he found himself often finishing his work before his peers and feeling bored while he waited to be allowed to move on to whatever was next. Second, if he wanted a break he could take one. At home, Sebastian appreciated that he did not have to complete his math work in one sitting but instead could engage with it more flexibly when he wanted to.

Pace and *schedule* came up in conversations with several students (n=6) as benefits of learning mathematics at home. Most of these students (n=4) emphasized that they liked not being rushed, being able to take breaks, and being able to do their work over a period of time that worked for them. Despite research that shows that timed activities in mathematics cause stress and anxiety, they remain prevalent in many classrooms (Boaler, 2014). Even beyond timed activities specifically, the way in which school is often structured leaves students required to complete math work within the very specific boundaries of math class. What if the timing of math class does not align with when a student feels mentally and emotionally prepared to focus and problem-solve? It is not unusual, for example, for literacy to be scheduled first thing in the morning in elementary classrooms, leaving math for later in the afternoon when students may be tired or hungry (Pope, 2016; Dunn et al., 1987). As it turned out, learning math from home helped to solve that problem for some students by allowing them to work at a pace and on a schedule that fit their needs.

Just as students talked about their *access to support* at school, some students (n=4) described feeling like they had *access to support* at home. Specifically, students talked about their family members helping them. All four of these students named not just their parents, but also their siblings. Kindergartener Claire explained that she likes being at home because she can get help from her "mom or dad or sister." Agnes shared that while she could do most of her work independently, she was glad to be able to have both her mom and her older sister closeby if she had "a really hard question." Other students described navigating which of their family members were available at different times. Families have always played a crucial role in children's mathematics learning, and yet their role is often underappreciated (e.g. Civil & Bernier, 2006; Civil, 2020). For these students, transitioning to learning from home highlighted for them the value of their families' support.

Four students felt like they had more *access to learning* from home than they did in school. Their reasons for feeling this way varied. One student shared that his teacher had been sending him personally more challenging work, which she had not done when schools were open, and this was allowing him to learn more. In other words, he appreciated that his teacher was differentiating for him more than she used to. Another student felt like she was better able to learn because she had fewer distractions at home. For two second grade students, Kira and Asher, they felt like they were better able to learn and be successful because of the way their work at home was being evaluated. In our interview, Asher talked a lot about the stress that the quizzes in school caused him. He said that when he turned them in, he always knew that he was going to get a bad grade. Now, at home things were different. With the app that he was using at home, if he got answers wrong or did not finish on time, he had the chance to try again and submit new answers. Because of this, Asher felt like he could actually learn and improve, and he felt like he could be successful in a way that he could not when he was at school. Relatedly, Kira shared that at school sometimes she had to do her work all over again if her teacher thought that it was not neat enough. But at home, she explained, “They won’t judge you on your handwriting.” Kira felt like she could focus on learning math instead of handwriting, and she was glad to have the opportunity for success.

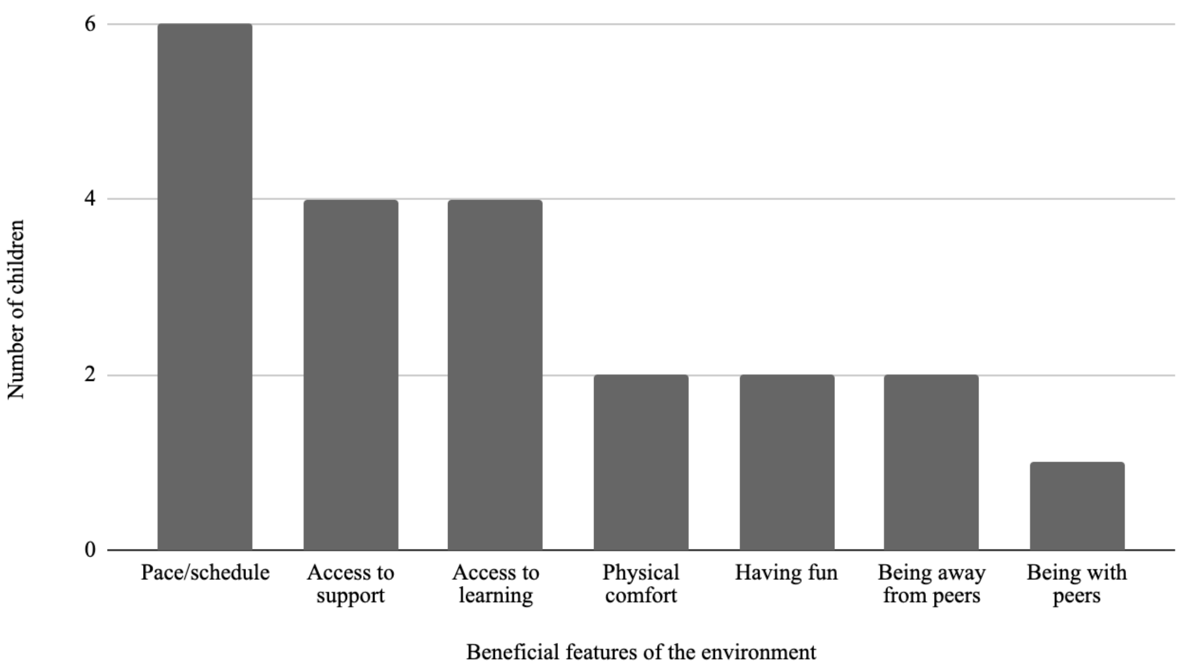
Less common were students who said that *having fun* (n=2) and *physical comfort* (n=2) were strengths of learning mathematics from home. The students who thought that it was more fun to learn math at home enjoyed the apps and games they were being assigned. The students who named physical comfort as a benefit talked about having seating choices at home that they did not have at school. One student talked about being able to move around and switch seats whenever she wanted. Ivy, a first grader, shared that she liked learning math work from home because she could “lay down on the floor,” which she thought was “actually very, very, very comfortable.”

Finally, *being with peers* also emerged in conversations about what it was like to learn from home. One student explained that even though she could not be with her friends in person, she still could see them through the screen which was “also great!” However, for two other students, it was actually the sub-code of *not* being with peers that was the benefit of learning math from home instead of from school. Second grader Ling explained, “Sometimes when I’m doing math, I like doing it alone.” She said she did not get the chance to work alone at school, but now she did. For Kindergartener Owen, being without peers was also a benefit of being at home. In his words, “I don’t have to be at school where every kid is yelling in my face like, ‘Hey that’s not the right number!’” Even though he still did not like math, Owen much preferred working with his mom from being with his peers.

In summary, many students noted benefits of learning mathematics at home. Specifically, 18 students responded that there was something (or some things) that they liked better about learning math remotely. Of those 18 students, 15 of them named one feature and three named two features. Having more control over their *pace* and *schedule* was the most commonly named feature, followed by *access to support* and *access to learning.* The features that students described as benefits of learning mathematics at home are displayed in Figure 4.2 from left to right in order of prevalence.

Figure 4.2

Benefits of Learning Mathematics at Home, According to Study Participants



Just as students named some challenges with learning from school, students also named challenges with learning from home. Again, I will reiterate that in the interview protocol I asked students about the differences between the contexts and about the benefits of each context, but did not specifically prompt for drawbacks or challenges, so these emerged organically. Interestingly, the students who described things that they found difficult about learning math from home mentioned the same features that other students saw as benefits. Several students (n=6) described feeling like they had limited access to support from home. Some of these students (n=2) specifically talked about wanting easier access to their teacher. Other students (n=2) shared that their family was home with them but were too busy to help or did not have the right skillset. Still other students talked about not having the support of peers (n=1) or of classroom materials (n=1) like manipulatives, posters on the wall, or printers to print out activities.

There were also several students who felt that they had less access to learning at home for a variety of reasons. Three students felt like there were too many distractions at home. As first grader Ivy explained, “I do all my math in my room, and that's mostly where my toys are, so I get distracted by my toys." Two other students thought that they were learning less because their teacher was sending them easier work. Second grader Torrin explained that the attendance in his virtual classroom seemed less predictable, so he imagined that was why his teacher was focusing on reviewing things they had already learned. Finally, second grader Miguel just thought that it was harder to learn online. “I’m more used to doing it with pencil instead of typing,” he said.

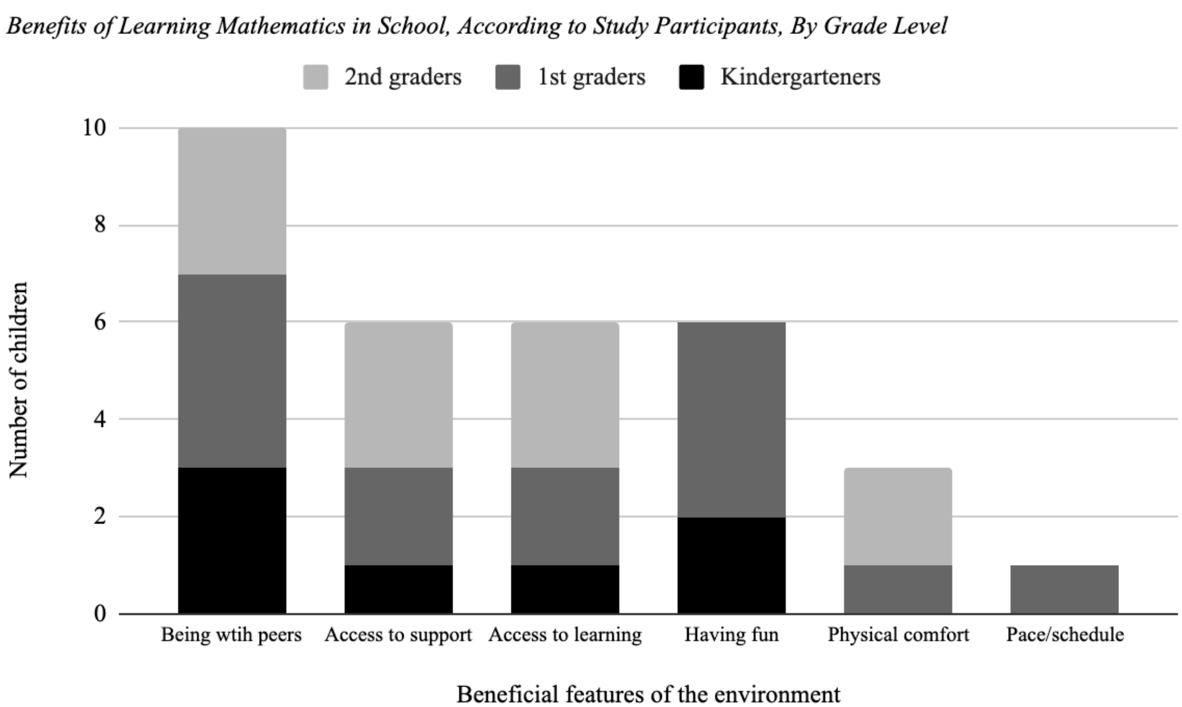
Lastly, two students found it less physically comfortable to work at home, three found the pace or schedule of being at home to be a challenge, and four students talked about being away from peers as drawbacks of learning mathematics outside of their physical school buildings. Again, it is notable that in all of these cases, there were students who said exactly the opposite; what these students saw as drawbacks to learning from home, other students saw as strengths.

Looking Across Grade Levels

In the previous chapter's analysis, there were a few notable differences across the grade levels of students participating in this study. In particular, the Kindergarteners were less likely to perceive, experience, or articulate their classrooms as collaborative environments than the students who began the study in first and second grade, and similarly they were less likely to perceive, experience, or articulate having access to mathematics as a domain that is meaningful and expansive outside of school. The Kindergarteners were also less likely to describe mathematics as a certain part of their future beyond school; doing so was nearly universal for the older students.

In this chapter, there were again some grade level differences, but they were more nuanced. For example, when discussing the benefits of learning mathematics in school buildings, approximately equal numbers of students in Kindergarten, first, and second grade talked about being with their peers. However, it was only older students who elevated that priority to the specificity of talking about collaboration and working together instead of just being together. And, while access to support and access to learning were discussed as important features of learning from school across the grade level groups, they were more common with the first and second graders than they were with the kindergarteners. Kindergarteners and first graders were more likely to mention having fun. These differences can be seen in Figure 4.3.

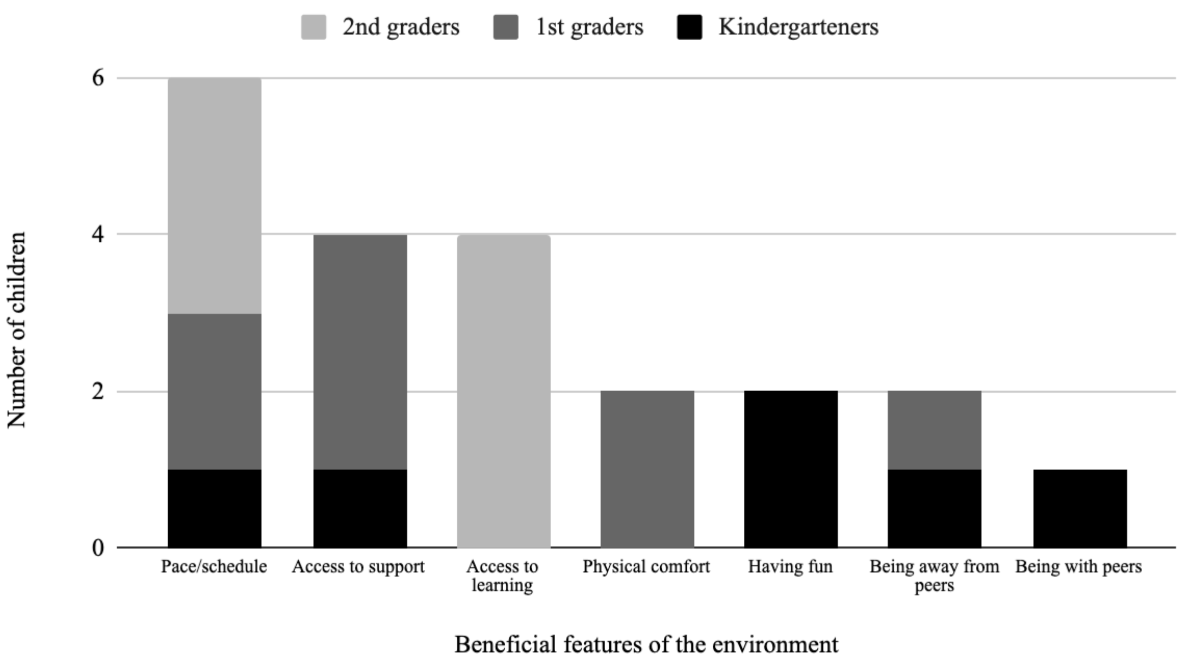
Figure 4.3



Similarly, students' comments about the benefits of learning from home often spanned the grade level groups, but in differing proportions. Though kindergarteners did mention having access to family support and appreciating the pace and schedule of learning from home, these features were more commonly named as benefits by older students. There were no kindergarten or first grade students who described having more access to learning while at home, and only older students talked about physical comfort in either setting. When it came to learning from home, only kindergarteners named having fun as a benefit. These grade level comparisons can be seen in Figure 4.4. Being *away* from peers was not shown on the previous graph because it was not named as a benefit of learning at school, but it was added to this graph because it was discussed by two students and was a distinct priority from being with peers.

Figure 4.4

Benefits of Learning Mathematics at Home, According to Study Participants, By Grade Level



Across discussions of both learning contexts, more older students focused on their *access to learning* and *access to support*. It may be that the older students, having generally had more experience with school-based mathematics, were more attuned to specific things that they needed in order to learn and be successful. In contrast, across both contexts the Kindergarteners were more likely to focus on *peers* and on *having fun*. These differences may reflect differences in how learning is organized for Kindergarten in comparison to first grade and beyond. Kindergarten often carries some of the spirit of play-based learning from early childhood, and that may dissipate even one year later. Despite these differences, this data highlights that students across all three grade level groups share several priorities for their learning environments. The study participants in Kindergarten, first, and second grade all emphasized a desire for agency over their activities, their interactions with peers, and the physical configurations of the rooms in which they learn. In addition, this data reiterates that even the youngest students can articulate a range of needs and desires for conditions that would make a learning environment supportive of positive learning experiences for themselves.

Discussion: Differing Responses But Shared Priorities

When asked to compare their mathematics learning environments in the classroom and at home, the students' responses were not uniform. In fact, often they appeared in direct contradiction to each other. One student would say that they learned math better from home because they were able to lay on the floor and shift seats when they wanted to, and then the next student would describe how helpful it was to have a variety of types of seating in their classroom. One student would talk about having more access to support at school because of the presence of her teacher, followed by another student who felt more supported at home with his family around. One student would describe learning more at home because his teacher was differentiating more in remote learning than she had previously, while the next student would share that she had access to more challenging work and therefore more learning when in the classroom.

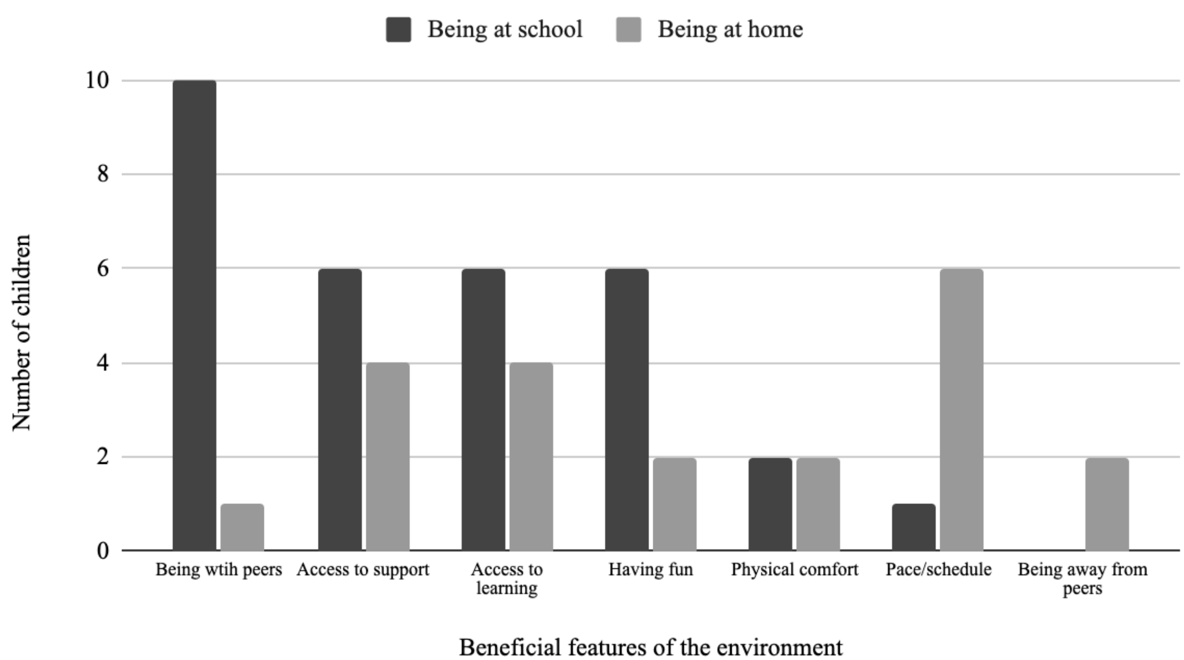
These types of individual differences were particularly evident within a small group of five students who specifically said that overall they preferred learning mathematics at home; that home is the context they would choose. I did not ask students to pick a preferred context in the interviews, but these five students on their own accord said that they liked learning mathematics at home better. However, from that point on their responses were quite varied. They all had different reasons for feeling that way. One student enjoyed using the app he was asked to use at home. A second student was glad to escape the social dynamic at school. The third student appreciated being able to take breaks. The fourth student found doing math at his table at school to be physically uncomfortable and he was glad to have second chances on quizzes at home. And finally, the fifth student was thankful that his teacher was sending him more challenging work at home than she used to give him at school. Though these students shared the perspective of preferring to learn math from their homes, they named a wide diversity of reasons. Each of these students would have needed something different in order to make learning mathematics in school a more positive experience.

It is not surprising that the young students in this study were not monolithic. Each student's experiences were their own and their stories were their stories to tell. However, despite this variation, themes emerged across students' responses that highlighted shared priorities. Whether learning mathematics in their classrooms or their living rooms, the young participants in this study cared about their relationships with their peers. They wanted support when they felt stuck, access to the people and tools they needed in order to learn, physical comfort, and control over the pace and schedule at which they did their work. They also wanted to have fun while learning. The students expressed a desire for agency; they wanted to be able to make decisions that would help them learn mathematics and enjoy doing so.

Sometimes, these needs were better met for students when they were physically in their classrooms. Students named being with peers, access to support, access to learning, and having fun as primary beneficial features of learning mathematics at school. But in other ways and for other students, these needs were better met when students were at home. Specifically, students described having control over their pace and schedule, having access to support via family members, and having access to learning as beneficial features of learning mathematics at home. Put otherwise, what students cared about crossed the boundaries of the physical contexts in which they were learning. Figure 4.5 shows what students described as benefits of each setting, side-by-side.

Figure 4.5

Benefits of Learning Math at School and at Home, By Study Participants



The priorities that emerged in conversations with students in this study resonate with what elementary school teacher and teacher educator Olga Torres found mattered to her third through fifth grade math students (2020). Together, she and her students created a document they called *The Rights of the Learner.* The list named what her students needed, and therefore deserved, in order to learn and be successful in mathematics. It included "the right to feel confused, to make mistakes, to do and represent only what makes sense to you, to engage in conversations that allow you to ask questions, share ideas, and listen to the thinking of others to support your learning, and to feel safe and respected," (Torres, 2020). Though Torres' students were a bit older than the students in this study, several of these rights align with the responses students in this study offered around having access to support and access to learning. Also, both groups of students shared the value of having agency and meaningful peer relationships in math class. It may be that the priorities expressed by the students in this study align with what other students, even in other settings, would also find important for mathematics learning environments.

Early in this chapter I emphasized that unpacking the features of the contexts in which students learn mathematics is important in part because of the history of dehumanizing math classroom environments. Rehumanizing the spaces in which students learn mathematics requires "recoupling mathematics with connection, joy, and belonging." (Gutiérrez, 2018, p. 4). The *Rights of the Learner* that Torres' (2020) students named speak to these requirements. They do so through a focus on mathematical sensemaking; they ensure that students can think and express ideas without fear of negative judgment. This is key to a sense of belonging; respecting a person's ideas is a way of showing that the person belongs, of showing that they and their ideas are worthy of being in the community. In addition, nurturing the kind of shared discourse that Torres' students name is a way of forming connections. This involves learning to listen to each other deeply. Similarly, in my interviews, students told me that their experiences learning with mathematics were more positive when they felt supported- surrounded by the people, materials, time, and space that they need to be successful. In other words, they were seeking connection and belonging. Additionally, the students in this study described more positive mathematics learning experiences when they had fun and when the activities they were engaging in brought them joy. Is there any reason that these needs cannot be met both in a physical classroom and in online spaces for learning? I argue no.

When schools closed in the spring of 2020, the change to remote learning was in the context of a global pandemic involving a new and unknown illness and its corresponding anxiety and trauma. Neither school leaders, teachers, parents, nor students had time to prepare for the switch. This led to many challenges including unreliable internet and device access, unclear expectations for participation, and complex balancing of school, work and family life (Jessup et al, 2020). The rapid, emergency shift in learning context was a situation that none of us chose and that none of us hope to have to replay. But, that does not mean that we cannot learn from what happened and use students' perspectives on their experiences to imagine possibilities for what learning online and in-person could look like for students in the future. Instead of focusing on returning to what math classes looked like before the pandemic, we can use students' experiences during this unique moment in time to grow and improve towards something better.

If we know that relationships matter to students–between students and students, students and teachers, students and families–, how can we leverage the spaces students are learning in to strengthen those relationships? Online, teachers can create multiple pathways for students to be in relationship with them and with each other including collaborative documents, breakout rooms, opportunities to chat via text and voice, and longer form discussion forums (Borba et al., 2021; Jessup et. al., 2020). Learning online can also open up new opportunities for families to be present with the mathematics learning as it is happening. Both online and in classrooms, teachers can offer opportunities for working alone, in partnerships, small groups, and as a whole class. Educators can support students in developing the skills needed for positive and productive collaboration (Webb et al., 2019; Kasemi & Hintz, 2014) and can offer a variety of ways of participating both verbally and nonverbally, sharing authority across members of the classroom community (Gutiérrez, 2018).

If we know that students want agency over their schedule and the pace at which they work, how can educators offer that control to students both in classrooms and online? Research on asynchronous courses notes that when students are learning on their own, they can take the time that they need to process the content (Hodge-Zickerman, et. al., 2021). But what about when students are learning synchronously either virtually or in school? Eliminating strict time limits on particular assignments would be a move in the right direction. Low-floor high-ceiling tasks offer all students the opportunity to enter the task and productively problem-solve while allowing students to extend their thinking in ways that fit their skills and make sense to them (Boaler, 2022). Bringing more low-floor high-ceiling tasks into the math classroom, whether virtual or in-person, would help shift the goal from having to finish a certain amount of problems within a specific time frame to contributing meaningfully to collective understanding. Teachers could offer students more agency over the order in which they complete assignments, and how and when they extend those assignments based on their own interests.

The contexts in which students learn mathematics are influential on students' learning of mathematics content and on the mathematics learners that they have the opportunities to become (e.g. Boaler & Greeno, 2000; Cobb et. al. 2009; Nasir & Hand, 2008). When a student does not experience connection, belonging, and joy in mathematics learning spaces, they may be less likely to take on learning mathematics as a meaningful part of who they are (Gutiérrez, 2018). Further, the opposite can happen, with students feeling actively excluded from the discipline. The ways in which features of learning environments come together is part of what creates the potential for or absence of those feelings of connection, belonging, and joy. The findings in this chapter highlight that it is specific features, not location, that matter. So much discourse surrounding schooling during the COVID-19 pandemic has asked (or assumed) why it is better for children to be learning in classrooms than at home, but that is the wrong question. Instead, we should be asking how the features that we know to be supportive of students' learning can be integrated into any learning setting. The students in this study described shared priorities for these features that across the board emphasized students' desires for agency. Young children know a great amount about what they need to learn and be successful in mathematics; and as educators, we can learn from their knowledge.

As Rochelle Gutiérrez wrote, "Not until we seek to stand in the shoes of our students, to understand their conceptions, will we be on the path toward recognizing and embracing their humanity," (2018, p. 2). The findings described in this chapter only scratch the surface of doing this, but they are a start. In order to rehumanize mathematics learning spaces, whether online or in-person, it is critical that we listen to and learn from students' stories. In doing so, we can move towards designing environments for learning mathematics that open up opportunities for more young students to form positive relationships with mathematics and that expand the possibilities for the mathematics learners that young students can become.

Chapter 5
"Well, what I like doing is crazy math problems": Exploring Child-Initiated Mathematics Activities

Though he was in second grade at the time we met, Sebastian–the student quoted in the title of this chapter–recalled an example of one of his "crazy math problems" that he made up and solved on his own the previous year. "I remember the divisor, it was 3,776, I will never forget that, and it was divided by like 64…I did that all in my head." Sebastian leaned forward from his reclined position on the couch and started typing. "Oh yes, it was 59, that's right," he told me, after having checked on the computer's calculator. Sebastian also shared that once he had figured out the answer, he had challenged his dad with the same question. "He got it right, [but] it took him a while," he said with pride.

Given how pervasive it is for people to carry negative mathematics identities and for people to disassociate from mathematics when they are beyond mandatory schooling (Boaler & Greeno, 2000), one might expect students to close the doors on mathematics when the school day is done. And, that may be the case for many students. But when I asked the young students in this study if they ever do math "just because they feel like it, not because anyone is telling them to," over half of them (n=17), including Sebastian, said yes. In this chapter, I dive into these positive responses and investigate the types of mathematics activities that the children in this study initiated on their own time, or what I call, *child-initiated mathematics activities.* These activities showcase students' conceptions of mathematics because they reveal what students consider to be mathematical, and they highlight what types of mathematics students choose to engage in when they are the authors of their experiences.

When these students opted to engage in mathematical activity on their own time and in their own ways, they exercised agency over their mathematical experiences. In mathematics classrooms, the ways that students can express agency are impactful on the collective norms of the space, on students' relationships with those norms, and on their relationships with discipline of mathematics (Cobb et al., 2009). Classrooms that limit students' expressions of agency to reproducing and using established solution methods maintain a more narrow view of competency, thus limiting students' opportunities to develop a sense of personal success (Cobb, et al., 2009). However, in classrooms in which students have the opportunity to express conceptual agency, where students can explore and develop meanings and connections between concepts and choose and share problem-solving methods that make sense to them, mathematical competency is opened up, allowing more students to see, value, and be appreciated for their own mathematical contributions (Louie, 2018; Ruef, 2021; Torres, 2020; Cobb et al., 2009). Further, in mathematics classrooms, the activities in which students engage and the ways in which they are asked to engage in them contribute both to their perceptions of what it means to learn mathematics and what it means to be a mathematics learner (Lave & Wenger, 1991; Nasir, 2021). When mathematics activities emphasize and require rote procedures, students who are seeking spaces in which thought and agency are valued reject the discipline as not for them because they come to see mathematics as a passive activity (Boaler & Greeno, 2000). Students are more likely to embrace mathematics as a part of who they are if they are given the opportunity to express themselves fully and contribute meaningfully to the collective learning in their mathematics classes (Nasir & Hand, 2008).

The findings discussed in the previous two chapters reiterate the importance of agency in mathematics for the early elementary school students who participated in this study. In the first findings chapter, I showed that whether or not students felt that they had conceptual agency in mathematics class appeared impactful on students' relationships with mathematics when those relationships were evaluated by whether or not students liked math and whether or not they opted to engage in math outside of school. Twenty of the students in the study described experiencing conceptual agency in their classrooms; this typically looked like getting to choose the strategies they used when problem-solving and/or getting to share and compare their thinking with their peers. Of those 20 students, 17 said they liked math, and 12 of those students also responded that they opted to do math on their own. These numbers represent larger percentages than the percentage of students falling into those same categories who did not describe experiencing conceptual agency and in the study population at large. This can be seen in Table 5.1.

Table 5.1

Students Who Described Experiencing Conceptual Agency

	Liked math	Opted to engage in math 'just because'	Liked math + opted to engage in math 'just because'
Students who described experiencing conceptual agency *n=20*	17 (85%)	12 (60%)	12 (60%)
Students who did not describe experiencing conceptual agency *n=10*	7 (70%)	5 (50%)	3 (30%)
Study population *n=30*	24 (80%)	17 (57%)	15 (50%)

Described in the reverse, students who perceived themselves as experiencing conceptual agency accounted for 71% of the students who liked math, 71% of the students who engaged in child-initiated mathematics activities, and 80% of students who fell into both of those categories.

In the second findings chapter, I delineated priorities for mathematics learning environments that emerged through conversations with students around the benefits of learning mathematics at home and learning mathematics at school. Several of these priorities involved other expressions of agency. For example, students wanted to be able to move and choose seating that was comfortable for them. Students also wanted to be able to determine a schedule and work flow that allowed them to focus when they felt focused and take breaks when they needed breaks. In addition, students expressed the desire to ask questions and access the support they needed when they felt stumped, both from people and materials.

Thus far, I have centered the discussion of agency on what is happening in school in mathematics classrooms, but of course, mathematics lives beyond classrooms. A significant body of literature has taken a sociocultural approach to explore the ways in which mathematics is embedded into everyday activities and cultural practices. For example, children make complex calculations while selling candy on the streets of Brazil (Saxe, 1988), players engage in problem-solving and strategy development while playing games such as dominoes (Nasir, 2005), and families participate in mathematics through gardening, construction, the arts, and other household daily activities (Civil, 2002). Further, mathematics is integral to many after-school activities including basketball (Nasir & Hand, 2008), tinkering (Echevarria, 2021), and digital design (Pinkard et al., 2017). Like in classrooms, agency is also important in these settings; when participants have the opportunity to make choices around their participation that reflect their needs, desires, and identities, they experience increased motivation and engagement (Hull & Greeno, 2002; Nasir & Hand, 2008).

All 30 students who participated in this study undoubtedly engaged in mathematics and mathematical thinking in a multitude of ways in their lives outside of school including through daily activities at home, creative play, or athletics. These experiences are valuable, meaningful, and impactful on the development of their mathematics identities (Hull & Greeno, 2006). However, understanding the richness of these activities would have required other methodologies such as ethnographic observations that were outside of the scope of this study. Instead, here I investigated a more narrow slice of participants' mathematical activity: the times in which students described actively and knowingly opting to engage in mathematics just because they wanted to. These child-initiated mathematics activities reveal displays of children's agency. They also offer us a window into children's conceptions of mathematics because in describing these activities, children themselves are defining and explaining what they see as mathematical. In this chapter, I explore what it was that students described doing in these moments and their reasons for doing so in order to continue building an understanding of how young children are conceiving of mathematics and their relationships to it. Specifically, the findings in this chapter respond to the research question: What can we learn about early elementary school children's emerging mathematics identities from the mathematics activities that they initiate? I uncover in what ways, for what purposes, and with whom young children opt to engage in mathematical activity outside of school and look at how these child-initiated mathematical activities compare to students' descriptions of the activities in their mathematics classes.

To answer these questions, I first noted which students, in any of the three interviews, said that sometimes they did math just because they wanted to, not because anyone was telling them to. There were 17 students who responded that yes, sometimes they did math “just because.” For these students, I gathered their responses about what those self-initiated activities were, both when school was and was not in session. For the same set of students, I also collected their responses to the questions, “Can you tell me about what math time is like in your classroom?” and “What activities do you do during math time?” These questions were asked in the spring and fall interviews, but not during the summer when students were on summer break.

I began analysis by identifying the types of activities that students described engaging in, both on their own and in school, in each relevant quote. I clustered similar activities into groups, and these groups became codes (Miles et al., 2020). I iterated between the student quotes and the codes to clarify the code definitions until I had assigned a specific code to each student quote. The final codes for the mathematical activities that emerged from student responses are found in Chapter 2, Table 2.10 along with definitions and examples of student quotes for each code. These codes include both activities from students’ descriptions of the mathematics they opted to do and the mathematics that was happening in their classrooms. They describe ten activity types: *solving equations, creating equations, counting, asking questions, discussing/explaining, drawing, building, measuring, telling time,* and *estimating*.

Next, I returned to the original student quotes to gather information on students’ purposes for engaging in the activities that they chose on their own time. Sometimes students offered this information organically, and other times I asked this as a follow-up question. This part of the analysis was only relevant to the child-initiated mathematics activities; I did not ask students to hypothesize about the purposes to their classroom activities since they did not choose them. Like with the analysis of activities, I clustered similar purposes into groups, which became codes, and I iterated between the student quotes and codes to clarify definitions. The final codes for students' purposes for engaging in mathematical activities on their own are found in Chapter 2, Table 2.11 along with definitions and examples of student quotes from each code. They describe eight distinct purposes: *for fun, because it's embedded in something else, to teach or challenge others, to practice or improve, to cure boredom, to learn something new, for a challenge*, and *to prepare for the next grade*. To further understand students' motivations, I categorized these student-named purposes into three umbrella categories: meeting school-based expectations, being embedded in a desired activity, and being driven by pleasure or curiosity.

Then, I returned one last time to the original student quotes and gathered the information about with whom students were engaging in mathematical activity (when it was available). In their descriptions of child-initiated mathematics activities, children often described bringing immediate and extended family members into the activity with them, but the majority of students did not specify with whom they engaged in mathematical activities in their classrooms. There are several possible reasons for this. First, the interview protocol did not specifically ask for this information, so it only emerged when it was noted organically by the participant. Second, it could reflect that in classrooms, often students worked alone. It also could be that students' work with classmates was routine enough that it did not seem salient to them to mention the co-participation of their peers. Still, on occasion students did describe working in various social configurations including partners, small groups, and the class as a whole. During the fall interviews, three students specifically mentioned working with peers in virtual breakout rooms, an adaptation that teachers may have made to keep groupwork alive despite the challenging circumstances.

With these three sets of information in hand–activity type, purpose, and co-participants–I compared frequencies across contexts and across grade level groups so as to understand the landscape of the child-initiated mathematical activities in comparison to children's descriptions of the activities in their math classes. Through my analysis, I found that on their own terms, students engaged in mathematics for a variety of purposes, some of which were tightly connected to school, like preparing for the next grade level, while others were not. The majority of the time, students were driven to engage in mathematics by pleasure or curiosity. Students also participated in a variety of mathematics activities, some of which mirrored the activities they described in their classrooms, while others did not. Overall, the children initiated both more and a wider range of mathematics activities than they described doing in their classrooms. In particular, creating rather than solving equations emerged as a frequent, choice activity outside of school. This again highlights the value of agency as the children became the producers, not just the receivers, of mathematics.

In the sections that follow, I illustrate these claims in detail. First, I describe the child-initiated mathematics activities, including the purpose for those activities and with whom children did them. Then, I compare those activities with how students described the mathematics happening in their classrooms. I note similarities and differences overall and within specific activity types. Finally, I end with key takeaways from this analysis for researchers and educators about the overlaps of student agency and mathematical activity along with relevant ongoing questions for future work.

Doing Math 'Just Because': Child-Initiated Mathematics Activities

Seventeen of the thirty students who participated in this study responded in at least one interview that sometimes they do math just because they feel like it, not because anyone is telling them to. These seventeen students included six students who entered the study as Kindergarteners, five as first graders, and six as second graders. Demographically, this group was representative of the study population overall, with similar breakdowns between students of color and white students, female and male students, and multilingual students and students who speak only English. These statistics can be seen in Table 5.2.

Table 5.2

Demographics of Students Who Said They Did Math 'Just Because'

	Race		Gender		Home language	
	Students of color	White students	Female students	Male students	Multilingual students	English- only students
Students who did math 'just because' *n=17*	9 (53%)	8 (47%)	10 (59%)	7 (41%)	6 (35%)	11 (65%)
Study population *n=30*	15 (50%)	15 (50%)	18 (60%)	12 (40%)	8 (27%)	22 (73%)

One of these students, Deja, explained during the spring of her second grade year that she did math on her own when she "played school." She told me that she used to play school a lot with her neighbor, but that had lessened with the onset of the pandemic. She still played school, but now stuffed animals and imaginary people played the roles of students. She assigned them math problems, and then sometimes helped the animals solve them (when they needed help). A few months later during the summer, Deja shared that she had done math while scooping out ingredients to bake brownies, which came out great. She also found herself practicing addition, subtraction, and multiplication in her head. The following fall, Deja continued to engage in math on her own. She told a story of going to Target and looking for something to buy. First, she counted her coins, which totaled to 99 cents. Also, with her newfound third grade knowledge, Deja opted to improve on previous work by correcting pages from her second grade math book. “It’s really off!” she exclaimed.

What mathematics activities did children initiate?

Across all three interviews, Deja described engaging in a range of mathematical activities of her own volition. When playing school she both *created* and *solved equations*, and she solved other equations when correcting her old schoolwork. While baking, she engaged in *measurement*. She also *counted* when totaling the coins in her pocket. The mathematical activities that Deja chose to do on her own were reflective of common trends in the data. *Solving equations* (n=14) was the most common activity named by students. Some students, like Deja, solved their own equations. Others solved equations on extra worksheets in their school books, in spare workbooks they had at home, or that they requested their parents to make for them. *Creating equations* (n=8) and *counting* (n=8) were the next most commonly named activities. Deja was not the only student to create equations in the context of recreating a school-like environment. Also a second grader, Louise described “homeschooling” herself and sister. They each had a notebook in which they would create equations, and sometimes solve them. Other students described writing equations on scrap pieces of paper or on white boards. With regards to counting, some students like Deja counted money. Others counted their toys, like Kindergartener Valentina who counted her Paw Patrol figurines. There were also students who described practicing rote counting aloud. The number of students who described participating in each activity is displayed in Table 5.3. There are more activities than there are students in this data group because many students said that they initiated more than one activity. The range was between one and five activities named per student, with a mean of 2.5.

Table 5.3

The Number of Children Who Engaged in Each Child-Initiated Mathematics Activity

Activity	Number of students
Solving equations	14
Counting	8
Creating equations	8
Asking questions	3
Measuring	3
Drawing	2
Discussing/Explaining	2
Building	2
Telling time	1
Estimating	0

It was less common for students to describe *asking questions*, *discussing or explaining*, *drawing*, *building*, *measuring*, and *telling time*. Sometimes when students asked questions, they did so of a family member. But for two students, they also asked Youtube. Second grader Louise shared that when she wanted to know how to do something she typed her question into Youtube, and then watched videos to find out the answer. “It’s satisfying,” Louise said, “so it makes me want to do more math.” Darius, a first grader, also asked questions about mathematical ideas, and like Louise, he would sometimes turn to Youtube when his mom's responses wouldn't suffice. This happened once when he wanted to learn more about multiplication. He explained, "I was learning some math on Youtube…it was like times, I just don't know my time travels- my time tables, so I found [the video] myself…I learn myself how to spell math, and I [typed] it, and then I just learned."

Like Deja, the other two students who talked about *measuring* did so in the context of baking. With regards to *drawing*, Kindergartener Valentina shared, "Sometimes [when] I get math for fun, I like some of the math to draw stuff and I don't have to label stuff." Earlier in that same conversation Valentina had expressed frustration that she had to label her work during math class, so this distinction was important to her. Second grader Meera also described drawing as an example of the math that she initiated. She drew shapes "to make dresses." She explained, "I would draw a heart, now I draw a triangle to make a dress."

The two students who named *discussing* or *explaining* as mathematics activities that they initiated described doing so with family members or friends. One student tried to get her younger sister to understand how to add. Louise explained the premise of division to a neighbor in order to clarify for her how many donuts each member of their families could get out of the dozen that her dad had brought home earlier that day. In her words, "So my dad, he got today, if there are 12 donuts–1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12." She drew tallies on a folded piece of paper as she counted, and then continued. "This is what I taught her, and there are six people in our family, how many would each people get? So, I divided them up into different groups, and when I looked at it, each person could get two donuts. And it would be an equal share."

Finally, the two students who identified *building* named working with their Legos as an example of their mathematical activity, and the student who named *telling time* said that he did so to keep track of how much time he had before certain parts of his day. In each of these examples, the students recognized mathematics as living beyond their classrooms' walls and they exhibited agency in using mathematics in ways that were meaningful to them.

Students across all three grade level groups engaged in child-initiated mathematics activities. The data of activity type broken down by grade level group is displayed in Table 5.4. The sample size here is small, especially when broken down by grade level, so I make no claims about the statistical significance of these findings. Instead, they reveal possible patterns and raise questions about the relationship between age, or time spent in school, and children's engagement with mathematics.

Table 5.4

The Number of Students Who Engaged in Each Child-Initiated Mathematics Activity, By Grade Level

Activity	K *n=9*	1st *n=11*	2nd *n=10*	Total
Solving equations	5	4	5	14
Counting	2	3	2	8
Creating equations	2	1	5	8
Asking questions	0	1	2	3
Measuring	0	2	1	3
Drawing	1	0	1	2
Discussing/Explaining	1	0	1	2
Building	1	1	0	2
Telling time	0	1	0	1
Estimating	0	0	0	0

Solving equations was the most common activity named in all three grade level groups. *Counting* was also described by more than one student in all three grade levels. Though at least one student in each grade level group talked about *creating equations*, this activity was more prominent with the second graders. These are the students who have had the most experience with mathematics in school. The fact that they are creating equations may show that they see equations as an important part of mathematics, and by creating equations they are exercising agency and putting themselves in charge of the activity instead of at the receiving end of it. It was also the case that overall, the older students described engaging in more activities per student than the younger students. Table 5.5 illustrates this.

Table 5.5

The Number of Activities That Students Described Initiating, By Grade Level

	K	1st	2nd	Overall
Range of the number of activities per student	1-4	1-5	1-4	1-5
Median of the number of activities per student	2	2	3	2
Mean of the number of activities per student	2.2	2.4	3	2.5

The Kindergarteners on average described initiating 2.2 mathematics activities each, and the means for the first and second graders were 2.4 and 3, respectively. Though these differences are small, it is possible that they hint at a pattern of students expanding their types of self-initiated mathematics activities as they get older and have more experience with mathematics both within and outside of school.

Why did children initiate mathematics activities?

Just as students opted to engage in a wide range of mathematical activities on their own time, they also did so for a wide range of purposes. Mia, who began this study as a Kindergarten student, initiated mathematics activities for multiple reasons. When I first talked with her in the spring, Mia shared that her teacher had provided her with log-in credentials for a supplementary app and had given her the green light to move forward in her workbook as she wished. Mia emphasized that she did not *have* to do these things, and it was not that she necessarily wanted to. Rather, Mia opted to engage in these activities *for a challenge*, which she saw as important given her self-perceived ability to “go way ahead…past Kindergarten.” The following summer, Mia explained in detail to me that she had used math to determine how much food she needed to prepare for her tyrannosaurus rex figurines. “There were five [T-rexes] on Natural Island, which is my train table…[and] three T-rexes were already planning on going down. They wanted to have extra dinner. It’s a special thing they do.” In this story, Mia engaged in mathematical thinking because it was *embedded in something else*; there was a scenario that she was playing out with the dinosaurs and it involved calculations. Mia also told me that while she was at soccer practice, she sometimes made up math problems in her head, *just for fun*.

As was the case with Mia, it was common for a given student to initiate mathematics activities for multiple reasons. And in her case, she had different reasons for each activity she described. In other cases, a student named multiple reasons for a single activity. That was the situation with first grader Gal, who talked about playing math games at home. She shared that sometimes, she played math games *to cure boredom,* other times she chose her favorite ones *just for fun*, and still other times she played specific math games *to practice* because she wanted *to improve* before playing them again at school.

Gal and Mia opted to engage in mathematical activity for some of the most common purposes across the data set. Of the 17 students, 14 of them, spread across all three grade level groups, described doing math *for fun*. The number of students who cited each purpose for their mathematical activity is displayed in Table 5.6. There are more purposes than there are students in this data group because many students named initiating mathematics activities for more than one purpose. The range was between one and five with a mean of 2.5.

Table 5.6

The Purposes That Students Named for Initiating Mathematics Activities

Purpose	Number of students
For fun	14
Because it's embedded in something else	6
To teach or challenge others	5
To practice or improve	4
To cure boredom	4
To learn something new	4
For a challenge	3
To prepare for the next grade	2

Second grader Torrin's stories of doing math 'just because' were emblematic of the next most common purpose, which was *because it is embedded in something else* (n=6). Torrin shared that he found himself counting as a part of playing several games including counting the cards in UNO, the troops in Risk, the money in Life, and the clubs and balls while setting up for a game of golf with his mother. *To teach or challenge others* followed with five students participating in each. This sometimes involved students teaching a younger sibling or friend, and other times involved students offering what they saw as particularly challenging equations to older siblings or parents to test the limits of their knowledge.

To practice or improve, like when Gal played math games specifically so that she would know more for the next time she played with classmates, and the desire *to cure boredom* motivated four students each to engage in mathematical activity. As second grader Landon said, "I do math when I don't have anything to do." Four students also chose to do math *to learn something new*. For example, one kindergarten student described working to figure out what 100 plus 100 equaled, because she wanted to learn how to add larger numbers, and another student described asking his mom to teach him multiplication, because he wanted to learn about it. Three students described opting to do math *for a challenge*, typically beyond what they felt they were learning in school. And finally, two students described doing math to *prepare for the next grade*.

Students across all three grade level groups engaged in mathematical activities on their own time for a range of purposes. Engaging in mathematical activity *for fun* was the most common purpose named for Kindergarten, first, and second grade students. *Because it's embedded in something else* and *to learn something new* were the only other two purposes that spanned all three grade level groups. The data of activity purpose broken down by grade level group is displayed in Table 5.7.

Table 5.7

Students' Purposes for Initiating Mathematics Activities, By Grade Level

Purpose	K	1st	2nd	Total
For fun	5	5	4	14
Because it's embedded in something else	1	2	3	6
To teach or challenge others	2	0	3	5
To practice or improve	0	2	2	4
To cure boredom	0	1	3	4
To learn something new	2	1	1	4
For a challenge	1	0	2	3
To prepare for the next grade	1	0	1	2

Notably, second grade students were represented across every purpose. Like in the breakdown of activity by grade level, which showed that second grade students on average participated in a wider range of mathematical activities on their own time, the data here suggest that the second grade students also participated in mathematical activities for a wider range of purposes. While Kindergarten and first grade students described an average of 2.2 purposes, second grade students described a mean of 3.2. In other words, overall, each second grade student described more reasons for engaging in mathematical activity than the younger students. Again, these numbers are very small and therefore certainly not statistically significant, but they may suggest a trend of students expanding their conceptions and enactments of mathematics between Kindergarten and second grade.

To further understand students' purposes for initiating mathematics activities, I clustered the eight purposes that students described into three umbrella categories: meeting school-based expectations, being embedded in a desired activity, and being driven by pleasure or curiosity. The umbrella category of school-based expectations included the student-named purposes of *to practice or improve* and *to prepare for the next grade*. *Being embedded in a desired activity* remained a category of its own. The umbrella category of being driven by pleasure included the majority of the student-named purposes including *for fun, to teach or challenge others, to cure boredom, to learn something new,* and *for a challenge*. Of the instances of child-initiated mathematics activities, six children described their purpose as meeting school-based expectations, six described their mathematics activities as being embedded in other desired activities, and 30 described engaging in mathematics because they were driven by pleasure or curiosity. This is exciting data; for this group of students, not only did they initiate mathematics activities that were creative and expansive in nature, they also did so because it made them feel good.

With whom did children engage in mathematics activities?

When students described engaging in mathematics activities because they wanted to, many of them (n=15) shared that at least sometimes, they did math by themselves without the co-participation of anyone else. However, nearly as often (n=14) students described engaging in mathematical activity with family members, including mothers, fathers, sisters, brothers, cousins, aunts, and grandparents. Unsurprisingly, given the alignment of the timing of this study with the COVID-19 pandemic and restrictions on social gatherings, it was much less common for students to describe opting to do math with friends or neighbors (n=3). In each of these examples, the students used their agency not only to engage in mathematics in a way and for a purpose that was meaningful to them, but they also brought others into the activity with them. In this way, they took on a leadership role, positioning themselves as decision-makers. Co-participation in child-initiated mathematics activities highlights the social nature of mathematics and resonates with the literature that has documented the critical role that relationships play in learning. Interpersonal connections and opportunities for cooperation are important resources for learning and identity development (Nasir & Cooks, 2009; Rogoff, 2003).

Comparing Child-Initiated Mathematics Activities to Mathematics Activities in Classrooms

All 30 of the participants in this study described the activities in their mathematics classes in their first interviews during the spring of 2020, and during their third interviews, which took place the following fall. However, here I looked specifically at how the 17 students who, at some point, said that they opted to do math on their own outside of school, described their classes. Even though the remaining 13 students could have offered more data on classroom mathematics activities, I chose to keep the group of students consistent throughout this chapter's analysis in order to more clearly compare students' mathematical experiences within and outside of school. Further, my goal here is not to objectively delineate what was happening in each study participant's mathematics class. To do so would have required additional data collection sources including extensive classroom observations. Instead, my goal was to understand what was salient for these students at the times of our interviews–what stood out to them, what they remembered, what they noticed–because students' perceptions of their classroom activities matter for how they make sense of those activities (Lamont & Swindler, 2014). And, how students make sense of their classroom activities is a quintessential piece of their relationships with mathematics and their developing mathematics identities (Sfard & Prusak, 2005).

Mathematics Activities in Classrooms

In the spring of her first grade year, Suriyah described three different activities in her math classroom. First, she said that they were *explaining and discussing* their thinking during an activity called 'Which One Doesn't Belong.' Second, she said that they spent time *solving equations* on worksheets. And third, they engaged in *estimating* when their teacher would show an image of multiple items, such as apples, on a screen and ask them to estimate how many there were. The following fall, Suriyah only named *solving equations* on worksheets as how she spent her time in math class. She said, "We do our worksheets, some have multiple choice, but on the first page, it's usually the ones that we usually have to write out."

Solving equations was the most common activity named by students when asked what they did during math time for school. In fact, all 17 students in this group described solving equations as a part of their math class. *Counting* was the second most common activity described, with nine students reporting it as a part of math in school. Sometimes, students described counting using specific tools or diagrams like a hundreds chart, which lists all of the numbers one through 100 in rows of ten, or a ten frame, which shows two rows of five squares each and is typically used to help children identify, combine, or separate numbers within 10. Other times students described counting as a part of playing a game. The other mathematical activities that emerged in conversations with students were either uncommon, having only one or two students describe them as a part of their math class (*drawing, discussing/explaining,* and *estimating*), or did not emerge in the conversations about school at all (*creating equations,* *building, measuring, telling time,* and *asking questions*). Table 5.8 shows the number of students who described participating in each activity during math time at school. There are more activities than there are students because several students named more than one activity. The range was between one and three activities named per student with a mean of 1.8.

Table 5.8

The Number of Students Who Engaged in Each Mathematics Activity in Their Classes

Activity	Number of students
Solving equations	17
Counting	9
Creating equations	0
Asking questions	0
Measuring	0
Drawing	1
Discussing/Explaining	2
Building	0
Telling time	0
Estimating	1

Across the three grade level groups, the activities that students described doing in their classrooms were not meaningfully different. All of the students described solving equations, and three students in each grade level group described counting. Beyond that, there were only one or two other activities represented in each of the grade level groups, and each of those by only one student. Additionally, unlike in the home activities where it appeared that the second grade students participated in an increasing number of activities, here, the average number of activities named per student was rather consistent. Students named, on average, 1.7, 2, and 1.7 activities for Kindergarten, first, and second grade respectively. In summary, when describing their classroom mathematics activities, students described fewer and a narrower range of activities than they did when describing the math they initiated ‘just because.’

Activities Across Contexts

The differences between the activities that students described engaging in during their math classes and the activities that they described initiating can be seen more clearly when the activities are displayed side-by-side. Table 5.9 shows the number of students who named each activity in each context.

Table 5.9

The Number of Students Who Engaged in Each Mathematics Activity, By Context

Activity	‘Just because’	At school
Solving equations	14	17
Counting	8	9
Creating equations	8	0
Asking questions	3	0
Measuring	3	0
Drawing	2	1
Discussing/Explaining	2	2
Building	2	0
Telling time	1	0
Estimating	0	1

Each child, on average, described initiating more types of mathematics activities than they participated in during school, and additionally, the overall spread of mathematics activities that children initiated is wider than what was described as taking place in classrooms. Interestingly, several of the child-initiated mathematics activities that were not described as happening in classrooms are creative in nature. Children described creating equations, where they put themselves in the position of authoring mathematics. Asking questions about what they wanted to learn also puts children's ideas front and center. Drawing and building, though named by fewer students, are also creative activities. Engaging in creative activity is a manifestation of agency; when being creative, children make meaningful decisions about what a project is or will be, what it will look like, how it will come together, and what purpose it will serve. In contrast, creativity did not emerge as central to the mathematics happening in classrooms.

Solving Equations. Further, though solving equations was most the most frequently described activity both in child-initiated mathematics and in classrooms, the nature of how students were solving equations on their own time was different from how they were doing so in their math classes. To find this, I returned to the original student quotes and noted the details of what solving equations looked like in each context. In their math classrooms, students most commonly solved equations on worksheets. Of the 17 students who described solving equations as a school math activity, worksheets were mentioned by 14 of them. This came up again and again right away when I asked students to describe what typically happened during math time. Besides worksheets, it was also common for students to solve equations while using apps assigned to them as part of their math activities. This was the case for eight students. Five students described solving equations while playing math games in their classes. This was often mentioned as a part of math rotations, with students playing games as one out of a few stations during their math period. Five students also described solving equations on whiteboards. Sometimes, this was a part of a whole class discussion with the teacher calling on individual students to come and write on the board. Other times, students used small, personal whiteboards while sitting and working with the teacher. Finally, one student described the teacher asking the students in her class to solve equations mentally, and one student talked about solving equations on math tests.

When students talked about solving equations 'just because,' sometimes their activity mirrored the worksheets that students were completing in their classrooms. In fact, eight of the 14 students who described solving equations on their own at least sometimes did so with worksheets, which suggests that the structure of a worksheet filled with equations was a significant part of their conceptions of what it means to do mathematics. These students were spread nearly evenly across the three grade level groups, so prolonged experience in school was not requisite for this conception. Sometimes students asked their parents to make or print out worksheets for them, and other times students opted to do extra worksheets from their school's workbook.

However, when initiating mathematics activity, it was just as common for students to solve equations in the context of play or games as it was for them to use worksheets. One student solved equations in order to determine whether or not her toy dinosaurs would have enough to eat for dinner. Another student described solving equations in the format of a rock-paper-scissors game. Three students 'played school,' and solved equations while filling the role of the teacher, the students, or both. There were also three students who solved equations while opting to play specifically what they called math games, possibly games that they originally learned in school.

Though not as frequent as through worksheets or play, some students (n=5) described opting to solve equations mentally. Often, this looked like a parent or sibling offering a math problem aloud and the student figuring out the solution. Other times, this was paired with the act of creating equations and students would both make up and solve problems on their own in their heads. Four students described solving equations while using apps; three of these students named apps that they also use in school, but that they opted to spend additional time on. Finally, two students described solving equations on whiteboards in their homes, and one student mentioned using flashcards.

Table 5.10 shows the breakdown of what it looked like for students to be solving equations during child-initiated mathematics activity and in their math classes. There are more activities than students because several students described solving equations in more than one way.

Table 5.10

Solving Equations in Child-Initiated Mathematics Activity and in Math Class

Solving equations	Child-Initiated *n=14*	In math class *n=17*
Worksheets	8	14
Games and other play	8	5
Mental math	5	1
Apps	4	8
Whiteboard	2	5
Flashcards	1	0
Tests	0	1

This data reiterates that though some similar practices occurred both in school and at home, solving equations during child-initiated mathematics activities involved more creative play, while worksheets dominated classroom activity.

Discussion

I began this chapter by describing the importance of student agency for learning mathematics and for the development of mathematics identity. Further, I explained that mathematics lives both within the classroom where student activity is largely directed by a teacher and also outside of the classroom. In this chapter, I specifically elicited and investigated examples of when students seized agency over their mathematical activity outside of the classroom–moments when they chose to engage in mathematics on their own terms, in their own ways, ‘just because.’ I refer to these activities as child-initiated mathematics activities. These activities reveal how children are understanding what mathematics is, because through them we can see what they are labeling as mathematical, and they showcase the types of activities that students partake in when they have significant control.

Through analysis of child-initiated mathematics activities, I found that when students engaged in mathematics on their own terms, they did so in ways that were more creative and expansive than how they described participating in mathematics in their classrooms. Unlike in their classrooms, where activity was dominated by solving equations and counting, when children initiated mathematics activities they also created their own equations, asked questions, measured, drew pictures, built structures, and told time. Several of these activities are creative in nature. Also, even when it appeared that students were engaged in activities that mirrored school activities, like solving equations, in comparison to in their classrooms, on their own they more frequently did so through creative play. In child-initiated mathematics activities, children both exercised agency in choosing to engage in the activities in the first place, and they further reiterated their agency by shaping the mathematics in ways that centered their original ideas.

Additionally, though students sometimes engaged in mathematics activities alone, they often brought family members into the activities with them, which reflects their understanding of mathematics as a social practice.

Still, analysis of child-initiated mathematics activities also showed that the students in this study remained, in some ways, tethered to their in-school mathematics learning experiences. Though overall child-initiated mathematics activities were more expansive and creative than in-school mathematics activities, many students brought narrow activities out of school with them. It remained, for example, common for students to complete worksheets on their own; many of the students in this study seemed to understand mathematics as focused on solving or completing sets of equations on worksheets. Sometimes students did this because they wanted to improve on a specific skill or prepare for the next grade, but other times students said they did worksheets just because they thought it was fun to do so. I wonder to what extent the fun stems from the activity itself or if the sense of fun is more tied to the students' confidence, accomplishment, pride, or other positive feelings that may result from successfully completing such a task. Especially if the latter is the case, there are open questions as to what will happen in the future. The findings in this chapter suggested that child-initiated mathematics activities were slightly more expansive for the second graders than for the kindergarteners; will that expansiveness continue to grow? Will child-initiated mathematics activities and school-based mathematics activities become more or less aligned over time as children get older? If students experience struggle during math class in school, will they still find it fun to carry classroom activities out of the school building? If they do not, then will those activities be replaced by something else or dropped altogether?

It is also important to remember that the analysis in this chapter focused on 17 students. There were 13 students who participated in the study who said that they never did math 'just because.' As I pointed out early in this chapter, of course those students engage in mathematical activity throughout their daily lives, but that is distinct from actively, and knowingly initiating mathematics activities. Child-initiated mathematics activities specifically require students to seize agency and make decisions about what mathematics means and looks like to them. So, why did those 13 students not initiate mathematics activities? From mathematics identity research we know that students tend to dissociate from mathematics as they get older; have these 13 early elementary school students already begun that journey? What can be done to shift their trajectory?

Attending to child-initiated mathematics activities may be one way for educators to gain insights into how to encourage children's positive relationships with mathematics. Child-initiated mathematics activities allow us to see how children are processing and defining what is mathematical, and we can respond to and expand their understandings by offering new opportunities for mathematics activities in classrooms. They offer us a lens through which to see how children choose to exercise agency, and we can follow their lead and build related opportunities for agency into school-based mathematics learning environments. They also reveal who is not considering their activity to be mathematical, or who is not opting to engage in mathematics outside of school buildings, which offers an opportunity for reshaping learning environments to better reach these students.

Child-initiated mathematics activities is a construct that should continue to be researched as a unique piece of the puzzle in understanding mathematics identities. It may be that children who already have stronger mathematics identities are more likely to initiate mathematics activities on their own, but it may also be that initiating mathematics activities on their own may further strengthen children's mathematics identities. Understanding this cyclical relationship and what role educators can play within it could contribute to the field's ongoing challenge of expanding who gets to be and feel mathematical.

Chapter 6
“I'm going to get great, amazing, excellent, MATHEMATICIAN": Looking Back, Across, and Forward

In the monograph on the study of identity in mathematics education published by the National Council of Teachers of Mathematics, researchers Langer-Osuna and Shah (2021) expressed the need for the field to commit to “making visible the lived experiences of people as they do and learn mathematics” (p. 1). They emphasized that students in mathematics classrooms do not sit and passively receive mathematical knowledge (Belenky et al., 1986), but rather in mathematics classrooms, students both shape and are shaped by their contexts as they learn, grow, and develop in multifaceted ways. Our task as math identity researchers then is to unpack that development, and to clarify how students’ experiences learning mathematics relate to how students come to see themselves and are seen by others as mathematics learners. In doing so, we can better understand how to support more students to perceive themselves as mathematically competent and how to ensure that more students see mathematics as something that is theirs to do, use, and contribute ideas to across settings and time.

This dissertation responds to this call by making visible the lived experiences of *young children* as they do and learn mathematics; it explores the emerging mathematics identities of early elementary school students. Here, I defined mathematics identity as the stories children tell about their relationships with mathematics, and I defined relationships with mathematics as a combination of how children feel about their mathematics experiences and how they perceive themselves as learners and doers of mathematics both within and outside of the classroom, now and in the future. This definition builds on Sfard and Prusak’s (2005) theory of identity as narrative, which explains that as people tell stories, they describe and create who they believe themselves to be. It is also rooted in a sociocultural theory of learning (Lave & Wenger, 1991; Wenger, 1998) because it sees identity as malleable, human-made, and formed in community through participation in practices. In addition, this definition of mathematics identity grows from the work of Martin (2000) who defined mathematics identities as a set of personal beliefs about one's relationship to mathematics throughout one's life. And finally, like the work of Boaler and Greeno (2000), Cobb et al., (2009) and Nasir and Hand (2008), it emphasizes that features of learning environments afford particular opportunities for students to enjoy, opt to participate in, and affiliate with mathematics, both within and outside of school walls.

Specifically, this study asked three questions. First, which facets of early elementary school students' relationships with mathematics differentiate among their emerging mathematics identities? Second, what features of classroom environments contribute to the development of positive mathematics identities in this same group of students? And third, what can we learn about young children's emerging mathematics identities from the mathematics activities that they initiate? In the remainder of this chapter, I will provide an overview of the findings that respond to each of these questions. Then, I will discuss how the concept of agency emerged as a through line in the findings, and I will expand on the ways in which agency has been discussed in the mathematics education literature by describing five dimensions of student agency in mathematics classrooms. After that I will identify limitations to this study. Finally, I will end by offering theoretical, methodological, and practical implications of this work for ongoing research and for educators.

Relevant Facets of Young Students' Relationships with Mathematics

Previous research has studied students' mathematics identities by investigating specific facets of their relationships with mathematics. These have included students' affinity for mathematics (Boaler & Greeeno, 2000), their desire to continue learning mathematics in the future (Boaler & Greeno, 2000), whether they see themselves as successful (Cobb et al., 2009), and the extent to which they take on mathematics as a part of who they are (Nasir & Hand, 2008), with classroom obligations transforming into obligations to themselves (Cobb et al., 2009). In response to my first research question, I analyzed whether these facets of relationships to mathematics that were previously studied with older students were meaningful or useful in differentiating amongst the emerging mathematics identities of young children who were in their first few years of elementary school, and I found that some were while others were not.

First, the vast majority of the Kindergarten through second grade students in this study expected to continue learning and doing math in high school, as college students, and as adults. It may be that to many young students, continuing to learn math seems inevitable rather than a choice. Because of this, at this stage in children's lives it does not seem helpful to focus on whether or not they see mathematics as a part of their futures. Similarly, nearly all the participants in this study saw themselves as at least sometimes successful in mathematics, making perceptions of success limited in their usefulness for differentiating between young children's relationships with mathematics. In contrast, I found variety both in students' affinities for mathematics–whether they liked mathematics and specifically math class–and in their developing affiliation with mathematics. In unpacking what it meant for students to affiliate with mathematics, I looked at their affinity in combination with whether or not they opted to do mathematics on their own, outside of school. This revealed who felt compelled to do mathematics not only because they were told to do so, but also because they saw it as enjoyable or valuable for themselves. These facets can be helpful in understanding the range of young children's emerging relationships with mathematics.

Supportive Features of Learning Environments

In response to the second research question, I first investigated classroom features that have been previously documented in the literature to be supportive of older students' positive mathematics identities. These included the collaborative nature of classrooms (Boaler & Greeno, 2000), the extent to which students see themselves as sharing mathematical authority (Cobb et al., 2009) and playing integral roles (Nasir & Hand, 2008) in their classrooms, students' access to the domain of mathematics (Nasir & Hand, 2008), students' experiences of conceptual agency (Cobb et al., 2009), and students' opportunities for self-expression (Nasir & Hand, 2008). I found that some of these classroom features did not emerge as priorities for the younger students in this study. In particular, regardless of their feelings towards mathematics, Kindergarten students rarely saw their classrooms as collaborative or saw themselves as sharing mathematical authority and playing integral roles in their classrooms. It was also rare for the Kindergarten students to describe having access to the domain of mathematics as expansive, connected, and purposeful. In turn, it was not possible to clearly link these classroom features to students' developing relationships with mathematics.

In contrast, across the Kindergarten through second grade groups, students expressed value in experiencing conceptual agency and being able to choose problem-solving strategies that were meaningful and reasonable to them. Having opportunities for self-expression also emerged as a feature of learning environments that was relevant across the study participants, meaning that students who expressed being able to add their own flair to their work and make their learning spaces reflect who they are as people were more likely to feel positively about mathematics and about themselves as mathematics learners. Additionally, I investigated the features of learning environments that emerged as priorities for students' mathematical learning experiences through their reflections about their transitions to remote learning due to the COVID-19 pandemic. These included the ability to be with peers, to have access to support and learning, to be physically comfortable, to control one's work pace and schedule, and to engage in activities that one finds pleasurable. Though the students in this study experienced dramatic changes in their physical learning environments, it was not so much the physical space that mattered to them but rather the ways in which those spaces offered, or did not offer, a combination of these features. Whether students were learning in-person, remotely, or in a hybrid format, the students emphasized that they valued having agency in their mathematics experiences; students wanted to make decisions about their learning.

Child-Initiated Mathematics Activities

In response to the third research question, I found that when children seized mathematical agency and orchestrated mathematics experiences for themselves outside of their classrooms, they engaged in a broader range of mathematical activities than they reported engaging in during school. Further, many of these child-initiated mathematics activities were creative in nature. For example, children asked questions, drew pictures, and built structures. They created their own equations, choosing both the numbers and operations. Sometimes they solved these equations, but sometimes the activity ended with the creation. When children did spend time solving equations, they often did so through either game-based or imaginative play. However, there were also many examples of students mirroring school-based mathematics activities on their own. This was most prominent in the number of students who described completing worksheets and solving lists of equations. Sometimes students initiated these activities for purposes that were tied to school–for example to practice or prepare for their class or the next grade–, but more often students described doing these activities just for fun. Largely, students chose to engage in mathematics to satisfy their own curiosities or because they found doing so to be pleasurable. Though it was common for children to engage in self-initiated mathematics activities alone, sometimes they brought others into the activities with them, further reiterating their position as directing the mathematics. Overall, the analysis of child-initiated mathematics activities revealed students' conceptions of mathematics on their own to be more expansive and creative than in school, and again highlighted students' desires for agency as they became the producers rather than the receivers of mathematics. This opens the important question of how educators can design space for student agency and creative, expansive mathematics experiences in classrooms as well.

It is also important to note that there were 13 children in this study who responded that they did not initiate mathematics activities outside of school on their own accord. Given the ubiquitous nature of mathematics embedded in daily activity, from a researchers' point of view I absolutely could describe ways in which these children engaged in mathematics outside of school. From interviews with parents (that were not analyzed for this dissertation), it was also clear that in many cases parents may disagree with their children and name several ways in which their children initiated mathematics activities. But, my goal here was to reveal children's perspectives, to bring their understandings to the forefront. To these 13 children, mathematics lived within the walls of their classrooms and was not part of how they chose to spend their time. These students were very early in their schooling; without intervention, their current limited conception of mathematics will undoubtedly impact their learning experiences in the years to come. Part of making the field of mathematics more inclusive involves expanding these students' conceptions of mathematics so that they begin to recognize the mathematics and mathematical thinking that they already engage in and are capable of using to navigate their lives.

The Five Dimensions of Agency

Across these findings, agency emerged as an important through line. Repeatedly, and in multiple ways, the young children in this study expressed that they valued having ownership and choice in their mathematics learning experiences, and analysis showed that experiencing agency aligned with more positive emerging mathematics identities. In the literature, agency has often been described as being on a continuum, with students having or being able to exercise some amount or degree of agency (Ruef, 2021; Langer-Osuna & Esmonde, 2017; Cobb et al., 2009), and research has shown that when students have more agency in mathematics classrooms, they are more likely to enjoy mathematics, see it as a part of their futures, and develop strong relationships with the discipline (Boaler & Greeno, 2000). In this discussion, I shift from quantifying the amount of agency that students have and instead build on work by Pickering (1995) to understand the different types of agency that students may experience in mathematics classrooms.

In his study of professional mathematicians, Pickering (1995) described that mathematicians exercised what he called human agency when they created and developed new ideas. But at other times, mathematicians had to follow standard procedures and fit their work into existing constructs; in these cases it was essentially the discipline of mathematics that had the agency, and the discipline constrained what the mathematicians were able to do. Pickering's work was taken up by Cobb et al. (2009) in their distinction between conceptual and disciplinary agency. They defined conceptual agency as students "choosing methods and developing meanings and relations between concepts and principles," (p. 45). In contrast, they defined disciplinary agency as situations in which students were limited to using established solution methods. In their study, students who experienced conceptual agency in their math class perceived themselves and their peers as more successful and began to take on their classroom obligations as obligations to themselves; in other words, they found personal value in and connection to mathematics as they were experiencing it. This was not the case for students who were limited to experiencing disciplinary agency. Similar to Cobb et al.'s (2009) conceptual agency, Engle (2011) described intellectual agency as students being able to develop their own ideas about a problem as opposed to being concerned with what a teacher or textbook would identify as correct, and she emphasized that this kind of agency was critical for learning. These definitions of types of agency by Pickering, Cobb et al., and Engle all focus specifically on engagement with mathematical idea-generating and problem-solving, but there are many facets to the activities that take place in math classrooms. The findings from my study suggest that other types of agency are important to students in different ways and at different times while learning mathematics. In turn, how agency is conceived in mathematics education research and in mathematics classrooms warrants expansion. Children in this study expressed having, appreciating, or seeking five distinct types of agency as parts of their mathematics learning experiences: conceptual, creative, physical, temporal, and interactive. Together, I refer to these as the Five Dimensions of Agency. They extend the literature's previous focus on problem-solving strategies to better encapsulate the intellectual, emotional, physical, and social experiences of a student in a mathematics classroom.

The first dimension of agency, *conceptual agency,* comes directly from definitions by Cobb et al. (2009) and Engle (2011) that I previously described. It reflects opportunities for students to choose their own problem-solving strategies, which is something that the participants in this study appreciated. (See chapter 3 for analysis regarding conceptual agency.) Second grader Meera explained, "Yes, I always do [pick my own strategy]. Because, if we didn't, I don't think math would be as fun as it already is." To Meera, part of the pleasure of engaging in mathematics was choosing problem-solving methods. This sentiment was echoed by other students who emphasized the importance of being able to solve problems in ways that were easy to them, or that made sense to them. For many students, this meant using their fingers whenever possible. Other students named drawing out problems and 'counting on' as specific strategies that they found helpful. In contrast, students complained about being required to use particular methods that they did not find meaningful. First grader Benji declared, "I think there's no reason to use the bar model. And I think it just wastes time." Though he otherwise liked math and math class, being constrained to a specific method detracted from Benji's positive experiences learning mathematics.

Students also found value in being able to exercise *creative agency*. This second dimension of agency incorporates what Nasir and Hand (2008) described as having opportunities for self-expression. They described that for high school students, being able to bring their whole selves, their full personalities, onto the basketball court was a significant part of why the students identified with the practice of basketball. This was not the case with geometry; in geometry class the students' opportunities to be themselves were limited. The children in this study also appreciated when they could bring themselves into math class. (See chapter 3 for analysis regarding self-expression.) Alice, a kindergartener, shared that she found math to be more fun when she could do her writing in different colors. Another student described pleasure in getting to come up with a name for her math group. These moments of personalization may seem trivial, but for these young children they were far from it. They were opportunities to make their learning their own, or to take ownership over their learning. I also saw creative agency emerge as highly valued in the child-initiated mathematics activities. (See chapter 5.) Of the children who said that they engaged in mathematics on their own terms outside of school, half of them described creating equations as one of their activities. A literal act of creative agency, by creating equations of their own, these students authored their own mathematics.

The third dimension of agency, *physical agency*, expands descriptions of student agency into thinking about students' physicality. Similar to how Ruef (2021) incorporated students being able to "compose their bodies" (p. 153) as a component of agency, physical agency refers to students having control over the physical space in which they are learning and the ways that their bodies interact with that space. First grader Ivy said that at home, she liked being able to lay down on the floor or on her bed while doing math; she found it more comfortable. Second grader Kira appreciated the variety of seating in her classroom; some chairs bounced, leaned, or rotated. Other students emphasized that it was important to them to be able to move around. (See Chapter 4 for relevant analysis.) Schools are notorious for the ways in which they exert control on students' bodies (e.g. Young, 2017), and experiences in mathematics classrooms are no exception. Students described being stuck in a single seat as uncomfortable, frustrating, and boring. These sensations limited students' enjoyment of and therefore affinity for mathematics, which are important facets of students' relationships with the discipline.

The fourth dimension of agency, *temporal agency*, emphasizes having control over pace and schedule. Research has highlighted the problematic consequences of timed tests specifically (Boaler, 2014), and avoiding the stress from being rushed on timed activities emerged as one part of why students desired temporal agency. Lucas, a kindergartener exclaimed that math would be better "if it didn't have time, cuz it has time, and I have to get it or I do wrong and…it doesn't even let me think!" Second grader Asher expressed similar frustration with having to turn in timed tasks that he did not have time to finish. But, temporal agency also expands beyond these specific activities. In addition, it refers to students' desires to choose the flow of their work. Students appreciated when they were able to take breaks, slow down, or speed up. They wanted to work when they felt focused, forge ahead when they felt confident, and pause when they were frustrated. They also wanted to be able to dive into new content when they felt curious and ready. (See chapter 4 for relevant analysis.) Having more temporal agency emerged as a benefit of learning virtually, and it is worth considering how children can be granted greater temporal agency in classrooms as well.

Finally, the fifth dimension of agency, *interactive agency*, describes students having choice over how they learn in community with others. In multiple ways, students expressed wanting to learn with and from the people around them: their peers, teachers, and family. (See chapters 3 and 4 for relevant analysis.) However, students' needs and desires around what that looked like varied. First grader Gal liked playing math games, and she wanted to work with others, but she shared that she felt frustrated when she was paired with students who she thought were much “better” than her in math; they rushed her and made the games less fun. There were other students who similarly felt constrained by their limited agency in assigned groupwork. Sometimes, students shared that they found pleasure in simply being co-present with other people. This was especially common amongst the youngest students. Maulik, for example, said that learning math in school is “nice because you could see all your friends.” But other times, students wanted to be able to work on their own, or interact with their peers specifically when they felt stuck and needed help. First grader Suriyah actually missed teaching her peers when she was at home–this was another type of community interaction that she found meaningful. Overall, the young children in this study expressed the desire to be and learn in community, but they wanted agency in order to make their interactions with others feel supportive and productive.

The Five Dimensions of Agency–conceptual, creative, physical, temporal, and interactive–reflect important elements of control that the students in this study expressed having, appreciating, or seeking as parts of their mathematics learning experiences. By suggesting that these dimensions of student agency are key features of mathematics learning environments that support positive mathematics identity development, I am in no way eliminating or minimizing the role of teachers. In fact, this work suggests that educators have the critical obligation to teach students how to use their agency in increasingly productive ways over time. Young students require guidance and practice to learn how to exercise their agency in ways that are supportive of their learning.

The Relationship between Agency and Identity

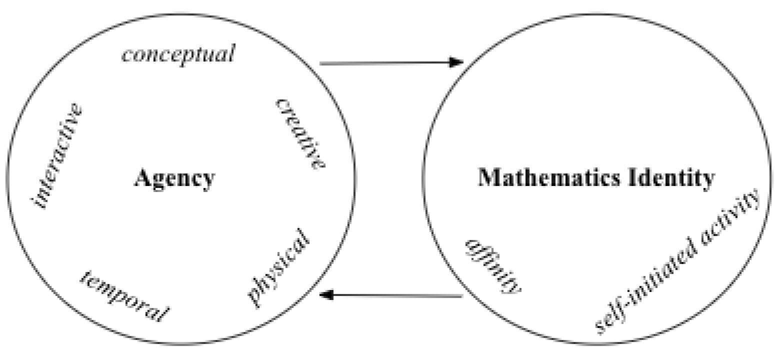
Berry (2021) described students' agency as "their identity in action" (p. vii). When students exercise any of the Five Dimensions of Agency, they are showing that they feel empowered. They feel empowered to make choices about mathematical ideas, about how they present themselves and their work, about how they situate their bodies, about how they organize their time, and about how they work with others. The very fact that they feel that empowerment reflects strength in their mathematics identities. In other words, students mathematics' identities allow them to exercise agency. When I asked second grader Gal to explain why she often opted to do "the harder things" in math class, she responded, "I'm that kind of person." In that moment Gal showed that her choices reflected who she believed herself to be. Hull and Greeno (2006) described this as having "identities full of agency," (p. 78).

At the same time, the reverse relationship is also powerful. Opportunities to experience agency can strengthen students' mathematics identities by showing them the ways in which they can have ownership over their learning experiences. Giving students the opportunity to make meaningful decisions reveals a level of trust in their sensibilities and respect for their humanity. When students are granted agency in mathematics classrooms, they are sent the message that who they are matters and that mathematics is for them. This opens up possibilities for how students see themselves. Louise, who was in second grade at the start of this study, told me a story about how in first grade, students would come into the room and complete a morning math assignment. She consistently finished her assignments with time to spare, and one morning her teacher offered her the opportunity to choose additional activities to try. Louise shared this story as an example of how she knew that math was "a good subject" for her. That morning in which her first grade teacher granted her agency impacted Louise's self-perception, and it stuck with her, even when she struggled in math the following year.

The relationship between agency and mathematics identity can be described then as reflexive, or mutually reinforcing. As students have opportunities to exercise agency in mathematics classrooms, they also strengthen their mathematics identities, developing a sense that they are competent mathematical thinkers who can contribute to and shape their own mathematics learning and the learning of others. And, students with stronger, more positive mathematics identities may exercise more agency, revealing that they have a sense of ownership over their learning experiences. This relationship is illustrated by Figure 6.1.

Figure 6.1

The Relationship Between Agency and Mathematics Identity for Young Children



The circle labeled "Agency" includes the Five Dimensions because these emerged as relevant to the stories that the elementary school participants in this study told. Similarly, the circle labeled "Mathematics Identity" includes both affinity and self-initiated activity because these were two facets of students' relationships with mathematics that were relevant in my data. It is certainly possible that other forms of agency, and other facets of mathematics identity could be important to other groups of students, and the figure could be easily updated to reflect that.

Limitations

This study has several limitations. Because I began right at the start of the COVID-19 pandemic, there was a lot about schooling that was, at the time, in flux. In turn, I was not able to work with specific schools or school districts, and I was limited to snowball sampling starting with my own personal networks to recruit study participants. Though the final group of participants did reflect some diversity across race, gender, and home language, white students for whom English was their primary language and who lived in the neighborhood where I previously taught were overrepresented. Relatedly, my analysis did not meaningfully grapple with differences in mathematics learning experiences along demographic lines. Previous math identity research has made clear that students are frequently marginalized in mathematics classrooms based on their race, gender, home language, and dis/ability status. This likely impacts students who are in the early elementary grades, and how it does so is worthy of further study. In addition, I was only able to speak with students virtually; being able to see students engaging in mathematics in classrooms or other settings that are undoubtedly influential on their mathematics identities would have enriched and potentially changed these findings. Finally, I was the sole coder of data for this dissertation; inter-rater reliability was not established.

Implications and Future Research

Theoretical implications

This dissertation offers several theoretical implications. First, it expands previous definitions of agency to encompass a broader range of young children's experiences learning mathematics, and it emphasizes the mutually supportive relationship between agency and mathematics identity. The Five Dimensions of Agency extend our understanding of the ways in which children can experience and express ownership over their mathematics learning. Second, and relatedly, this dissertation brings forth child-initiated mathematics activities as a meaningful site for exploring young children's conceptions of mathematics and their relationships to the discipline. Previous research has looked at how mathematics is embedded in everyday life and cultural practices (e.g. Saxe, 1988; Nasir, 2005; Civil, 2002), but child-initiated mathematics activities are distinct in that they showcase moments when children use their agency to choose to engage in something that they themselves identify as mathematical.

Third, my analysis revealed that of the many facets of mathematics identity previously studied, students' affinity for mathematics and their tendency to initiate mathematics activities on their own time are central to understanding young children's relationships with mathematics. Put otherwise, whether children like mathematics and whether they independently opt to engage in mathematics are meaningful expressions of how they perceive themselves as mathematics learners. This finding contributes to the field's ongoing goal of operationalizing mathematics identity. However, as I described in the limitations section, a key missing piece of understanding young children's mathematics identities is recognizing and unpacking how their mathematics identities interact with their other identities. This is critical because children with different, intersecting identities have different experiences in schools at large and in mathematics classrooms in particular. For mathematics identity research to contribute to more equitable mathematics education, it has to grapple with these differences.

There were glimmers of the relationship between students' mathematics identities and other identities in my data. For example, Ling, a second grader, explained to me that sometimes when she gets stuck on her school math work, she figures it out using "the Chinese way." She explained that she finds the Chinese way of doing math to be easier and to make more sense, but her teacher doesn't know about it, so after she figures out the solution, she erases her work and re-writes it using the strategy she's been taught in her American classroom. Ling seemed to see "American math" and "Chinese math" as two different disciplines, and had developed her own practices for moving between them. Another second grader, Deja, shared that some of her motivation to succeed in math comes from stories her grandmother told her about "people like her" who "were really good at math…like scientists, people who went to space, lots of people." Though not explicit, it seemed that Deja implied a cultural connection between herself and the characters in her grandmother's stories. In both of these examples, who these students were as people influenced who they perceived themselves to be as mathematics learners. In the future, I intend to explore developing a study that would more intentionally gather meaningful data on these important identity intersections.

Additionally, I began my analysis for this dissertation by investigating whether mathematics identity patterns in the literature that described older students were applicable or relevant to students in the first few years of elementary school. Now, I can ask the question in reverse. In what ways do the findings from this dissertation, specifically around the Five Dimensions of Agency, apply to older students? Do students in upper elementary school, high school, or college seek these same forms of agency in their mathematics learning experiences? Are there other forms of agency that become relevant as students get older? I intend to continue this line of inquiry across age groups to better understand mathematics identity development.

Methodological implications

This dissertation also has methodological implications. Previous child development literature shows that young children make sense of themselves in relation to world around them (e.g. Bigler & Liben, 2006; Rogers et al., 2012; Coll & Szalacha, 2004) and that young children can be valuable and reliable reporters about themselves and their experiences (Sabol, Busby, & Hernandez, 2020). The findings from this dissertation support this work and show that interviews with children as young as Kindergarten can be fruitful for understanding their perceptions of and relationships with mathematics. Further, though conducting interviews over Zoom was a result of the COVID-19 pandemic rather than an intentional choice from the onset, this study showed that young children can communicate meaningful information about themselves through virtual conversations. Virtual interviews offer several affordances, including eliminating geographic constraints and ease of scheduling for both researcher and participants, that should be considered in the design of future studies.

Though the interview methodology was powerful for collecting data on children's relationships with mathematics in this study, further work is warranted to better capture change over time. Students' mathematics identities, like other types of identities, develop and shift. They are influenced by, amongst other contextual factors, classroom changes from year to year (e.g. Horn, 2008). I designed this dissertation study with the intent of noting changes in students' mathematics identities across two school years, but I found that most of the students showed stability in how they described their relationships with mathematics across the interviews. Though this is an interesting thing to note, I am cautious to draw any conclusions from it. There were a few students who specifically described feeling differently about mathematics across different years, noting for example that math was easier, or more fun in a previous grade. A future study of change in students' mathematics identities over time would likely involve increasing the study's duration, adapting the interview protocol to more more thoroughly address potential change, and possibly adding observations of students into the data.

The interview-based methodology could also be expanded to gather perspectives from other people who play important roles in children's development. As Sfard and Prusak (2005) explained, identities are not just stories that people tell about themselves, but they are also stories told by others about them. I have already collected stories from participants' caregivers via surveys and interviews. I plan to analyze that data and compare my findings to the first person narratives shared by the students. In the future, I would also like to conduct a study that brings teachers' perspectives into the mix because teachers undoubtedly have a significant influence on how children see themselves as mathematics learners.

Practical implications

Finally, the findings from this dissertation also have practical implications for teachers, and especially for teachers of young children. The findings highlight that if teachers want to get a sense of their young students' mathematics identities, they may find it helpful to attune specifically to whether or not their students like math and whether or not they can describe ways they do math on their own outside of school. Mathematics identity is complex, but these two facets are approachable for teachers and can be ascertained through simple conversations with children. By gathering this information, teachers can build a sense of which of their students needs more support to feel positively about themselves in relation to mathematics.

Further, this dissertation shows the importance of creating opportunities for student agency in mathematics learning spaces. Teachers have significant power in classrooms, including the ability to make decisions about how they organize both lessons and students (Ruef, 2021). The Five Dimensions of Agency offer a way for teachers to reflect on the multiple opportunities they can create for student choice. It may not be possible in every moment for children to engage in all five types of agency, but by intentionally embedding opportunities for children to have ownership over how they solve problems, express themselves, physically situate their bodies, organize their time, and work with others, teachers can empower students to see themselves as mathematically competent and to see mathematics as a part of who they are. If early elementary school teachers do this, it may have long lasting impacts on students' trajectories in mathematics.

Conclusion

The quote in the title of this chapter, "I'm going to get great, amazing, excellent, MATHEMATICIAN!" comes from first grader Hiba. She was describing to me that already, in her first two years of school, she had improved a lot in math: "First I was okay, then not bad, then I got good." She went on to say that as she continued to practice, she expected to get better and better. In fact, she saw herself on a trajectory towards being a mathematician. But many children do not see themselves on such a trajectory. Research has repeatedly documented that schooling perpetuates the belief that only some people can achieve success in advanced mathematics, that it is a field for the elite, not for the masses. As students progress through school, many opt out and are pushed out of mathematics, thus limiting their access to a variety of school and career opportunities. This problem is heightened in the mathematics learning experiences of students of color, girls, students for whom English is not their first language, and students with disabilities.

All children deserve the opportunity to feel and achieve success in mathematics. Mathematics opens up academic and career opportunities, but just as importantly, it is a way of seeing the world, a way of problem-solving, finding patterns, and understanding systems. Mathematics can be an act of joy, and that joy should be for everyone to share. This dissertation contributes to a body of research on the relationship between the contexts in which children learn mathematics and children's relationships with the discipline. It elevates the voices of early elementary school students who are at the beginning of their academic journeys and reveals experiences that are relevant and impactful to them as they begin to develop mathematics identities. In doing so, this dissertation joins the movement to reimagine mathematics classrooms as spaces for simultaneously learning meaningful mathematics and supporting positive mathematics identity development. By hearing and learning from the stories of young children, we can help shift the paradigm of mathematics education from one that excludes to one that includes, creating space for more students to feel empowered as mathematical learners, thinkers, and doers and for more people to find confidence, knowledge, and joy in the discipline of mathematics.

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Appendix A: Child Interview Protocols

Child Interview Protocol: Round 1

Opening:
Hello! My name is Mari, what's your name? It's so nice to meet you, ___. We're going to do a little thinking together and talk about math if that's ok. I'm excited to talk with you and learn from you about math. I am going to record our conversation so that I can watch it later and so that I can share what I learn from you with other people who are also interested in learning about how kids feel about math.

You don't have to talk to me. It's totally up to you. If you ever change your mind and want to stop, all you have to do is tell me. No one will be mad at you if you decide that you don't want to talk any more. Also, if you ever aren't sure how to answer one of my questions, or if you don't want to answer one of my questions, that's totally ok- just tell me and we'll move on.

Before we get started, do you have any questions for me?

Math activity:
First, I want to do a little bit of thinking together with you. How does that sound? Let's look at this picture of four things [show WODB picture].

1. Can you tell me what you notice about the picture?
2. Which things do you think go together? Why?
3. Which one doesn't belong? Why?

What is math?:
Thanks for sharing your thinking with me! That was really interesting. Now I'd like to ask you some questions about math.

1. Can you tell me, what is math?
2. How can you tell if someone is doing math?
   a. What does doing math look like?
   b. Can you tell me about a time recently when you saw someone doing math?
   c. Do you know anyone who does a lot of math? If so, what kinds of things does that person do?
3. How can you tell if someone is doing math really well?
   a. Have you seen anyone doing that? Who?
   b. Can you tell me about a time when you felt like you were doing math really well?
4. Are there some parts of math that you feel strong with?
   a. What are those parts?
   b. Why do you feel that way?
5. Are there other parts of math that you feel less strong with?
   a. What are those parts?
   b. Why do you feel that way?

Math at school:

Now I am going to ask you some questions about what math was like when your school was open. Do you remember all the way back into March when you were still going to school? Can you picture where you used to sit? Do you remember some of the things that were hanging on the wall?

1. Can you tell me a story about what your math class was like?
    a. What happened in your math class when your school was open?
    b. What kinds of things did your teacher do during your math class?
2. What kinds of things did your teacher expect you to do in your math class?
3. Did you like math class? What parts did you like or dislike?
    a. Can you tell me about one of your favorite things you learned this year in math class?
4. How did you solve math problems at school?
    a. Why did you pick those ways to solve problems?
    b. What did you do if you got stuck?
5. If there was something new you wanted to learn, what did you do?

Math at home:

Since your school has been closed, you've been doing lots of things at home. I'd like to ask you a little bit about the math you've been doing lately.

1. What kinds of math does your teacher ask you to do at home now?
    a. Can you tell me about one of the activities you've done recently?
2. Do you like doing the math that your teacher asks you to do? What parts do you like or dislike?
3. What is different about doing math at home and at school?
    a. Are there things you like better about doing math at home?
    b. Are there things you like better about doing math at school?
4. How do you solve math problems at home?
    a. Why do you pick those ways to solve problems?
    b. What do you do if you get stuck?
5. If there is something new you want to learn, what do you do?
6. Do you ever do math just because you feel like doing math not because someone is telling you to?
    a. If so, what kinds of things would those be?
7. Since school is just about over for the year, do you think you're going to do any math over the summer?
    a. What kinds of things might you do?

Closing:

Thank you so much for sharing your ideas! It was very interesting for me to learn from you. Is there anything else about math that you think I should know that you haven't had a chance to tell me? Do you have any other questions for me? I look forward to talking to you again in a couple of months. Have a great summer!

Child Interview Protocol: Round 2

Opening:
Hi, _____. It's so nice to see you again! How has your summer been? Can you tell me about something fun you've done recently?

I'm excited to talk with you more about math today if that's ok. Just like last time, I am going to record our conversation so that I can watch it later and so that I can share what I learn from you with other people who are also interested in learning about how kids feel about math.

And before we start, I want to remind you that whether or not you talk to me today is totally up to you. If you ever change your mind and want to stop, all you have to do is tell me. No one will be mad at you if you decide that you don't want to talk any more. Also, if you ever aren't sure how to answer one of my questions, or if you don't want to answer one of my questions, that's totally ok- just tell me and we'll move on. Do you have any questions that you want to ask me first?

Math activity:
Now, just like last time, I want to do a little bit of thinking together with you. Do you remember that I showed you a picture of four things last time and we talked a bit about them? I am going to do that again but with another set of pictures. Let's look at this picture of four things [show WODB picture].

1. Can you tell me what you notice about the picture?
2. Which things do you think go together? Why?
3. Which one doesn't belong? Why?

Math in the summer:
Thanks for sharing all those ideas with me! Now I'd like to go back to talking a bit about your summer. You told me that one fun thing you did was _____. That sounds great!

1. Have you been doing any math this summer?
    a. Did you do math when you were doing any activities that *you* chose to do?
        i. Can you tell me about some of those activities?
    b. Did you do math when you were doing any activities that *someone else* (like your parents) asked you to do?
        i. Can you tell me about some of those activities?
    c. When you did math this summer, did you do it by yourself or with other people?
        i. Who did you do it with?
2. Have you learned any new math this summer?
    a. What have you learned?
    b. Why did you learn it?

    c. How did you learn it?
3. How can you tell if you're doing math?
4. How can you tell if you're learning math?

What is math?:

So, you told me that some of the math activities you did this summer were ______. Now, I'd like to hear a little bit more from you about what makes those activities math.

1. Can you tell me, what is math?
2. How can you tell if someone is doing math?
    a. What does doing math look like?
    b. Can you tell me about a time recently when you saw someone doing math?
    c. Do you know anyone who does a lot of math? If so, what kinds of things does that person do?
3. How can you tell if someone is doing math really well?
    a. Have you seen anyone doing that? Who?
    b. Can you tell me about a time when you felt like you were doing math really well?
4. Are there some parts of math that you feel strong with?
    a. What are those parts?
    b. Why do you feel that way?
5. Are there other parts of math that you feel less strong with?
    a. What are those parts?
    b. Why do you feel that way?

The upcoming school year:

Now, it's almost a new school year! How do you feel about becoming a [1st grader/2nd grader/3rd grader]? I'd like to ask you some questions about what you think learning math is going to be like once school starts.

1. Is there anything about school starting that you're looking forward to?
    a. Is there anything about school starting that you're feeling nervous about or that you're not looking forward to?
2. What do you think math class might be like this year?
    a. What kinds of activities do you think you might do?
    b. What kinds of things do you think you might learn?
3. What do you *hope* your math class is like this year?
    a. Is there anything you would really like to do?
    b. Is there anything you would really like to learn?
4. Last time I talked to you, we talked about what your math class was like in [K/1st grade/2nd grade]. Do you still remember some of the things you did in math last year?

5. What do you think might be different about math class this year compared to last year?
    a. What do you think might be the same?
6. You've told me a lot about the math you did this summer. I'm wondering how that might be different from the math that you're going to do in school this year.
    a. What do you think might be different about doing math for school this year compared to what doing math was like over the summer?
        i. Is there anything you think you might like better about doing math over the summer?
        ii. Is there anything you like better about doing math during school?

Closing:
Well, thank you again so much for sharing your ideas! It was very interesting for me to learn from you. Is there anything else that you want to tell me? Do you have any other questions for me? I look forward to talking to you again in a couple of months. I hope you have a great end of your summer and beginning of the school year!

Child Interview Protocol: Round 3

Opening:
Hi, _____. It's so nice to see you again! How are you doing? Can you tell me about something fun you've done recently?

I'm excited to talk with you more about math today if that's ok. Just like last time, I am going to record our conversation so that I can watch it later and so that I can share what I learn from you with other people who are also interested in learning about how kids feel about math.

And before we start, I want to remind you that whether or not you talk to me today is totally up to you. If you ever change your mind and want to stop, all you have to do is tell me. No one will be mad at you if you decide that you don't want to talk any more. Also, if you ever aren't sure how to answer one of my questions, or if you don't want to answer one of my questions, that's totally ok- just tell me and we'll move on. Do you have any questions that you want to ask me first?

Math activity:
Now just like the other times we've talked, I want to start by doing a little bit of thinking together. Do you remember that I showed you a picture of four things last time and we talked a bit about them? I am going to do that again but with another set of pictures. Let's look at this picture of four things [show WODB picture].

1. Can you tell me what you notice about the picture?
2. Which things do you think go together? Why?
3. Which one doesn't belong? Why?

What is math?:
Thanks for sharing that thinking with me! Now, I'd like to talk together about math.

1. Can you tell me, what is math?
2. How can you tell if someone is doing math?
    a. Can you tell me about a time recently when you saw someone doing math?
    b. Do you know anyone who does a lot of math? If so, what kinds of things does that person do?
3. How can you tell if you're doing math?
    a. How can you tell if you're learning math?
4. How can you tell if someone is doing math really well?
    a. Have you seen anyone doing that? Who?
    b. Can you tell me about a time when you felt like you were doing math really well?
    c. What do you think it is about you that makes you able to do math really well?

5. Are there some parts of math that you feel strong with?
    a. What are those parts? Why do you feel that way?
6. Are there other parts of math that you feel less strong with?
    a. What are those parts? Why do you feel that way?
7. Do you do math by yourself or with other people?
    a. If you do math with other people, who do you do math with?
8. Do you like math?
    a. What do you like about it?
    b. What don't you like about it?
9. How do you feel when you do math?

Math at school:

Last time we talked, school hadn't started yet, and now you've had a couple of months in ___ grade! I'm excited to hear how it's going! Can you tell me something that you like about school this year?

1. Can you tell me about what math is like in school this year?
    a. What kinds of activities do you do during math time?
    b. What kinds of things have you been learning?
    c. Can you tell me about something you did in math recently?
2. Do you like math class? What parts do you like or dislike?
    a. Can you tell me about your favorite thing that you've done in math this year?
3. What does your teacher do during math time?
4. What does your teacher expect you to do during math time?
5. When you are working on math, how do you decide how to solve math problems?
    a. How do you pick what strategy to use?
    b. What do you do if you get stuck?
6. What happens if you make a mistake in math?
    a. Can you tell me about a time when you made a mistake?
    b. How do you feel when you make mistakes in math?
7. If there is something new you want to learn in math, what do you do?
8. If you were in charge of math time, what would you do?
    a. What kinds of activities would you have all the students do?
    b. What kinds of things would you have all the students learn?
9. Last year you were in ____ grade. What is different about math this year from last year?
    a. What is the same about math this year from last year?
    b. Are there things you liked better about math last year?
    c. Are there things you like better about math this year?

Math beyond school:

1. Do you ever do math with your family?
    a. What kinds of math do you do together?
2. Do you ever talk about math with your family?
    a. What kinds of things do you talk about?
3. Have you ever learned math from people in your family?
    a. Who have you learned from?
    b. What have you learned?
4. Do you ever do or learn math with other people in your community?
    a. Who have you done or learned math with?
    b. What have you done or learned?
5. Do you ever do math just because you feel like doing math, not because someone is telling you to?
    a. If so, what kinds of things do you do?
6. When you're older, do you think you're going to keep learning math?
    a. What kinds of things do you think you'll learn when you're older?
    b. (In high school? In college? As a grown-up?)
7. When you're older, do you think you're going to use math?
    a. How might you use math when you're older?
    b. (In high school? In college? As a grown-up?)

Thank you so much for sharing all of those ideas with me! I am so glad that I got to learn from you. Is there anything else that you want to tell me about math before we finish up? Do you have any questions for me? I hope that the rest of your school year is great!

Appendix B: Caregiver Surveys

Caregiver Survey 1

Thank you for agreeing to participate in this research study! We are excited to learn from you and your child. If you have any questions about the survey or about the study at large, feel free to reach out by email at marialtshuler@u.northwestern.edu.

1. Please enter your first and last name.
2. Please enter your participating child's first and last name.
3. What is your child's current grade level (2019-2020 school year)?
    a. Kindergarten
    b. 1st grade
    c. 2nd grade
4. What school is your child currently enrolled in?
5. How has your child's school organized remote learning? *For example: Is your child meeting live with their teacher? Are there whole-class meetings or small groups? Are assignments mostly completed online or offline?*
6. What is your child's teacher expecting that your child does on a weekly basis? *For example: How many assignments is your child asked to complete? How frequently are assignments turned in? How much time is your child expected to spend on school?*
7. How does your child seem to feel about remote learning overall? Why?
8. What **math** is your child's teacher expecting that your child does on a weekly basis? *For example: Does your child have any math classes that are live? How many math assignments is your child asked to complete? What do those assignments look like? How frequently are assignments turned in? How much time is your child expected to spend on math?*
9. How does your child seem to feel about doing math schoolwork at home? Why?
10. How did your child seem to be doing in math in school prior to school closures? How could you tell?
11. How does your child seem to be doing in math in school during remote learning? How can you tell?
12. Math happens in schools, but also happens in many other parts of our lives. How does your child seem to feel about math overall, regardless of where it is happening? Why?
13. How did you feel about helping your child with math school work prior to school closures? Why?
14. How do you feel about helping your child with math school work during remote learning? Why?
15. How did you feel about math as a kid?

    a. Loved it
    b. Liked it
    c. Felt neutral
    d. Didn't like it
    e. Hated it

16. How do you feel about math now?
    a. Love it
    b. Like it
    c. Feel neutral
    d. Don't like it
    e. Hate it
17. What is your zip code?
18. What is your race? Select all that apply?
    a. American Indian or Alaskan Native
    b. Asian
    c. Black or African American
    d. Hispanic or Latinx
    e. Native Hawaiian or Other Pacific Islander
    f. White
    g. Multiracial
    h. Prefer to self-describe:
    i. Prefer not to say
19. What is your participating child's race? Select all that apply.
    a. American Indian or Alaskan Native
    b. Asian
    c. Black or African American
    d. Hispanic or Latinx
    e. Native Hawaiian or Other Pacific Islander
    f. White
    g. Multiracial
    h. Prefer to self-describe:
    i. Prefer not to say
20. What is your gender? Select all that apply.
    a. Female
    b. Male
    c. Non-binary/Third-gender
    d. Prefer to self-describe:
    e. Prefer not to say

21. What is your participating child's gender? Select all that apply.
    a. Female
    b. Male
    c. Non-binary/Third-gender
    d. Prefer to self-describe:
    e. Prefer not to say
22. What is the primary language spoken in your home?
23. What other languages are spoken in your home? If only the primary language is spoken, enter N/A.
24. What's your preferred means of communication with the research team?
    a. Phone (if so enter # below)
    b. Text (if so enter # below)
    c. Email

Thank you! We'll be in touch shortly to schedule our first interview with your child!

Caregiver Survey 2

Once again, thank you for agreeing to participate in this research study! We are excited to continue to learn from you and your child. If you ever have any questions about the survey or about the study at large, feel free to reach out by email at marialtshuler@u.northwestern.edu.

1. Please enter your first and last name.
2. Please enter your participating child's first and last name.
3. What is your child's upcoming grade level (2020-2021 school year)?
    a. 1st grade
    b. 2nd grade
    c. 3rd grade
4. What school will your child be attending in this upcoming school year?
5. To what extent would you say that your child has done math this summer?
    a. None
    b. A little bit
    c. Some
    d. A lot
6. Please describe the kinds of math activities that your child has done this summer.
7. Of the math activities that your child has done this summer, did your child initiate any of them? If so, which?

8. Of the math activities that your child has done this summer, did you initiate any of them? If so, which?
9. What role have you played in the math activities that your child has done this summer?
10. Do you think that your child feels differently about math during the summer than they do during the school year? If so, why?
11. How does your child seem to feel about math over the summer? Why?
12. How does your child seem to feel about math overall? Why?
13. As of today, what do you know about your school's plans for the fall? (e.g. Is school starting remotely, in-person, or a combination? Will your child have contact with a teacher each day on the computer?)
14. What are your goals for your child in math this upcoming year?
15. What do you hope your child will learn in math this upcoming year?
16. How are you feeling about supporting your child in math this upcoming year?
17. What is your preferred means of continued communication with the research team?
    a. Phone (If so enter # below)
    b. Text (If so enter # below)
    c. Email (If so enter best email below)

Thank you! We'll be in touch shortly to schedule our second interview with your child!

Caregiver Survey 3

Once again, thank you for agreeing to participate in this research study! We are excited to continue to learn from you and your child. If you ever have any questions about the survey or about the study at large, feel free to reach out by email at marialtshuler@u.northwestern.edu.

1. Please enter your first and last name.
2. Please enter your participating child's first and last name.
3. What is your child's current grade level (2020-2021 school year)?
    a. 1st grade
    b. 2nd grade
    c. 3rd grade
4. What school is your child enrolled in this school year?
5. What format(s) is your child's school experience this year thus far? Select all that apply.
    a. Fully remote/virtual
    b. Fully in-person
    c. Hybrid
6. IF HYBRID: Please describe how your child's school has structured hybrid learning. For example, is your child at school half days every day? Or two days per week?

7. IF MORE THAN ONE: Please explain why you selected more than one school format. Did the school year start in one format and then transition to another?
8. How does your child seem to feel about math in school this year? How can you tell?
9. Does your child get assigned math homework?
   a. Yes
   b. No
10. How does your child feel about doing math homework? How can you tell?
11. What is your perception of how your child is doing in math in school? Why do you think so?
12. Has anything changed about your child's experiences in math in school this year compared to last year? If so, what?
13. What are your hopes for your child in math this year?
14. Do you have any concerns about your child in math this year? If so, what are they?
15. How do you feel about helping your child with math? Why do you feel that way?
16. Math happens in schools, but it also happens in many other parts of our lives. How does your child seem to feel about math overall, regardless of where it is happening? Why?
17. Does your family engage in math together? If so, in what ways?
18. Would you have any interest in participating in a follow-up parent interview? Selecting yes here is in no way a commitment to participating in the future should the opportunity arise.
    a. Yes
    b. No
19. What's your preferred means of communication with the research team?
    a. Phone
    b. Text
    c. Email

Thank you! We'll be in touch shortly to schedule our third interview with your child!

Appendix C: Caregiver Interview Protocol

Opening:
Hello! It's so nice to see you again. The opportunity to talk with your child over the past several months was truly a pleasure, and I really appreciate you helping to arrange those interviews. Now, I'm looking forward to learning more from your perspective.

First, I want to make sure that you know that if you ever don't want to answer a question of mine, that is completely ok, just tell me and we'll move on. You can also always ask me to clarify if a question of mine isn't clear. Further, if you want to end the interview at any point in time, you are welcome to do so, just let me know. Before we get started, do you have any questions for me?

Caregiver's relationship with math:
All along I've been talking about your child's experiences with math, but if you don't mind, I'd love to actually start today by hearing a little bit about your experience with math.
*Depending on what the parent discusses without prompting, follow-up with any of the following:

1. Reflecting back, can you think of an experience with math you had early in your life?
2. How did you feel about learning math in school when you were a kid?
3. How do you feel about math now as an adult?
4. And to clarify, what do you think of as math?

Their child in school this year:
Thank you for sharing those stories with me. Now, I'd love to shift and talk about your child. What impressions do you have about what learning math in school has been like for your child this year?
*Depending on what the parent discusses without prompting, follow-up with any of the following:

1. What does your child say about their math time?
2. How does your child seem to feel about their math time?
3. What is your sense for how your child is doing in math this year?
4. How do you feel about supporting your child with their math learning this year?
5. How do you feel about your child's experiences with math this year?

Their child in school in the past:
Thanks for sharing that- I know this year has been challenging for all of us for a variety of reasons! Now, I'd love to hear from you about what math in school has been like for your child in the past, before Covid.

*Depending on what the parent discusses without prompting, follow-up with any of the following:

1. Do any of your child's experiences with math in school pre-covid stand out to you (positive or negative)?
2. Pre-covid, how has your child seemed to feel about math time in school?
3. What is your sense for how your child has done in math in the past?
4. How have you felt about supporting your child with their math learning in the past?
5. How do you feel like your child's experiences with math in school have compared to your experiences learning math in school as a child?

Outside of school:
When I talked to your child, I always asked them about the math that they do outside of school with their families. Do you see your child doing math outside of school at home?
*Depending on what the parent discusses without prompting, follow-up with any of the following:

1. Who usually initiates the activities that use math at home?
2. Who else in your family participates in activities that use math?
3. How does your child seem to feel about engaging in activities that use math outside of school?

Hopes for the future:
Thank you for sharing a bit about your family with me. It's so interesting to hear the ways you engage in math together at home. Finally, before we wrap up, I'd love to hear a little bit about your hopes for your child in the future, specifically those that are related to math. What do you hope your child's math learning experiences will be like moving forward?
*Depending on what the parent discusses without prompting, follow-up with any of the following:

1. What kinds of things do you hope your child learns in math this year or in the future?
2. In what ways do you imagine your child might use math in their future?
3. How do you hope your child will feel about math in the future?

Closing:
Thank you so much for taking the time to talk with me today. I appreciate you sharing your family's experiences with me. Do you have any questions before we wrap up? Feel free to reach out if you have further questions in the future.