NORTHWESTERN UNIVERSITY

AN AIR TRAVEL DEMAND MODEL
FOR A SINGLE MARKET

By

Gerard Damay
and
Niels de Terra

A THESIS

SUBMITTED TO THE GRADUATE SCHOOL
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

for the degree

MASTER OF SCIENCE
in
TRANSPORTATION

Evanston, Illinois
December 1972

ACKNOWLEDGMENTS

John A. Bailey, Director of the Transportation Center at Northwestern University, assisted our efforts with introductions in industry and government. Professor Duane Marble, Director of Academic Programs, was instrumental in the provision of generous amounts of computer time.

Professors Peter Watson and Richard Westin, our thesis advisors, were sources of penetrating advice and moral support throughout. Professor Edward K. Morlok was kind enough to review and comment on an early draft.

We are deeply grateful to the carrier that provided us with unpublished data, and in particular, to those executives who were so generous with their time. This study would not have been possible without this assistance.

Thanks are also due to Mr. Knut Hammarskjold (IATA), Mr. Bjorn Elle (ICAO), Mr. Carl Swerdloff (DOT), Messrs Samuel Brown and Wayne Watkins (CAB), Ms. Joan Turek (NPA), Mr. George James (ATA) for their time and advice.

Gerard Damay
and
Niels de Terra

Evanston, Illinois
December 1972

TABLE OF CONTENTS

A B S T R A C T

" AN AIR TRAVEL DEMAND MODEL FOR A SINGLE MARKET "

An intensive empirical investigation is conducted into the factors influencing demand for coach-class reserved-seat air travel in a single pleasure market. An econometric model, with a forecasting capability, is developed through multiple regression analysis performed on a six year time series of weekly frequency (1966-1971). The best results are achieved with a multiplicative model. A multi-stage procedure of estimation is employed in order to avoid auto-correlation of residuals.

Demand is found to be significantly related to price,income, and seasonality. Price elasticity is equal to -1.6. Lagging of the price and income variables produces marked improvements in correlations with the dependent variable. Seasonality is incorporated into the model by means of a set of binary dummy variables. A forecast for the first quarter of 1972 is made and compared with actual traffic data not used in estimating the parameters of the equation. The error is only -2.65%.

LIST OF FIGURES

Fig. Page

LIST OF ABREVIATIONS

ATA - Air Transport Association (Washington, D.C.)

BLS - Bureau of Labor Statistics

CAB - Civil Aeronautics Board (Washington, D.C.)

CPI - Consumer Price Index

DOT - Department of Transportation (Washington, D.C.)

GNP - Gross National Product

GPFI - General Passenger Fare Investigation (by CAB)

IATA - International Air Transport Association (Montreal)

ICAO - International Civil Aviation Organization (Montreal

NPA - National Planning Association (Washington, D.C.)

OAG - Official Airline Guide

O-D - Origin to Destination

PAX - Passenger

SMSA - Standard Metropolitan Statistical Area

SST - Supersonic Transport

CHAPTER I

INTRODUCTION

The analysis of the demand for passenger air transportation has received increasing attention during the last ten years. The interest has, in part, been stimulated by attempts of the Civil Aeronautics Board (C.A.B.) to refine demand analysis to the point where it might be used to aid the regulatory decision-making process, and, in part, by academic researchers seeking to bring modern econometric analytical techniques to bear on an important and complex problem in demand analysis.

The industry itself has not found it necessary, until very recently, to consider the use of sophisticated analytical tools in studying the demand for its product. A market that was experiencing a healthy secular growth did not need to be subjected to searching analysis. Furthermore, the domestic airline industry, which has been accurately described as a "homogeneous oligopoly", is not motivated to affect total demand for its product. With price competition non existent and other forms of competition aimed at increasing market shares rather than the overall level of demand, the emphasis has been on market research rather than theoretical economic demand analysis.

Purposes

The purposes of this study are fourfold:

1. To identify the factors influencing the demand for non-business travel in a single market. While this has been attempted previously, and with some success, this study undertakes a test of those relevant factors for which data exists, it analyzes the phenomenon of seasonality to a degree that is unique, and it goes into a detailed empirical investigation of lagged variables.
2. To construct an econometric demand model and then to estimate the structural parameters of the equation for coach-class reserved-seat travel in a single city-pair market. This has not previously been attempted with data of weekly frequency.
3. To obtain estimates of the price and income elasticities.
4. To develop a model with a practical and tested forecasting capability.

Methodology

Chapter II comprises a review of relevant existing literature. This review led the authors to conclude that both the concept of the value of time and the use of distance as a variable influencing the demand for air travel were not appropriate to this study. Price, income and seasonality were chosen for inclusion on the basis of the results of previous studies, which were also consistent with a priori knowledge. Finally, the review also prompted the authors to experiment with both multiplicative and additive models.

Chapter III presents the mathematical technique of regression analysis, with all its assumptions.

Chapter IV describes the basic hypothesis under which the study was approached. It details the data requirements for the hypothesis, as well as actual data obtained. It goes on to discuss the use of dummy variables for seasonality, and of lagged variables for price and income. Finally, it describes the process by which the best measures of the variables influencing demand were selected.

Chapter V summarizes the experimental selection of the best form of the equation, and goes on to present the values of the structural parameters. It explains in detail the procedure used to avoid auto-correlation of the residuals.

Chapter VI discusses the implications of the results, with respect to price and income elasticities. Traffic for the first quarter of 1972 is forecasted using the model calibrated with data from January 1966 to December 1971, and then compared to actual traffic data.

Chapter VII presents the conclusions drawn from the work, as well as the authors' views on where research of this type might next be directed.

CHAPTER II

REVIEW OF THE LITERATURE

Fifteen papers, theses and dissertations on demand were reviewed and summarized in the course of the authors' work. They are:

Gronau:	Values of Time (1967)
Brown & Watkins:	Elasticities (1968,1971)
ATA Model:	Airport Traffic Forecasting (1969)
ITA Model:	Regional Growth Rates of Air Traffic (1971)
Bjorkman:	European Air Transportation (1970)
Asher:	SST (1971)
Quandt:	Abstract Mode Model (1971)
Mathematica Studies:	Air Travel Demand (1965)
Kessler:	Air Travel Demand (1965)
Port of New York Authority:	Forecasts of Domestic Traffic (1969)
Verleger:	Point to Point Model (1971)
Stuart:	Cross Sectional Model of the Hawaiian Market (1968)
Joun:	Air Travel Demand (1966)
Taneja:	North Atlantic Travel Demand (1971)
Schultz:	Marketing Planning Model (1971)

Five of them have been selected as being relevant to this particular study, and they are presented at length below:

Section 1: Joun

Section 2: Stuart

Section 3: Taneja

Section 4: Verleger

Section 5: Schultz

Section 6 summarizes briefly the findings for that review of the literature.

Section 1: Young Pyo Joun

The demand for Air Travel, (University of Washington, Ph.D. 1966, 90 pages).

1.1 Purpose

The primary purpose of the study was to construct a theoretical air travel demand model and to obtain empirical estimates of the parameters of the air travel demand function.

1.2 Theoretical Approach

The value of time was a starting point. Recognition of the value of time and time constraint produced a somewhat different equilibrium condition for consumers and firms from the traditional demand theory. To avoid a specification bias, a speed variable was introduced.

1.3 Mathematical Formulation

The time series demand models contained 4 explanatory variables:

$$Y_t = AP_{at}^{\beta_1} \; P_{tt}^{\beta_2} \; X_t^{\beta_3} \; S_t^{\beta_4} \; e^{rt} \qquad (2\text{-}1)$$

where:

Y_t = per capita scheduled U.S. domestic airline revenue passenger miles at time t.

P_{at} = the weighted price index of the air fare at time t, deflated by the BLS consumer price index.

P_{tt} = the yield on first class passenger railroad travel, in cents per mile at time t, deflated by the BLS consumer price index.

X_t = the per capita disposable personal income at time t, deflated by the GNP.

S_t = the average speed of the U.S. domestic trunk lines at time t, in miles per hour.

e^{rt} = the exponential growth factor, a surrogate variable for consumer learning and the expansion of the air travel to previously unserved regions, etc.

The coefficients β_1, β_2, β_3, β_4, are respectively the price elasticity, the cross-elasticity, the income elasticity, and the speed elasticity.

The coefficient r measures the constant autonomous growth factor.

The demand functions for business and non-business air travel are assumed to have the same form of functional relationship as the aggregate demand equation, the interpretation of the estimated parameters being different.

The cross section air travel demand function considers current prices and speed as constant for a given time period and can therefore be written as:

$$Y_{it} = e^{(\alpha_1 + \alpha_2 R_{it} + V_{it})} \cdot X_{it}^{\alpha_1} \tag{2-2}$$

where:

Y_{it} and X_{it} are defined as in the time series model.

R_{it} = a demographic dummy variable representing the taste of passengers.

V_{it} = the error term.

1.4 Data

All data were yearly. Traffic data were obtained from the O-D survey of the CAB. Because there is no published data for the number of business and the number of non-business air passengers, specific city-pairs were assumed to be "business traffic" and others "non-business traffic."

Fares and speed of airplanes came from the O.A.G. The fare used was a weighted average of 1st class and coach prices. Income and population figures came from "Sales Management."

For the cross sectional model, the National Market Survey data collected by the University of Michigan's Survey Research Center was used.

1.5 Results

In spite of statistical problems such as multicollinearity, serial correlation and measurement errors in independent variables, estimates of parameters were found. The estimated aggregate price elasticity was greater than 1 (in absolute value) and the aggregate income elasticity was close to unity. When omitting the speed variable, the income elasticity was overestimated and the price elasticity was underestimated. The estimation of business and non-business price elasticity showed that price elasticity is larger for non-business air travel than business air travel, which conforms to a priori theoretical expectation. The aggregate time series income elasticity was not in the range between the business and non-business time series income elasticity!

1.6 Comments

The data base can obviously be criticised, and especially the price variable. The statistical results are poor, some standard errors being bigger than the coefficients themselves, and, as Mr. Joun concludes himself: "It would be of interest to experiment with different functional forms instead of limiting the choice of functional form to log-linear functional relationship."

However, the theoretical specification of the model, including "the value of time concept" is interesting. The use of the "Theil and Nagor test of the independence of regression disturbances" when the Durbin-Watson test is inconclusive is also worth noting.

Section 2: Merril M. Stuart

The use of a Gravity Model in interpreting air passenger traffic between Hawaii and the Continental United States. (Columbia University, Ed.D, 1968, 153 pages).

2.1 Purpose

The purpose of the study was to examine the hypothesis that air passenger traffic to the Hawaiian Islands is a function of distance and population distribution.

2.2 Theoretical Approach

A gravity model was the main analytical tool used in the study. It was used primarily to interpret the relationship of traffic, distance and population involved in the pattern of air passenger traffic between urban areas in the USA and Honolulu.

2.3 Mathematical Formulation

The basic gravity model was altered to form the following equation, which is compatible with the particular over-water and mainland originating air traffic being investigated.

$$\log \frac{T_i}{P_i} = \log K - \alpha \log D_i \tag{2-3}$$

where

i = index identifying the traffic generation markets.

T_i = the number of air passenger trips between Honolulu and an ith urban area.

P_i = population of an ith urban area.

K = constant.

α = exponent of distance.

D_i = air distance between Honolulu and a ith urban area.

2.4 Data

Traffic data came from the International O-D survey of U.S. flag airline passenger traffic, conducted semi-annually, for the months of March and September, by the C.A.B.

Population data, for the 1955 data, came from a geometric average of the 1950 and 1960 decennial censuses, and for the 1965 data from "Sales Management, the magazine of marketing."

2.5 Results

The analysis of the gravity model revealed that the volume of air passenger traffic between Honolulu and 72 Standard Metropolitan Statistical Areas (SMSA) was highly related to the population size and distance to Hawaii of the SMSAs. The exponents derived for air passenger travel between Honolulu and urban areas in the USA were relatively high ($4.35 \leq \alpha \leq 4.99$), compared to those found in other studies using gravity models to analyze travel within conterminous areas.

The gravity model was secondarily employed to identify the residual or anomalous SMSAs producing more or fewer air passengers than would be expected in view of their population size and distance to Hawaii. From such an analysis, it was found that the volume of traffic generated from a SMSA was highly related to the existence at the SMSA of through-service with connections to Hawaii. In addition, it was also found that the volume of traffic generated from a SMSA was highly related to the income and occupational nature of the SMSA.

2.6 Comments

The data base seems rather poor, and the model does not seem good for forecasting purposes. However, the use of the model as a diagnostic tool is very interesting. One must not forget that the Hawaiian market is a very special market, because of the 2400 miles of ocean that separate it from the conterminous United States, and because of the fact that traffic generation is concentrated on the mainland.

Section 3: Nawal K. Taneja

A model for Forecasting Future Air Travel Demand on the North Atlantic (M.I.T., Research Report, 1971).

3.1 Purpose

The purpose of the study is threefold:

a. To identify, explain and evaluate the critical factors which have influenced North Atlantic air travel demand in the past and those which may become important in the next 15 years.

b. To develop an analytical model to estimate the structural parameters of the North Atlantic air travel demand equation.

c. To make a reliable and realistic forecast of the air travel demand on the North Atlantic for the period 1970 - 1985.

3.2 Theoretical Approach

The basic hypothesis is the existence of a functional relationship between the total North Atlantic passenger travel volume and socio-economic and transport characteristics on both sides of the Atlantic. The model takes into account both European and U.S. socio-economic data, weighted by the average proportion of traffic generated by each country during the period 1951-1969. The transport variables are "cost", "speed" and "comfort/safety".

3.3 Mathematical Formulation

Four slightly different multiplicative models were tried. The best one was:

$$T_t = K \; D_t^{\beta} \; F_t^{\gamma} \; V_c^{\delta} \; (1 + g)^{\epsilon t} \qquad (2\text{-}4)$$

where

T_t = total traffic between U.S. and Europe

D_t = composite national income per capita

F_t = average cost of transportation

V_c = average cruise speed of aircraft

$(1+g)$ = function of time trend...a natural growth term. This factor represents such factors as population, improvement in service etc.

3. 4 Data

Yearly data are used simultaneously in time series and cross sectional forms. The time series traffic data was taken from annual

IATA reports. The cross sectional data was taken from the annual reports of Immigration and Naturalization Service. The national income data came from United Nations' Statistical Abstract. It was expressed into each country's national currency, then converted in U.S. dollar using the prevailing official exchange rate, then divided by population to determine per capita income. In order to calculate the composite national income per capita, the traffic breakdown was used. The cost variable has been assumed to be the average New York - London fare obtained by multiplying the yield by the length of haul. The cruising speeds were taken from the CAB Handbook of Airline Statistics.

3.5 Results

The numerical results were:

β	= .587	se = .243	t: 2.41
γ	= -.271	se = .396	t: -.68
δ	= .089	se = .216	t: .41
ϵ	= .064	se = .012	t: 5.35
log k	= 3.982	se = 4.374	t: .91

$R^2 = .99$ $F = 1081$

$DW = 2.62$ $(4-d_u = 2.13;\ 4-d_L = 3.18$ at 5%)

The forecast gives, for the period 1970-1985, an average increase of traffic of 10.7% per year, with 10.9% and 8.1% as the optimistic and pessimistic forecasts.

3.6 Comments

Although the R^2 and F tests are very good, the t tests are bad, even at the .10 or .50 levels of confidence, and the Durbin Watson Ratio give an inconclusive result to the question of independence of residuals. Similarly, the value of the price elasticity ($\delta = -.27$) leavesus perplexed; one understands that Taneja says: "It is the model which has to be justified on the basis of validity and forecasting capabilities rather than the regression coefficients or elasticities."

However, while the empirical results are disappointing, the methodology used in this report is very good, and the deep study of the factors affecting the demand is the best of those reviewed.

The North Atlantic is a very complicated market, and a point-to-point, fare-by-fare analysis with data of higher frequency might have been more rewarding.

Section 4: Philip K. Verleger

A Point to Point Model of the Demand for Air Transportation (M.I.T., Ph.D., 1971, 312 pages).

4.1 Purpose

The thesis attempts to determine whether air travel can be treated as a homogeneous commodity and thus attempts to determine whether the estimation of aggregate demand functions for air travel is econometrically correct.

4.2 Theoretical Approach

The U.S. Civil Aeronautics Board has collected point to point traffic data on a sample basis since 1950. Since 1960, this data has been collected on a continual cross sectional basis. Therefore, it is possible to consider a point-to-point type of model. If the parameters of the same model estimated for a large number of city-pairs of diverse characteristics are approximately the same, then the aggregate model may be acceptable. The models developed total travel between each city-pair without regard to the composition of travel (business vs. pleasure), general economic conditions, etc.

4.3 Mathematical Formulation

In the simplest form, the model is specified as:

$$T_{ijt} = a \ P_{ijt}^{\alpha} \cdot M_{it}^{\beta_1} \ M_{jt}^{\beta_2} \cdot \epsilon \qquad (2\text{-}5)$$

where:

T_{ijt} is the travel from i to j at time t.

P_{ijt} is the price prevailing at t.

M_{it} and M_{jt} are mass variables in i and j.

From that simplest form, three basic theoretical forms can be estimated, depending on the specification of the mass variable:

a. $$T_{ij} = a(Z_i Z_j)^{\alpha} \cdot P^{\beta} \qquad (2\text{-}6)$$

b. $$T_{ij} = a(Z_i + Z_j)^{\alpha} \cdot P^{\beta} \qquad (2\text{-}7)$$

c. $$T_{ij} = a(Z_i^{\alpha_1} \cdot Z_j^{\alpha_2}) \cdot P^{\beta} \qquad (2\text{-}8)$$

The mass variable Z_{it} is

$$\int_0^{\infty} f_t^i \,(y)\ h_t^i \,(y)\ dy \qquad (2\text{-}9)$$

where

f_t^i (y) represents the income distribution at time t in i and h_t^i (y) the propensity to travel function. In discrete terms, it is approximated by:

$$Z_{it} = \sum_{k=1}^{n} N_{it}^{k} \cdot H_{it}^{k} \qquad (2\text{-}10)$$

where

N_{it}^{k} is the number of households with income Y within arbitrary limits Y_{j-1} and Y_j and H_{it}^{k} is the number of trips taken by an average household with income in that interval.

The price variable used is:

$$P_{ijt} = \left(\frac{TF}{TT}\right)_{ij}^{67} \cdot PF_{ijt} + \left(\frac{TY}{TT}\right)_{ij}^{67} \cdot P^{*}_{y\ ijt}\left(WY_{FF}^{ij} + WY_{FP}^{ij} \cdot FP_{dt} + WY_{DA}^{ij} \cdot DA_{dt}\right)$$

where (2-11)

P_{ijt} = price index at time t

$\left(\frac{TF}{TT}\right)_{ij}^{67}$ = % of traffic in 1967 in first class on ij.

PF_{ijt} = first class fare in 1967 on ij.

$\left(\frac{TY}{TT}\right)_{ij}^{67}$ = % of traffic in 1967 in coach on ij.

WY_{FF}^{ij} = % of full fare coach traffic on ij in 1967.

WY_{FP}^{ij} = % of family fare coach traffic on ij in 1967.

WY_{DA}^{ij} = % of Discover America coach traffic on ij in 1967.

FP_{dt} = family plan discount (as % of full coach fare)

DA_{dt} = Discover America discount (as % of full coach fare)

$$P^{*}_{yijt} = P_{yt} \cdot WY_i + P_{Ft} \cdot WF_i \qquad (2\text{-}12)$$

where

P_y = full fare in coach

P_F = first class fare

WY and WF = rationing weights

4.4 Data

Quarterly data were used for the period 1960-1967. Income data came first from the Survey of Buying Power, then from the U.S. Census Bureau, and finally from the Internal Revenue Service. The traffic data came from the CAB O-D surveys. The percentage of traffic in each class came from the CAB "General Passenger Fare Investigation". The distribution by fare class for June 1967 was assumed to apply to the entire period where the fares existed.

4.5 Results

The results are very lengthy and therefore they cannot be totally reproduced here. The conclusion is threefold:

a. Travel demand is price inelastic.

b. Travel demand is greatly income elastic.

c. No similar pattern between different markets can be found. Therefore, the CAB should not rule on rate changes on a national basis.

4.6 Comments

Verleger's review of literature contains many interesting and constructive criticisms. The theoretical specification of the independent variables is good, but when it comes to the experimental procedures and results, there is room for improvement. His conclusions on price elasticities are difficult to accept with standard errors of coefficients larger (sometimes three times as large) than the coefficients themselves. The use of the 1967 GPFI to estimate percentage of traffic at each fare for the total period also seems open to question. Furthermore, the manipulations and changes in the income data leave us perplexed about the validity of the results.

However, the conclusion that no general pattern between different markets can be found is very interesting for it provides empirical support to the serious questions that have been raised with respect to aggregated (cross-sectional) demand analysis.

Section 5: Randall L. Schultz

The Development of a Marketing Planning Model Through Simultaneous - Equation Multiple Regression Analysis: An Airline Study (Northwestern University, Ph.D., 1971).

5.1 Purpose

The purpose is to develop a marketing planning model for airline passenger demand in one two-city market utilizing simultaneous-equation multiple regression analysis.

5.2 Theoretical Approach

The a priori assumptions underlying the model are that the demand for air travel is influenced by price, marketing effort, population, business income, personal income and changes in prices.

The study uses classical techniques of model-building by means of regression analysis and draws upon Kotler's formulations of market share theory and also Palda's single-equation, single-marketing variable approach.

Schultz chooses a single city-pair for analysis and asks what are the determinants of demand and market share. The demand analysis is based on single-equation multiple-regression, and the market-share analysis on the simultaneous equation multiple regression model.

5.3 Mathematical Formulation

The initial formulation of the model of total industry demand is

DEMAND = f [price, advertising, population, business income, personal income, discounts, time, seasonality, GNP]

5.4 Data

Schultz used the ex ante published coach fare as the price variable in the absence of available data on average passenger fares.

Demand	=	Total industry demand, CAB O-D quarterly data
Price	=	Coach fare/CPI
Advertising	=	Total industry advertising expenditures
Population	=	Total air passengers for both cities
Business Income	=	Pre-tax corporate income
Personal Income	=	Disposable personal income
Discounts	=	Dummy variable for discount fare use
Time	=	Trend
Seasonality	=	Seasonal variation
GNP	=	Gross National Product

He made a variety of experiments on the functional relationship of the model. The basic demand function was subjected to four experimental forms: linear, no lags; logs, no lags; linear, lags; and logs, lags.

5.5 Results

Both unlagged versions produced better results than the lagged versions. Personal income, seasonality and price were found to be significantly related to traffic. The price elasticity coefficient was quite plausible (-1.12), and the estimated income elasticity approximately 1.5, and the regressions exhibited no significant multicollinearity or autocorrelation of residuals.

Schultz proceeded to test the predictive power of this model by making comparisons with new actual data not used in the parameter estimations. The best forecast results were provided by the linear (lagged and non-lagged) models, and turning points were accurately reflected.

5.6 Comments

Schultz recognizes the fact that his dummy variable relating to discounts (discount fares) may not appear in the model because of the crudeness of the price variable, and he says that "replacement of the dummy variable with a more refined measure of discount use ... could lead to a statistically significant relationship" (p. 114).

Generally, within the limitations of the data, Schultz's work and results are sound. The failure of lagged variables to contribute to the model may in large part be due to the frequency of the data, and consequently, the period lagged. A priori knowledge would incline one to doubt that a time lag of one quarter would be the most likely period for the variables in this model

The major contribution of Schultz's thesis lies in its development of an overall marketing planning model. (The review has concentrated on aspects dealing with the demand model because of its relevance to the objectives of this study and has not attempted to cover the market share analysis).

Section 6: Conclusion

The preceding review led the authors to conclude that price, income, and seasonality were relevant factors influencing demand. On the other hand, the concept of the value of time was deemed inappropriate to the model. Consumer choice theory need only incorporate the concept of the value of time in situations of inter-modal choice and/or in situations where travel time varies over time. The time series used in this study (1966-71) is characterized by homogeneous travel times due to the exclusive use of jet equipment. In addition, distance was not required as a variable with only one market under study.

Although the literature review would have indicated a preference for a multiplicative model, it was decided to follow the suggestion of Joun (cited above) to experiment with additive forms as well.

CHAPTER III

MATHEMATICAL TECHNIQUE: THE REGRESSION ANALYSIS

Section 1: Definition

The simplest form of stochastic relation between two variables X and Y is called a simple linear regression model. This model is formally described as:

$$Y_i = \alpha + \beta X_i + \varepsilon_i \tag{3-1}$$

where Y is the dependent variable,

X is the explanatory variable,

ε is the stochastic disturbance,

α and β are the regression parameters, which are unkown.

The values of X and Y are observable, but those of ε are not. The stochastic nature of the regression model implies that for every value of X there is a whole probability distribution of values of Y. This means that the value of Y can never be forecast exactly. The uncertainty concerning Y arises because of the presence of the stochastic disturbance ε which, being random, imparts randomness to Y. Therefore, the full specification of the regression model includes not only the form of the regression equation as given in (3-1), but also five basic assumptions:

Normality:	ε_i is normally distributed	(3-2)
Zero mean:	$E(\varepsilon_i) = 0$	(3-3)
Homoskedascity:	$E(\varepsilon_i^2) = \sigma^2$	(3-4)

Non autoregression: $E(\epsilon_i \epsilon_j) = 0 \quad (i \neq j)$ (3-5)

Non stochastic X: $\frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^2$ is a finite number different from zero. (3-6)

A complete discussion of these assumptions can be found in Johnston, "Econometric Methods," pp. 106 sq.

Section 2: Estimation of Regression Parameters

The regression parameters α and β can be estimated in three different ways: (a) least square estimation; (b) best linear unbiased estimation; (c) maximum likelihood estimation.

Least Square Estimation:

The principle of the least square estimation involves minimizing the sum of squared deviations of the observed values from the mean:

$$\Sigma \epsilon_i^2 = \Sigma(Y_i - \alpha - \beta X_i)^2 \qquad (3\text{-}7)$$

Using the usual partial differentiation techniques, we will take the derivations with respect to α and then with respect to β.

$$\frac{d \Sigma \epsilon_i^2}{\partial \alpha} = -2 \Sigma(Y_i - \alpha - \beta X_i) \qquad (3\text{-}8)$$

$$\frac{d \Sigma \epsilon_i^2}{\partial \beta} = -2 \Sigma X(Y_i - \alpha - \beta X_i) \qquad (3\text{-}9)$$

We set each of the two resulting equations equal to zero, (putting a "hat" on α and β to indicate that the resulting equations are satisfied by the least square estimators of α and β, not by their true values):

$$\Sigma Y_i = \hat{\alpha} n + \hat{\beta} (\Sigma X_i) \tag{3-10}$$

$$\Sigma X_i Y_i = \hat{\alpha} (\Sigma X_i) + \hat{\beta} (\Sigma X_i^2) \tag{3-11}$$

These equations are generally known as the least square normal equations. They can be solved for $\hat{\alpha}$ and $\hat{\beta}$; the results are:

$$\hat{\beta} = \frac{n(\Sigma X_i Y_i) - (\Sigma X_i)(\Sigma Y_i)}{n(\Sigma X_i^2) - (\Sigma X_i)^2} = \frac{\Sigma (X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma (X_i - \bar{X})^2} \tag{3-12}$$

$$\hat{\alpha} = \frac{1}{n} (\Sigma Y_i) - \hat{\beta} \cdot \frac{1}{n} (\Sigma X_i) = \bar{Y} - \hat{\beta}\bar{X} \tag{3-13}$$

Best Linear Unbiased Estimation

The best linear unbiased estimation requires that the estimator be a combination of sample observations, that they be unbiased and that its variance be smaller than that of any other linear unbiased estimator. It then becomes a problem of constrained minimization, which can be solved with the help of the Lagrange Multiplier method. The final results are exactly the same as those given in equation (3-12) and (3-13). One advantage of the best linear unbiased estimation is the determination of the variance of α and β:

$$\text{Var } (\beta) = \frac{n \sigma^2}{n(\Sigma X_i^2) - (\Sigma X_i)^2} \tag{3-14}$$

$$\text{Var } (\alpha) = \frac{\sigma^2 (\Sigma X_i^2)}{n(\Sigma X_i^2) - (\Sigma X_i)^2} \tag{3-15}$$

Maximum Likelihood Estimation

The maximum likelihood estimation uses the idea that one must find the estimators of the parameters which would most often generate the observed sample. To find these estimators, one must determine the likelihood function for the observations in the sample and maximize it with respect to the unknown parameters, which can be done by means of partial differentiation.

The final results here are exactly the same as those given in equation (3-12) and (3-13). It is also worth noting that the variance of the disturbances is equal to the sample variance of the least square residuals.

In summary, under the assumption of the classical normal linear regression model, the least squares estimators of the regression parameters are equivalent to the best linear unbiased and the maximum likelihood estimators. Therefore, they are unbiased, consistent and efficient.

Section 3: Generalization

Let us consider a set of T observations on k + 2 variables. These variables are denoted:

$$\begin{array}{cccc} Y_1 & X_{1,1} - - - - & X_{1,k} & X_{1,k+1} \\ \vdots & \vdots & \vdots & \vdots \\ Y_T & X_{T,1} - - - - & X_{T,k} & X_{T,k+1} \end{array}$$

where the k+2th variable is a vector of 1.

It is supposed that these variables are related by a set of stochastic equations:

$$\begin{bmatrix} Y_1 \\ \vdots \\ Y_T \end{bmatrix} = \begin{bmatrix} X_{1,1} & \cdots & X_{1,k+1} \\ \vdots & & \vdots \\ X_{T,1} & \cdots & X_{T,k+1} \end{bmatrix} \cdot \begin{bmatrix} \beta_1 \\ \vdots \\ \beta \\ \beta_{k+1} \end{bmatrix} + \sigma \begin{bmatrix} \epsilon_1 \\ \vdots \\ \epsilon_T \end{bmatrix}$$

which may be written in matrix notation as:

$$Y = X\beta + \sigma\epsilon \qquad (3\text{-}16)$$

where

Y is the T x 1 vector of observations on the dependent variable or regressand.

X is the T x k+1 matrix of observations on the independent variable, or the regressor, the k first vectors corresponding to the regression coefficient $\beta_1 \ldots \beta_k$ and the k+1 vector corresponding to the constant β_{k+1}.

β is the (k+1) x 1 vector of the fixed, but unkown regression coefficients.

σ is the standard deviation of the disturbance term.

ϵ is a T x 1 vector of independent, identically distributed random variables with mean = 0 and variance = 1.

The least square estimator of β is then:

$$\hat{\beta} = (X'X)^{-1} X' Y \qquad (3\text{-}17)$$

given $\hat{\beta}$, we can compute the residuals $u = \sigma\epsilon$;

$$\hat{u} = Y - X\hat{\beta} \qquad (3\text{-}18)$$

and from that an unbiased estimate of σ^2, the residual variance

$$\hat{\sigma}^2 = \frac{\hat{u}'\hat{u}}{T-k-1} = \frac{Y'Y - Y'X\hat{\beta}}{T-k-1} \tag{3-19}$$

The square root of $\hat{\sigma}^2$ is the standard error of estimate.

$$S_{Y,X} = (\hat{\sigma}^2)^{1/2} \tag{3-20}$$

If the X is independent of ϵ, the covariance matrix of regression coefficients is estimated by:

$$\hat{\Sigma}_{\hat{\beta}} = \hat{\sigma}^2 (X'X)^{-1} \tag{3-21}$$

and the square roots of the diagonal elements of $\hat{\Sigma}_{\hat{\beta}}$ on the standard errors of the coefficients:

$$\hat{S}_{\hat{\beta}_J} = S_{Y\cdot X}\left[(X'X)^{-1}_{JJ}\right]^{1/2} \qquad (J = 1, \ldots, k+1) \tag{3-22}$$

The T ratios are:

$$t_J = \frac{\hat{\beta}_J}{\hat{S}_{\hat{\beta}_J}} \tag{3-23}$$

The partial correlations between Y and X_J, given all other variables are:

$$R_{Y,X_J,X_k} = \left(\frac{t_J^2}{t_J^2 + T - k-1}\right)^{1/2} \qquad (k \neq J) \tag{3-24}$$

(the sign of any R is the same as the sign of the corresponding β_J)

The squared coefficient of multiple correlation is:

$$R^2 = \frac{Y'X\hat{\beta} - T\bar{Y}^2}{Y'Y - T\bar{Y}^2} = 1 - \frac{\hat{u}'\hat{u}}{Y'Y - T\bar{Y}^2} \tag{3-25}$$

The F statistics for the test of the hypothesis that $\beta_1 = \ldots = \beta_k = 0$ is:

$$F = \frac{R^2/k}{(1-R^2)/(T-k-1)} = \frac{(Y'X\hat{\beta} - T\bar{Y}^2)(T-k-1)}{k(Y'Y - Y'X\hat{\beta})} \tag{3-26}$$

with (k, T-k-1) degrees of freedom.

CHAPTER IV

INPUT

Section 1: Hypothesis

In the authors' view there already exists conclusive support for the general proposition that the demand for non-business travel is significantly related to disposable personal income, price and seasonality. The works reviewed above have established the point.

The hypothesis of the present study is that further refinement of our understanding of the determinants of the demand for air travel is possible in the following areas:

1.1: Price. To the authors' knowledge all demand studies in this field have relied upon a single price variable. This has usually been the published coach class fare appearing in the Official Airline Guide. However, any time series of passenger traffic after 1968 cannot validly be related only to the published coach-class fare. The introduction and widespread use of discount and promotional fares since that time has meant that a very large proportion of passengers travelling in the coach compartment of an aircraft are not travelling at the coach fare, but at widely differing fares. This is particularly true of a route such as the one under study, which is characterized by an unusually high proportion of non-business travellers (78% vs. 55% for the country as a whole).

Thus, a rigorous examination of the relationship between price and traffic would require data on the number of passengers travelling at each fare and this could be, at minimum, ten fares, if one is speaking only about reserved-seat coach-compartment traffic. An improved method of studying correlation between traffic and fare must be found if the coefficients for price elasticity are to have meaning.

1.2: Income. Cross-sectional and time-series studies of the demand for air travel covering the entire market have utilized national disposable personal income statistics. Such statistics are not available in any frequency for individual Standard Metropolitan Statistical Areas (SMSAs). Substitutes may be found, and payroll statistics are available for most large SMSAs. However, the authors contend that additional measures may exist that will significantly add to an explanation of the consumers' expenditures on air travel. Personal expenditures for non-business air travel are likely to fall into that same category of discretionary income that might be measured by department store sales or the level of savings (time deposits). This study seeks to find the best available measures of the income-related variable determining the demand for non-business air travel through an exhaustive examination of all logical possibilities.

1.3: Climatic Effect. The market under study is a north-south pleasure market, and the primary motivation of non-business travellers is the fact that the weather at the southern point of the city-pair is far more temperate than the weather at the northern end. The authors intend to test the hypothesis that the demand for air travel will increase during those periods of the year when the temperature differences between the north and south cities are greatest.

1.4: Population. It might seem appropriate to include a measure of the potential that a city might have for producing traffic. This measure would not be the number of inhabitants, since this need not be directly related to the city's traffic generation. Marketing surveys performed for the carrier would enable one to estimate population segments or classes clearly identifiable as being traffic generating, (e.g., households with annual incomes over $25,000). However, the authors decided not to pursue this question in view of the fact that such a variable would almost certainly be strongly intercorrelated with the income and income-related variables described above in part 2 of section 1.

1.5: Seasonality. The effects of seasonality on traffic have not received close examination in previous studies, largely because data with a frequency shorter than quarterly periods has not been available. Since the problem of seasonality is one of immediate concern to carriers the authors propose to investigate its influence on traffic with a view to developing at minimum an explanatory, and at best, a predictive model that would take into account the full effects of seasonality. This requires very frequent observations if all the fluctuations attributable to seasonality are to be measured.

1.6: Time Lags. Traditional analyses of air travel have often assumed a world of instantaneous communication and consumption. It has now become accepted practice to introduce the concept of lagged variables into demand analysis. Non-business air travel is a service typically purchased well in advance of its consumption. For this reason, a study of the factors influencing the demand for air travel must examine closely the degree to which consumption phenomena recorded in one period (traffic) are influenced by events occuring in earlier periods (price, income, weather). It is the authors' contention that income and price-related variables, and possibly weather-related variables, will show a higher correlation with the dependent variable, traffic,when they are lagged. Because of the likelihood that relatively short time periods will contain significant changes in values we are faced with a requirement for data of at least weekly frequency.

Section 2: Data Required

This section deals with the data requirements corresponding to the hypothesis stated above. By concentrating on two similar city-pairs, which had a common southern destination city, the authors were able to undertake an exhaustive search of all available data. The hypothesis called for measures of traffic, price, income, and temperature differences. The effects of seasonality are accounted for by using dummy variables, the construction of which are described below in section 6.

2.1: Traffic

A number of previous studies in this field have made use of time series data that spanned major qualitative changes in the nature of the service offered to the customer. These changes introduced hidden and unmeasurable variables. For example, a time series of traffic that has its origin in a period when piston-engined single-class aircraft predominated, but runs into the period when jets and the introduction of coach-class service have fundamentally transformed the nature of the service provided through large scale price decreases, and increased levels of service and time savings, is open to serious question when used in model-building. For this reason, the authors established as a requirement for the traffic data that it be drawn from a homogenous period, relatively free from such distorting effects. From 1965 on, one could be certain that the changes in the air travel market brought about by jet equipment had made their full impact on the public. Thus, 1965 was taken as the earliest possible starting year for a time series running, if possible, to the present time.

The prevalence of considerable fluctuations in traffic resulting from seasonal influences, as discussed above, resulted in a requirement for traffic data of high frequency. Quarterly data, for example, would have obscured the very seasonal fluctuations that the authors hoped to incorporate in the model. Even monthly data would not have permitted as accurate an analysis of seasonality as a preliminary examination of the traffic indicated was necessary (see Fig. 4-1, 4-2).

It was also decided that only a time-series of reserved-seat coach compartment traffic would be required. The decision to ignore first-class traffic was based upon the fact that the proportion of business travellers on the routes under study was less than half the normal proportion for domestic U.S. air travel (22% vs. 45%). It is well accepted that business travel is price inelastic. Since the primary objectives

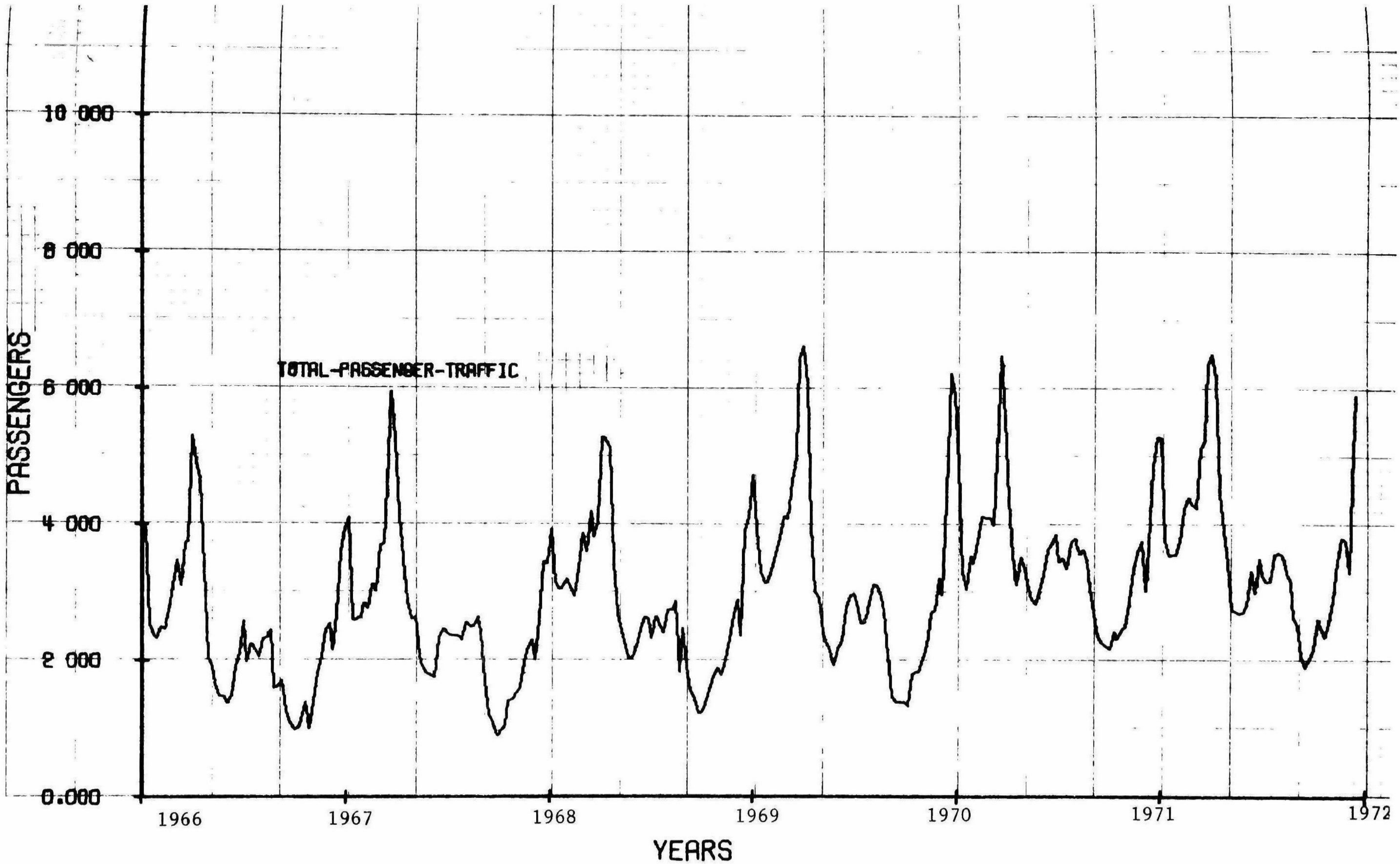


Fig. 4-1: TOTAL PASSENGER TRAFFIC
(Originating in City "A")

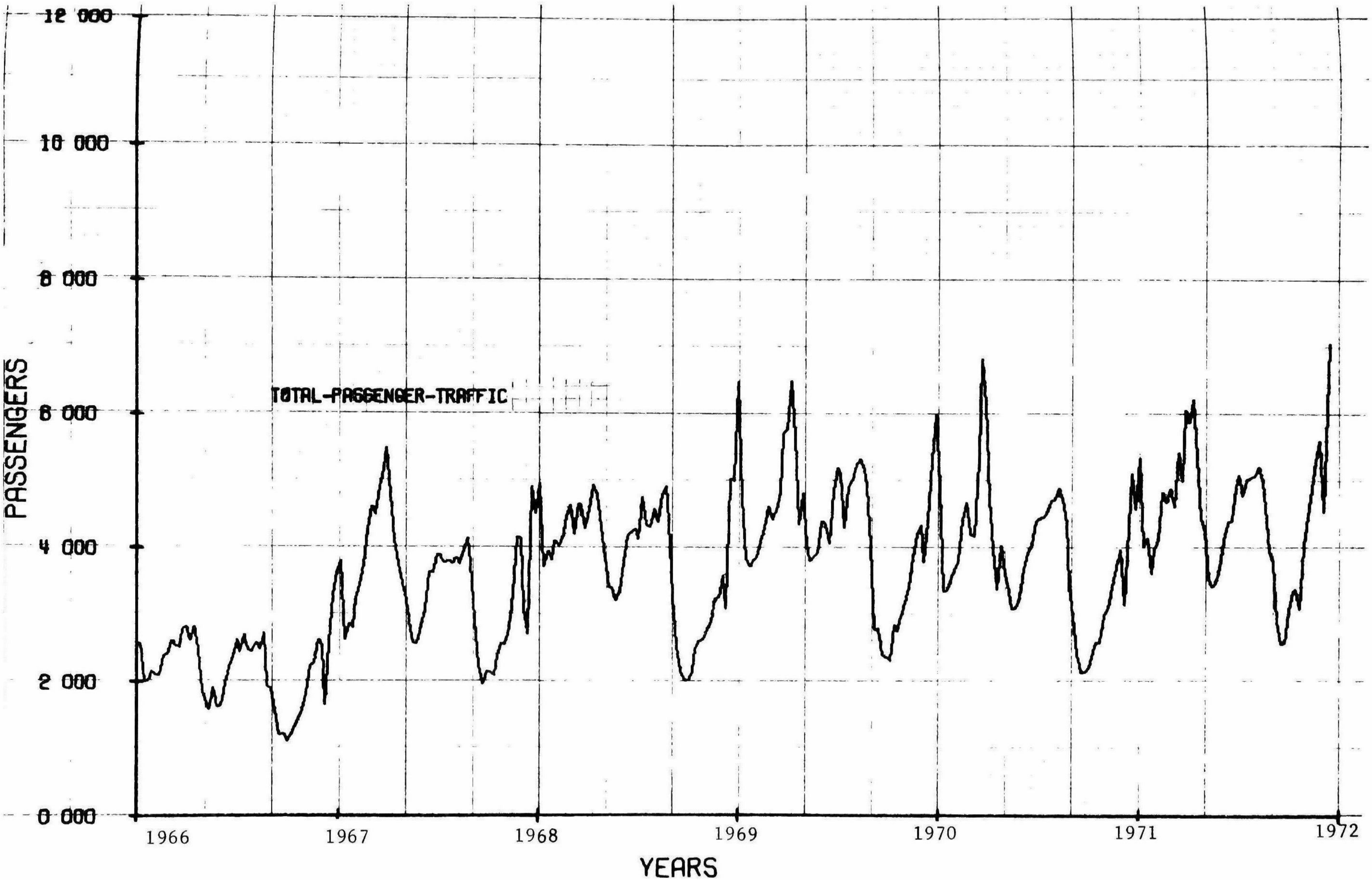


Fig. 4-2: TOTAL PASSENGER TRAFFIC
(Originating in City "B")

of this study relate to an examination of price elasticity, income elasticity, and seasonality, there was little to be gained by including first-class (business) travel. The exclusion of stand-by coach-compartment passengers is warranted by the assumption that this particular market, consisting of youth-fare and military traffic, is fundamentally different in economic terms from the reserved-seat coach traffic.

Thus, with respect to passenger traffic, the requirement was for a weekly time-series of passengers travelling on a reserved-seat basis in the coach-compartment of the aircraft. Furthermore, such data should, if possible, represent origin-destination traffic. This is because a number of the independent variables relate only to the two northern cities. Finally, in order to minimize the effects of major changes in air transportation the data series could not begin prior to 1965.

2.2 Income:

Ideally, the study required a time series covering disposable personal income in the two northern cities. Such data is not available, and the authors conducted an exhaustive examination of the income data available. It was decided to evaluate both direct measures of income, such as local and national payroll indices, and also indirect measures of income, such as department store sales and levels of savings deposits. It was also decided to test a measure of income expectation such as a common stock price index.

2.3 Weather:

In order to test the hypothesis that an important motivation for non-business travel to the southern city and its surrounding area lies in the seasonal climatic differences between the northern cities and the southern city, it was necessary to have a time series reflecting this phenomenon. Since it was assumed that the mean weekly temperature in the southern city remained relatively constant, it was not necessary to take north-south temperature differences. Instead, the absolute weekly mean temperature values for northern cities sufficed.

2.4 Seasonality:

The fourth major influence on traffic, was studied by means of dummy variables (see below, Section 4).

Section 3: Data Available

3.1 Traffic:

Any study of the demand for air travel between single city-pairs, as opposed to aggregate cross-sectional or time-series studies of the entire industry, is made difficult by the scarcity of accessible data sources. The Civil Aeronautics Board (CAB) published quarterly origin-destination (O-D) data covering travel on trunkline carriers between continental U.S. city-pairs. This quarterly series was published in a consistent format between 1960 and 1969. In 1969 the series was changed to include international travellers on domestic U.S. routes, and there were also alterations in the sampling methodology, which meant that the earlier nine-year series was not consistent with the post-1969 series. There is no published source of traffic data between U.S. city-pairs of a frequency less than quarterly.

Faced with a situation in which published data did not come close to meeting the requirements for the study, the authors were fortunate to receive the assistance and cooperation of a major U.S. carrier. Two options were available to the authors. The first involved carrying out a six-year weekly sample of individual ticket coupons in order to obtain the required data on coach-compartment traffic broken down by each coach-class fare. Such an approach would have required very large resources. The second option available was to make use of an existing time series prepared by the carrier for internal analytical purposes.

It should be noted that this data was of the type known as "on-board" rather than O-D. That is to say, it represented all persons travelling in the coach-compartment of the aircraft between the northern and southern cities studied, regardless of where their journeys might have commenced. As a test of the validity of the assumption that, in this case, "on-board" traffic data could be used as a substitute for O-D data, the authors compared a series of quarterly O-D totals with "on-board" data and found that the relationship was reasonably constant. On-board data remained close to 15% above comparable O-D figures. The use of on-board data is also a reasonable approximation of O-D data in view of the fact that virtually all flights between the city-pairs under study are non-stop and none of the three cities is an important transit city, such as Chicago or St. Louis.

3.2 Price (Fare):

The treatment of the price variable in the study of air travel demand presents a complex problem. Previous studies involving single city-pair markets have relied upon the fares published in the Official Airline Guide. However, in referring to traffic within the past few years, it is a distortion to equate the published coach-class fare with the price paid by all passengers travelling on a reserved-seat basis in the coach compartment of the aircraft. The use of coach fares in demand studies is common because of the absence of accurate data, but the fact remains that this fare may not even come close to the fares actually paid. The proliferation of discount and promotional fares, especially since 1969, has meant that average fare received by the carrier from coach-compartment passengers is often well below the published coach fare.

The authors originally sought to obtain traffic data at each distinct fare. However, such figures are available neither from the CAB nor from the carriers. Certain airlines have now begun to assemble such data, but to the authors' knowledge no historical time series on such a basis exists. It would have to be constructed by sampling ticket coupons, which is an immense task.

One measure that may be obtained from the records of a carrier is that of revenue per passenger mile (yield) by city-pair. The coach yield provides an average price that is already weighted, by definition, by the number of passengers travelling at each fare. The yield is not a price perceived by any single passenger. It is a measured price. For this reason, the economist may have some hesitation in accepting yield as the price variable. In the authors' view, there does in fact exist satisfactory justification for using yield over the published coach fare price. The yield, artificial though it may be, does represent the cumulative effect of the mix of prices and marketing strategies offered by the carrier. It is an amalgam of perceived prices, and it must be accepted as being superior to the published coach-class fare in the absence of fare-by-fare traffic data.

It seems reasonable to assume that as a new coach fare is intro-

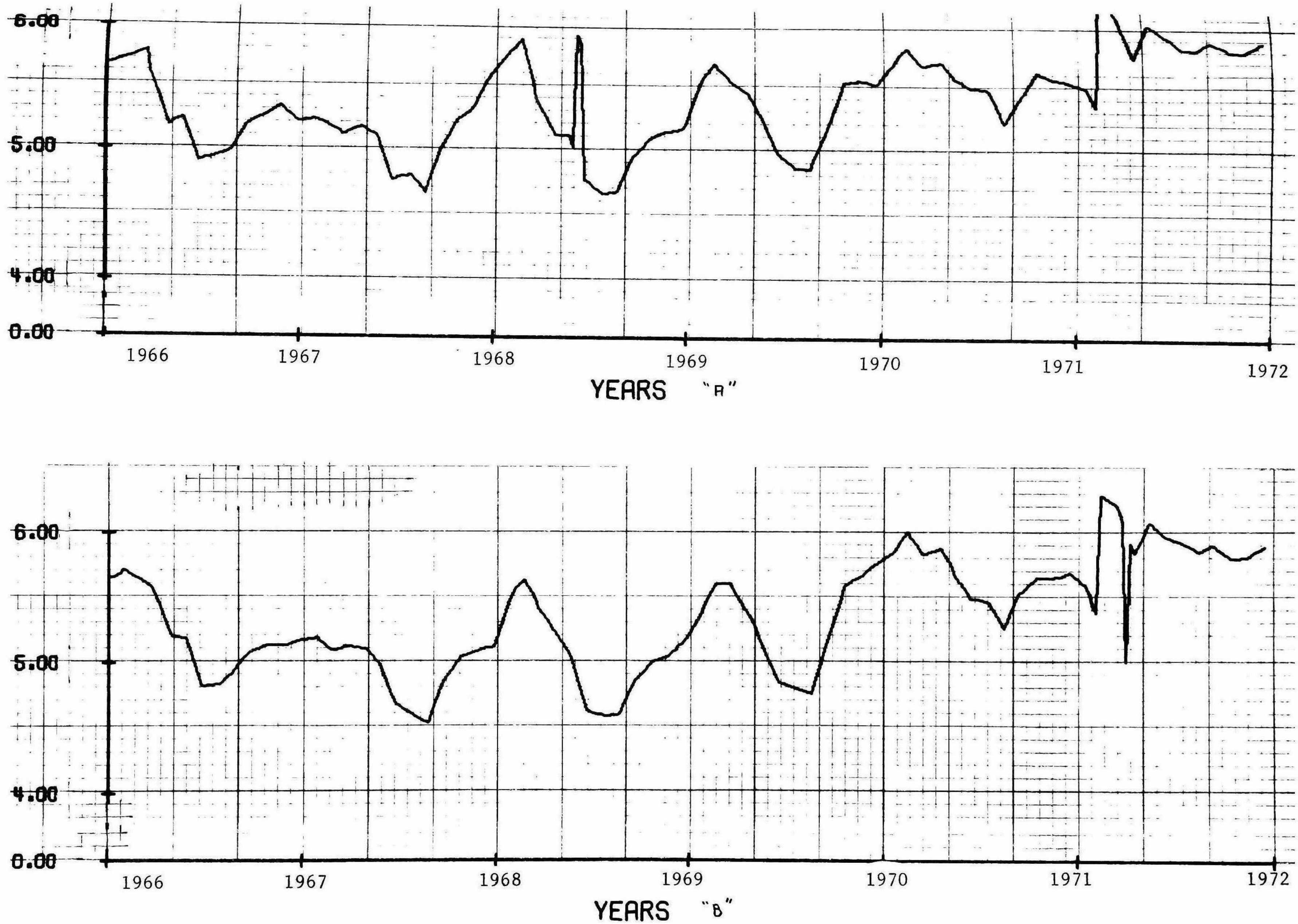


Fig. 4-3: YIELD (cents per mile)

duced (a discount or promotional fare) it will have an impact on the prevailing yield.

In this study, the published OAG coach fare was initially selected as being the only available measure of the price variable. However, it proved quite unsatisfactory in early correlation tests. When later, during the course of the study, it proved possible to obtain yield figures from data provided by the carrier,this was substituted (see Figure 4-3) and the OAG fare was discarded from the study.

The weekly national consumer price index was included in the data base so that the price variable could be expressed in constant dollars. This was accomplished by dividing the price variable by the consumer price index.

3.3 Weather:

The mean weekly temperatures for the northern cities were taken from the Weekly Weather and Crop Bulletin published by the Department of Agriculture.

3.4 Income and Income-Related Variables

Local: Four indices of payrolls are available for one of the cities. These provide measures of payrolls for the manufacturing, construction and service sectors, as well as a fourth for total payrolls. One income-related measure of discretionary income is the level of department store sales. It was thought that a relationship might exist between discretionary expenditures for air travel and those for department store purchases, and this was included in the study.

The best available measure of payrolls for the other city was found in a time series of employment levels for the manufacturing and non-manufacturing sectors. Department store sales for this city were also obtained. The total number of toll calls from this city was included for examination as a possible alternative measure of the level of business activity, and therefore, indirectly, of income. Finally, the production index for the second city was included as a further measure of changes in the disposable income of the residents of the second city's region.

Regional: Two measures were used to provide alternative sources for disposable income in the region of the northern cities. The first was the level of time deposits in the Federal Reserve District in which both cities are located. The second measure was the National Steel Production Index, which by its nature, was thought to bear a high correlation with the level of economic activity, and hence disposable income, in the northern cities.

National: Two national indices were incorporated into the study. Non-agricultural income as reported by the Bureau of the Census was used in the event that it would perform better than local measures of income. This possibility appeared less likely, although it was thought necessary to test this hypothesis.

Two Standard and Poor Indices were also tested, primarily because stock prices have some value as a measure of expected income.

Section 4: Dummies

It may be readily seen from an examination of Figures 4-1, 4-2, and 4-4, that the annual traffic pattern is subject to great seasonal fluctuations. The causes of these variations are difficult to pinpoint and impossible to quantify. We may consider that such factors as school holidays play an important part in shaping the seasonality of traffic on a route that is primarily pleasure travel. However, other habits and preferences of consumers of air travel services are certainly also at work here. All we can say with certainty is that there is a consistency in the pattern of seasonality based upon a study of the plotted traffic over a six year time period. Seasonality may be described as an artificial independent variable made up of a variety of known and unknown variables such as school holidays, religious holidays, traditions, weather, etc.

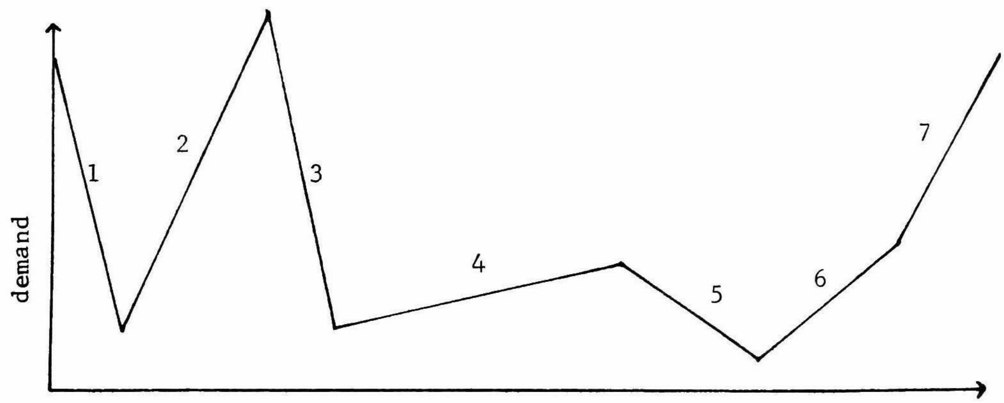


Fig. 4-4: Traffic Pattern

It is through the use of dummy variables that the effect of a variable such as seasonality may be brought into the equation. Binary dummy variables, taking on the value of 0 or 1, may be introduced into the model in eigher additive or multiplicative forms.

4.1: Additive Dummy Variables:

Given a model where total traffic T_t is a linear function of the trip price, and is also dependent on the season of the year (peak or non-peak). A simple formulation of the model might be

$$T_t = \beta_1 + \beta_2 P_t + \beta_3 \partial + \epsilon_t \qquad (4\text{-}1)$$

where ∂ is a dummy variable that takes a value of 0 or 1 depending on whether we are in a peak or non-peak period. Thus, we have either:

$$T_t = \beta_1 + \beta_2 P_t + \epsilon_t \quad \text{(peak)} \quad (4\text{-}2)$$

or

$$T_t = \beta_1 + \beta_3 + \beta_2 P_t + \epsilon_t \quad \text{(non-peak)} \quad (4\text{-}3)$$

Figure 4-5 illustrates that what we have accomplished is a shift in the curve according to the season.

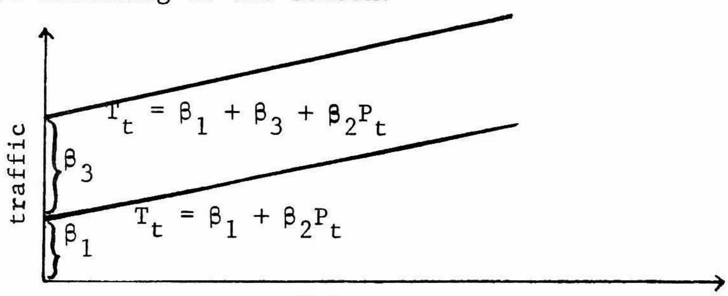


Fig. 4-5

4.2: Multiplicative Dummy Variables

If we postulate that the peak period affects traffic in such a way that the slope and not the intercept of the curve is affected, then the effect of seasonality appears as follows in the regression model:

$$T_t = \beta_1 + \beta_2 P_t + \beta_4 \delta P_t + \epsilon_t \quad (4\text{-}4)$$

In this case we have either

$$T_t = \beta_1 + \beta_2 P_t + \epsilon_t \quad \text{(peak)} \quad (4\text{-}5)$$

or

$$T_t = \beta_1 + (\beta_2 + \beta_4) P_t + \epsilon_t \quad \text{(non-peak)} \quad (4\text{-}6)$$

Figure 4-6 illustrates that the slope has altered.

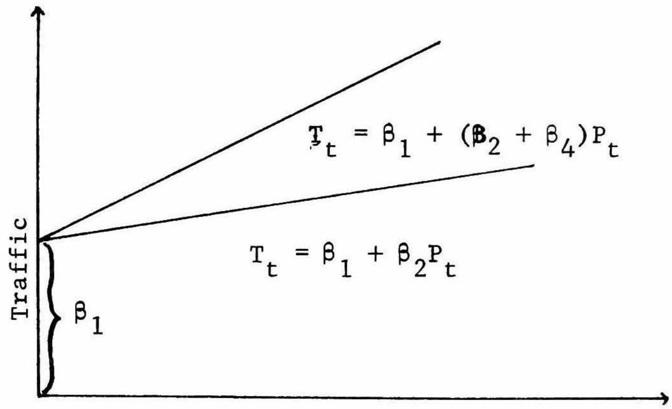


Fig. 4-6

4.3: General Formulation

The generalized model combines the additive and multiplicative dummy variables, so that the slope and intercept are subject to change according to the season: $T_t = \beta_1 + \delta\beta_3 + \beta_2 P_t + d\beta_4 P_t + \varepsilon_t$ (4-7)

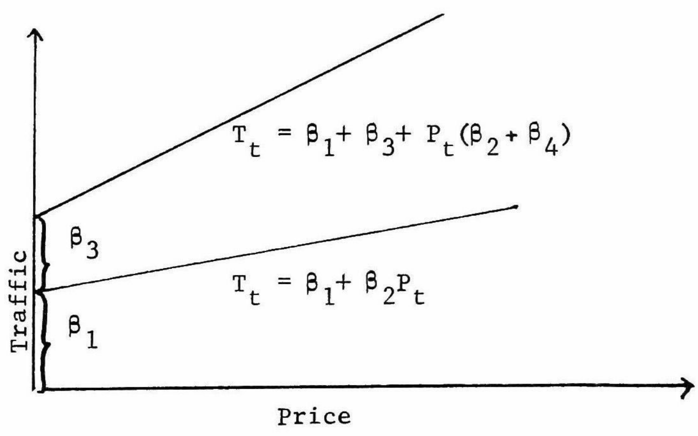


Fig. 4-7

We then have either

$$T_t = \beta_1 + \beta_2 P_t + \varepsilon_t \quad \text{(peak)} \qquad (4\text{-}8)$$

or

$$T_t = \beta_1 + \beta_3 + P_t(\beta_2 + \beta_4) + \varepsilon_t \quad \text{(non-peak)} \qquad (4\text{-}9)$$

An examination of the seasonality patterns in the traffic plots under study showed a total of seven major periods that were repeated each year. A set of dummy variables was established. The spacing of the time periods to be allocated to the dummy variables was derived from the plots and is summarized in Fig. 4-8. During any single time period or "season", only one of all the dummy variables will take on a value of 1, while all others will take on the value of 0.

By introducing a set of dummy variables it was possible to take into account the rather complex seasonality pattern and to improve the explanatory power of the model. It was found that introduction of the dummy variables to account for the seven seasonal periods described above improved the R^2 by 60%. Because of this it was decided to retain the "seasonality" dummy variables in the model.

It should be noted that another set of dummy variables introduced to explain the effects of the introduction and publicizing of promotional fares on the routes under study did not improve the results, and was dropped. One possible explanation for this failure of fare dummy variables lies in the complexity of the fare structure at any single point in time, and in the possible misspecification of these "fare dummy variables".

Dummy #	1	2	3	4	5	6	7
Symbol on P-O.	M	N	O	P	Q	R	S
1965							1
1966	4* 4	8 12	4 16	16 32	6 38	9 47	5 52
1967	4 4	7 11	6 17	16 33	5 38	9 47	5 52
1968	4 4	9 13	5 18	14 32	7 39	9 48	4 52
1969	4 4	9 13	6 19	13 32	7 39	8 47	5 52
1970	4 4	8 12	6 18	15 33	7 40	8 48	5 53
1971	4 4	9 14	6 20	13 33	6 39	9 48	3 51

Fig. 4-8 Dummy Table:

*Number of weeks for which Dummy #1 = 1; Dummy #1, Dummy #2, etc. are sequential.

Section 5: Lagging Variables

The variables affecting the demand for air travel may, on the basis of a priori knowledge, be assumed to influence demand with some time delay. An examination of these possibilities is essential to a study of the subject, since lags may significantly increase the explanatory power of the model. In addition, the introduction of lags in a time series model may add a forecasting capability. Kmenta has stated the general case for lagging variables:

> "In setting up the regression equation in this way [$Y_t = \alpha + \beta X_t + \epsilon_t$] we are, in fact, assuming that the current value of Y may depend on the current value of X but not on any of the past values of X. A more general formulation, which would allow for the current as well as the past values of X to affect Y, would be written as
>
> $$Y_t = \alpha + \beta_o X_t + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \ldots + \beta_m X_{t-m} + \epsilon_t.$$ [1] "

However, this formultaion, which leads to either a geometric or a Pascal lag, presents an obvious problem of multicollinearity between the independent variables X_t, X_{t-1}, $X_{t-2}, \ldots, X_{t-m}$. Such a geometric formulation is appropriate to demand situations in which the dependent variable at time t is affected by a series of varying past values of the independent variable at times t-1 to t-m. An illustration of this might be the demand for a speculative stock where the demand would be a function of expected variations in future price values based upon changes in past price values.

It will be clear that the case under study is inherently different from the case calling for the above formulation. A marketing survey

[1] (p. 473, Jan Kmenta, Elements of Econometrics, Macmillan , N.Y., 1971.)

performed on-board by the carrier on the north-south routes showed that 62% of coach-class passengers had decided upon travel within 30 days of departure and 75% within 60 days of departure.

Therefore, the authors concluded that since there was a clear lag between the consumer's decision to purchase air travel services and the actual date of departure (i.e., the date on which the travel would be recorded as a traffic statistic), the passenger traffic during a given time period should depend, inter alia, on past values of a number of variables.

An inclination to travel is combined by the consumer with a response to the perceived price at the time the services are purchased. Although air fares do not vary from month to month, the availability of discount and promotional fares does vary with the seasons. These variations result in continuously fluctuating average yields per passenger.

Thus, we do not require a geometric formulation of lags because consumption is not a function of expected changes in the price of air travel. Rather than the geometric formulation, we now require a simple formulation $Y_y = \alpha + \beta X_{t-m}$ and our problem can be reduced to that of establishing the value of (t-m).

The independent variables were first considered with a view to selecting those that might be expected to show improved correlations with the dependent variable when lagged. Thus, incomrelated variables could reasonably be expected to show higher correlations when lagged for the reasons given above.

The variables to be tested for lagging were each lagged 22 times, from 1 to 22 weeks. The simple correlation coefficients between the dependent variable and the lagged independent variable were arrayed in tabular form so that the highest correlation coefficient could be identified along with the "best" (t-m) value.

The study of the correlation patterns emerging from the study of lags showed three distinct patterns. Certain functions, when lagged, produced a clear peak of correlation coefficients. Where these seemed to have a logical explanation (e.g., in the case of yield)[1], the peak was used to select the lag period to be used at a later stage in the regression analysis. An example of this first case is given in figure 4-9:

Lag 0	.499
1	.472
2	.508
3	.542
4	.571
5	.583
6	.578
7	.555
8	.522
9	.483
10	.440
11	.397
12	.355
'	'
'	'
'	'
22	.272

Fig. 4-9: Pattern of the correlation coefficients between yield and total traffic when lagging the yield.

Where the peak did not appear to have a logical explanation (e.g., payroll-services still increasing after a 22-week lag), the use of a lagged

1 cf 46-47; Also provides a suitable forecasting lead. It is not implied that the lag reflects a lack of information.

variable was not considered. An example of this case is given in Fig. 4-10:

Lag 0	.295
1	.297
2	.299
3	.303
4	.306
5	.309
6	.310
7.	.311
8	.312
9	.313
10	.314
11	.317
12	.322
13	.343
14	.352
15	.361
16	.369
17	.376
18	.383
19	.389
20	.396
21	.403
22	.408

Fig. 4-10: Pattern of correlation coefficients between payroll-services and total traffic when lagging payroll services.

A second type of function remained constant and it was assumed that there was no valid reason for lagging such functions. (e.g., non-agricultural income). An example of this type is given below in Fig. 4-11.

Lag 0	.345
1	.344
2	.345
3	.345
4	.347
5	.347
6	.347
7	.344
8	.343
9	.343
10	.343
11	.344
12	.344
'	'
'	'
'	'
'	'
22	.343

Fig. 4-11: Pattern of correlation coefficients between non-agricultural income and total traffic when non-agricultural income is lagged.

The third type of function resulted in cyclical fluctuations in the correlation coefficients (e.g., steel production). Since no logical explanation could be found for this phenomenon, these functions were also not considered in their lagged forms. An example is given in Fig. 4-12.

Lag 0	.047
1	.064
2	.088
3	.114
4	.137
5	.146
6	.140
7	.124
8	.097
9	.069
10	.058
11	.017
12	.033
13	.089
14	.086
15	.116
16	.135
17	.159
18	.176
19	.200
20	.215
21	.206
22	.202

Fig. 4-12: Pattern of correlation coefficients between steel production and total traffic when steel production and total traffic when steel production is lagged.

The following variables were studied for the possible introduction of lagged version into the general equation:

Var. No.	City A	City B
#5	Weather	Weather
#6	U. S. Steel Prod. Index	U. S. Steel Prod. Index
#7	Dept. Store Sales	Dept. Store Sales
#8	Payrolls-Mfg.	Citywide Production Index
#9	Payrolls-Services	Toll Calls from City B
#10	Payrolls-Construction	Employment-Mfg.
#11	Payrolls-Total	Employment-Non-Mfg.
#12	Consumer Price Index	Consumer Price Index
#20	Average Coach Yield	Average Coach Yield
#21	U.S. Non-Agric. Income	U.S. Non-Agric. Income
#22	Time Deposits(Regional)	Time Deposits(Regional)
#24	Standard & Poor 425 Ind.	Standard & Poor 425 Ind.
#25	Standard & Poor 500	Standard & Poor 500

Fig. 4-13

The results are given for city A in Fig. 4-14 and for City B in Fig. 4-15.

#5	Temperature	*6-week lag : increase of 35%
#6	Steel Production	Useless (3 wks. + 8%)
#7	Dept. Stores Sales	*15-week lag: + 225%
#8	Payrolls-Mfg.	Useless (5 wks: + 10%)
#9	Payrolls-Services	Useless (22 wks.:+ 16%)
#10	Payrolls-Construction	Useless (6 wks.:+2%)
#11	Payrolls-Total	*12-week lag: + 20%
#12	Consumer Price Index	Useless: Constant
#20	Yield	*5-week lag: + 17%
#21	U.S.Non-Agric. Income	Decreasing
#22	Time Deposits(Regional)	Decreasing and then Increasing
#24	Standard & Poor 425 Ind	Decreasing and then Increasing Slowly
#25	Standard & Poor 500	Decreasing and then Increasing Slowly

Fig. 4-14: City A results of lag study.

City B:

#5	Temperature	* 7 week lag: increase 114%
#6	Steel Production	Useless-non-monotone.
#7	Dept. Store Sales	* 15 week lag: + 70%
#8	Production Index	* 15 week lag: + 600%
#9	Long Dist. Phone Calls	Useless (2 wks: + 8%)
#10	Employment - Mfg.	* 12 week lag: + 62%
#11	Employment - Non-Mfg.	Useless (12 wks: + 4%)
#12	Consumer Price Index	Useless: Constant
#20	Yield	* 6 week lag: +38%
#21	Non-Agriculture Income	Decreasing
#23	Time Deposits	Useless: Constant
#24	Standard & Poor 425 Ind.	decreasing and then increasing slowly
#25	Standard & Poor 500	decreasing and then increasing slowly

Fig. 4-15: City B: Results of lag study

The introduction of lags in a time series model has an additional benefit in that they add a forecasting capability to the model. This may be illustrated by the following example. If an established relationship exists between a dependent variable X at time t and an independent variable at time t-5, then this enables one to forecast the value of the variable T, under the assumption that the value of the variable X is known without appreciable delay at time (t-5).

Section 6: Data Selection

Having assembled on punched cards a data base representing all available measures of price and income over a period of six years, that were believed to be valid indicators of the factors influencing the demand for air travel,the authors were faced with the question of determining which individual data series were the best measures of the effects they represented. This section deals with the process of selecting the best measures from the data base.

6.1: Correlation Matrix

A valuable tool in the selection of data is the simple correlation coefficient matrix. This matrix presents correlation coefficients between the dependent variable and all independent variables as well as between all independent variables.

The values of simple correlation coefficients present certain problems of interpretation because they are not measured on a ratio scale (i.e., a value of .4 is not twice as good as a value of .2). However, the square of the correlation coefficient R^2 is linear, and therefore may be used for making comparisons between coefficients.

It should be noted that such coefficients are not adjusted for the degrees of freedom lost in measuring the estimation equation. Hence, they are larger than they should be. Since these are simple correlation coefficients, they can be made unbiased by multiplying them by (N-1)/N, with N being the number of observations (in this study N = 312 for non-lagged runs).

The procedure followed in selecting the price variable to be used is illustrative of the use of the correlation matrix in the process of data selection.

Two distinct measures of price were available to the authors. Each one of these measures could be used in current or in constant dollars deflated by using the consumer price index. This made for a total number of four possible price variables (OAG clach class fare and average yield, both in current or constant dollars).

Previous studies involving city-pair time series analysis have relied upon the coach-class fare as published in the OAG. As discussed above, the use of this measure is open to serious questions since the coach fare is no longer an approximation of the fare paid by the majority of coach-compartment passengers. Nevertheless, at the outset of this study it was the only price measure available to the authors, and it was incorporated in the experimental examination of variables. However, both in current and in constant dollars the OAG coach fare was poorly correlated with traffic, the dependent variable. The authors were faced with a situation in which an unsatisfactory measure was the one existing for one of the most important variables in the study.

Discussions with the carrier resulted in the offer of unpublished data from which a weekly time series of yield could be calculated. The substitution of a new price variable represented by yield (in constant and current dollars) resulted in a substantial improvement in the correlation with traffic. The differences may be seen from the results shown below that were taken from the correlation matrices.

	R^2
OAG Coach Current $	.203
OAG Coach Constant $	.347
Yield Coach Current $	.429
Yield Coach Constant $	.307

Fig. 4-16

It was, therefore, decided to use Coach yield in current dollars as the price variable in the study.

6.2: Multicollinearity

The generalized linear regression model requires that there be no exact or approximate linear relations holding among the observed values of the independent variables. If such relations do exist, it becomes very difficult to assess the partial relationships between an independent variable and the dependent variable. Thus, in the process of calculating the β vector, a singular matrix appears, which cannot be inverted. Another consequence is that the independent variables tend to duplicate each other and hence contribute little more in combination than they do separately.

"Classical" solutions to multicollinearity between several independent variables are either to discard all multicollinear variables except one (whichever is the most important in terms of its relationship with the dependent variable) or to use a new single variable which is a combination of the multicollinear variables.

Both the above solutions have been used in this study. The first solution was used to eliminate intercorrelation between "non-agricultural income" and "time deposits" - two variables related to income. The second solution described above was used as a basis for selecting one of four income-related variables - total payroll index over manufacturing payrolls, services payroll, and construction payroll.

CHAPTER V

OUTPUT

Section 1: Possible Mathematical Forms of Regression Equation

The least square procedure can be used to estimate non-linear equations whose coefficients are linear. These are called intrinsically linear equations:

$$Y = \alpha\ e^{\beta X} \qquad (5\text{-}1)$$

$$Y = \alpha\ X^{\beta} \qquad (5\text{-}2)$$

$$Y = \alpha + \beta \cdot \frac{1}{X} \qquad (5\text{-}3)$$

They can be respectively transformed into:

$$\log_e Y = \log_e \alpha + \beta\ X \qquad (5\text{-}4)$$

$$\log_e Y = \log_e \alpha + \beta\ \log_e X \qquad (5\text{-}5)$$

$$Y = \alpha + \beta\ Z \text{ with } Z = \frac{1}{X} \qquad (5\text{-}6)$$

From the analysis of literature the authors found that the two general types of equation used in similar cases were either the simple additive formulation:

$$Y = \alpha + \sum_{i=1}^{n} \beta_i\ X_i \qquad (5\text{-}7)$$

or the multiplicative formulation:

$$Y = \alpha \prod_{i=1}^{n} X_i^{\beta_i} \qquad (5\text{-}8)$$

It was decided to test these two possibilities with the same set of independent variables. In all cases, it appeared that the multiplicative model gave better results. Therefore, it was used in preference to the additive.

One advantage of the multiplicative model is the fact that, if X_i is the price(or income) variable, the corresponding parameter β_i is the price (or income) elasticity, as can be shown:

$$Y = \alpha \cdot X^{\beta} \tag{5-9}$$

$$\frac{dY}{dX} = \alpha \cdot \beta \cdot X^{\beta - 1} \tag{5-10}$$

$$\text{Elasticity} = \frac{dY}{dX} \cdot \frac{X}{Y} = \alpha \cdot \beta \cdot \frac{X^{\beta-1} \cdot X}{Y} = \frac{\alpha \cdot \beta \cdot X^{\beta}}{Y} \tag{5-11}$$

Introducing (5-9) in (5-11) we get:

$$\text{Elasticity} = \frac{\alpha \cdot \beta \cdot X^{\beta}}{\alpha \cdot X^{\beta}} = \beta \tag{5-12}$$

Such a constant elasticity assumption can be criticized as not corresponding to a priori knowledge, and a mixed model of the form $Y = \alpha \cdot X^{\beta_1} \cdot e^{X\beta_2}$ might be prefered because the elasticity is then $\beta_1 + \beta_2 X$, as shown below:

$$Y = \alpha \cdot X^{\beta_1} \cdot e^{\beta_2 X} \tag{5-13}$$

$$\frac{dY}{dX} = X^{\beta_1} \cdot \beta_2 \, e^{\beta_2 X} + \beta_1 \, X^{\beta_1 - 1} \, e^{\beta_2 X} \tag{5-14}$$

$$\frac{dY}{dX} = e^{\beta_2 X} \left(\beta_2 \, X^{\beta_1} + \beta_1 \, X^{\beta_1 - 1} \right) \tag{5-15}$$

$$\text{Elasticity} = \frac{dY}{dX} \cdot \frac{X}{Y} = \frac{e^{\beta_2 X} \left(\beta_2 X^{\beta_1} + \beta_1 \, X^{\beta_1 - 1} \right) \cdot X}{\alpha \cdot X^{\beta_1} \, e^{\beta_2 X}} \tag{5-16}$$

$$\text{Elasticity} = \frac{\beta_2 X^{\beta_1+1} + \beta_1 X^{\beta_1}}{X^{\beta_1}} = \beta_2 X + \beta_1 \qquad (5\text{-}17)$$

However, it was found that the joint use of a multiplicative model and of seasonality dummy variables allowed different levels of elasticities depending on the period of the year, and therefore it was not necessary to use a mixed model as described above.

Section 2: Numerical Results of the Multiplicative Model

The basic variables appearing in that model were:

Var #1:	Time 1 to 52	:	Seasonality
Var #7:	Department Stores Sales	:	Income
Var #11:	Total Payrolls	:	Income
Var #20:	Yield	:	Price
Var #26:	Time 1 to 312	:	General Increase
Var #13-19:	Dummies for seasonality	:	Seasonality

These first five variables were not intercorrelated, as can be shown in the following partial correlation table:

	1	2 dep. var.	7	11	20	26
1	/////					
2 (dep. var)	<u>.446</u>	/////				
7	-.192	<u>.529</u>	/////			
11	.053	<u>.354</u>	.468	/////		
20	-.215	<u>.611</u>	.330	.432	/////	
26	.008	<u>.434</u>	.449	.557	.484	/////

Fig. 5-1: Partial Correlations

The results were: $R^2 = .87$

$F = 124$ with 15 and 281 degrees of freedom

Residual Variance: .022

t test significant for all variables in equation

Section 3: Study of Residuals

However, the residuals seemed seriously autocorrelated:

	R^2
t-1	.47
t-2	.10
t-3	-.01
t-4	-.01
t-5	.08
t-6	.07
t-7	.01
t-8	-.01
t-9	.01
t-10	.07

Fig. 5-2: Autocorrelation of residuals (Stage 0)

To know if the autocorrelation was significant, the Durbin-Watson ratio was used. This ratio is the basis of a test for autoregression in the residuals. It is calculated as follows:

$$d = \frac{\Sigma\ (e_{t+1} - e_t)^2}{\Sigma\ e_t^2} \tag{5-18}$$

where

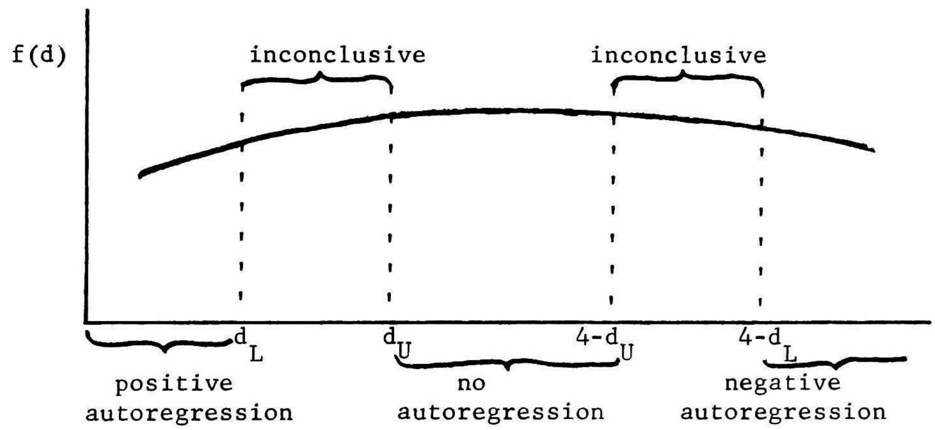
e_t = residual at time t

d = Durbin-Watson ratio

If there is zero autoregression, d will equal 2; if there is perfect positive autoregression d will equal zero, because e_t will then be a constant. If there is perfect negative correlation, d will equal 4.

The table for testing d is set up on the assumption that the residuals in the universe are uncorrelated and normally distributed. It then gives two values d_L (for "lower limit") and d_U (for "upper limit").

Kmenta gives a diagrammatic representation of the test:



In our case, d = 1.04 with a sample of size N=312-15=297 and with five explanatory variables.

The table was not available for such a large sample size. However, for N = 100

5 explanatory variables

1% level of confidence

we had

$d_L = 1.44$

$d_U = 1.65$

It was legitimate to use N = 100 instead of N = 297, because d_L and d_U were becoming increasingly stable as N increased. Therefore, it was concluded that a significant positive autoregression existed in the residuals, which meant that the formulas for estimating sampling errors ("F" tests, "t" tests) were in error, since the residuals were not independent of each other.

The answer to that problem was to use a multi-stage procedure, taking advantage of the stationary autoregression scheme.

The basic equation is:

$$\epsilon_t = \rho_1 \epsilon_{t-1} + v_t \tag{5-19}$$

with

$$v = \eta\ (0, \sigma_v^2)$$

Thus, in a univariable model,

$$Y_t = \alpha + \beta X_t + \epsilon_t \tag{5-20}$$

and

$$Y_{t-1} = \alpha + \beta X_{t-1} + \epsilon_{t-1} \tag{5-21}$$

Because $\rho_1 \neq 0$, (5-21) can be written:

$$\rho_1 Y_{t-1} = \alpha\rho_1 + \beta\rho_1 X_{t-1} + \rho_1 \epsilon_{t-1} \tag{5-22}$$

Subtracting (5-20) from (5-22) we get:

$$Y_t - \rho_1 Y_{t-1} = \alpha(1-\rho_1) + \beta(X_t - \rho_1 X_{t-1}) + (\epsilon_t - \rho_1 \epsilon_{t-1}) \tag{5-23}$$

Combining it with (5-19), it gives:

$$(Y_t - \rho_1 Y_{t-1}) = \alpha(1-\rho_1) + \beta(X_t - \rho_1 X_{t-1}) + v_t \tag{5-24}$$

If we define

$$Y^* = Y_t - \rho_1 Y_{t-1} \tag{5-25}$$

$$X^* = X_t - \rho_1 X_{t-1} \tag{5-26}$$

We see that the new model is of the form:

$$Y_t^* = \alpha' + \beta X_t^* + v_t \tag{5-27}$$

which can also be estimated by the least square procedure.

After such a transformation of variable, the following results were obtained:

$\rho_1 = .47$

$R^2 = .73$

F = 54 with 15 and 280 degrees of freedom.

"t" tests significant.

These results cannot be compared to the initial results, since the variables in the equation are transformed variables. The interesting point is the fact that they are statistically good in their own right.

Looking at the autocorrelation of residuals, better results were realized:

	R^2
t-1	.19
t-2	-.01
t-3	-.07
t-4	-.10
t-5	.03
t-6	.01
t-7	-.04
t-8	-.09
t-9	-.07
t-10	.04

Fig. 5-3: Autocorrelation of residuals (Stage 1)

The Durbin-Watson ratio was d = 1.58, which is located in the inconclusive part of the test. It was, therefore, decided to go one stage further.

From (5-27) we get:

$$(Y_t^* - \rho_2 Y_{t-1}^*) = \alpha(1-\rho_1)(1-\rho_2) + \beta(X_t^* - \rho_2 X_{t-1}^*) + v_t \qquad (5\text{-}28)$$

Developing and replacing, it gives:

$$[Y_t - \rho_1 Y_{t-1} - \rho_2(Y_{t-1} - \rho_1 Y_{t-2})] = \alpha(1-\rho_1)(1-\rho_2) + \beta[X_t - \rho_1 X_{t-1} - \rho_2(X_{t-1} - \rho_1 X_{t-2})] + v_t \qquad (5\text{-}29)$$

$$Y_t - (\rho_1 + \rho_2)Y_{t-1} + (\rho_1\rho_2)Y_{t-2} = \alpha(1-\rho_1-\rho_2+\rho_1\rho_2) + \beta[X_t - (\rho_1+\rho_2)X_{t-1} + (\rho_1\rho_2)X_{t-2}] + v_t \qquad (5\text{-}30)$$

$$Y_t = (\rho_1+\rho_2)Y_{t-1} - (\rho_1\rho_2)Y_{t-2} + \alpha(1-\rho_1-\rho_2+\rho_1\rho_2) + \beta[X_t - (\rho_1+\rho_2)X_{t-1} + (\rho_1\rho_2)X_{t-2}] + v_t \qquad (5\text{-}31)$$

If we generalize (5-31) to the multivariable case,we get:

$$Y_t = (\rho_1+\rho_2)Y_{t-1} - (\rho_1\rho_2)Y_{t-2} + \alpha(1-\rho_1-\rho_2+\rho_1\rho_2) + \sum_{i=1}^{n} \beta_i\,[X_{i,t} - (\rho_1+\rho_2)X_{i,t-1} + (\rho_1\rho_2)X_{i,t-2}] + v_t \qquad (5\text{-}32)$$

But we were working with natural logs; therefore:

$$Y_t = \mathrm{Log}_e Z_t \qquad (5\text{-}33)$$

$$X_t = \mathrm{Log}_e u_t \qquad (5\text{-}34)$$

and (5-32) can be written:

$$Z_t = \frac{Z_{t-1}^{(\rho_1+\rho_2)}}{Z_{t-2}^{\rho_1\rho_2}} \cdot e^{\alpha(1-\rho_1-\rho_2+\rho_1\rho_2)} \cdot \frac{\prod_{i=1}^{n} u_{i,t}^{\beta_i} \cdot \prod_{i=1}^{n} u_{i,t-2}^{\beta_i\rho_1\rho_2}}{\prod_{i=1}^{n} u_{i,t-1}^{\beta_i(\rho_1+\rho_2)}} \qquad (5\text{-}35)$$

With such a two stage procedure, the results became:

$R^2 = .77$

F = 62 with 15 and 279 degrees of freedom.

"t" test significant for all the variable in the equation

We had then $\rho_1 = .55$ and $\rho_2 = -.16$.

The autocorrelation of residuals became:

	R^2
t-1	.12
t-2	.07
t-3	-.02
t-4	-.07
t-5	.03
t-6	.03
t-7	-.01
t-8	-.02
t-9	-.03
t-10	.07

Fig. 5-4: Autocorrelation of residuals (Stage 2)

and the Durbin-Watson ratio d = 1.72, which was in the "nonautoregression" zone.

One important result of this study of the residuals is that ρ_1 was equal to .47 in the first stage and then equal to .55 in the second stage. This shows that the classical method used to take into account autocorrelation of residuals, namely the use of first differences, could not be used here as it was in the CAB or Schultz studies, because the use of first differences implies that the value of ρ is equal to one.

Section 4: Final Equation

Equation (5-35) provides the basic form of the model. The variables used in the final formulation are presented on the next page together with their coefficients and significance tests (Fig. 5-5).

As can be seen, all "t" tests are significant.[1] The first variable that entered the equation was "yield" and the second one, "Department Store Sales". These two variables alone explained 42% of the variation of the traffic. This shows that the traffic is highly related to the air ticket price and to the level of expenditures of the customers.

[1] Although in a predicitve model, the requirements on the "t" test are less strong than those in an explanatory model, the authors decided to keep the strong requirements in order to have both an explanatory and predictive model and to prevent the number of variables to become unmanageable.

Lags:	Department Store Sales:	15 weeks
	Payroll:	12 weeks
	Yield:	5 weeks

$\rho_1 = .55$ $\rho_2 = -.16$

	Coefficients	Standard Error	T Value
Constant	α = 3.71	.48	7.7
Time 52	β_1 = - .05	.015	-3.6
Dept. Stores	β_2 = .19	.040	4.8
Dum 2	β_3 = 2.72	1.03	2.6
Dum 5	β_4 = 9.38	2.29	4.0
Dum 6	β_5 = -16.76	1.62	-10.3
Yield	β_6 = - 1.63	.28	- 5.7
Time 312	β_7 = .12	.02	5.5
T52. D1	β_8 = - .11	.37	- 2.9
T52. D2	β_9 = .60	.11	5.2
T52. D4	β_{10} = .42	.10	6.0
T52. D5	β_{11} = - 3.76	.51	- 7.2
T52. D6	β_{12} = 4.31	.42	10
YLD. D2	β_{13} = - 2.35	.65	- 3.6
YLD. D4	β_{14} = - .91	.19	- 4.6
PAY. D5	β_{15} = .73	.29	2.4

Fig. 5-5: Final Results

CHAPTER VI

IMPLICATIONS

Section 1: Elasticities

The multistage procedure used to deal with autocorrelation of residuals is introduced because otherwise the F test and t tests are not reliable. However, the basic structure of the model (i.e., multiplicative model) being estimated is not changed. The coefficients can therefore still be interpreted as elasticities, as shown in Chapter V, Section 1 for an univariable model.

6.1: Price Elasticities

Seasonality dummies influence the price variables only during periods 2 and 4. Therefore, elasticities are shown below:

periods 1, 3, 5, 6, and 7:	-1.63	s.e = .28	t: -5.7
period 2 :	-3.98	s.e = .65	t: -3.6
period 4 :	-2.54	s.e = .19	t: -4.6

Fig. 6-1: Price Elasticities

These elasticities have the expected negative signs and their respective values for the different periods are quite plausible and in line with other studies in the field.

From these values, one sees that a decrease in yield will result in an increase of traffic. However, since yield is determined by the outcome of various marketing efforts, revenues might be changed by either selective or "across the board" decreases in fares, and nothing in this study allows the authors to suggest one or the other method.

Previous studies have produced a variety of estimates of price elasticity for air travel. In the authors' view, all such measures are approximations, and it is not possible to base precise forecasts of changes in traffic resulting from variations in fares only on these elasticities.

The findings of the present study show that the price elasticity of non-business travel is between -163 for 30 weeks of the year and -3.98 and -2.54 for two other periods of 9 and 13 weeks, respectively. This indicates that the short-run demand for non-business travel is strongly price elastic. These findings also support the introduction of lower promotional fares during dummy periods 2 and 4, (cf. Fig. 4-4).

6.2: Income Elasticity

The income elasticity, measured by the department store sales, is .19. This means that an increase of 10% in the public's income will generate an increase of 1.9% in the airline traffic. This seems very little, especially on a pleasure market. However, this results may be explainable by the choice of the income variable itself. Department

Store Sales are a function of the disposable personal income of a larger population than the population from which air travellers originate. Market surveys have shown that families with annual income below $20,000 do relatively little flying, yet these families certainly consume a proportionate amount of goods sold in department stores. As stated in Chapter IV, Section 2, it would have been preferable to use an income variable relating only to that segment of the population using air transportation, but this was not available.

Section 2: Forecasting

The forecasting ability of a model may be assessed by means of R^2 and other tests. However, where this is possible, the best test is the empirical one of using the model to make a real forecast, and then to compare the forecast with actual data. The equation and parameters of the model presented above were established with data running from January 1966 to December 1971. Using that equation and those parameters, the authors carried out a forecast of weekly and monthly traffic for the first quarter of 1972. At a 99% level of confidence, the standard error of estimate was $\pm$ 18%, which is within the range of short term equipment flexibility of any airline. This forecast was then compared with the actual figures which had not been used in estimating the original equation. The results of the forecast are shown in Fig. 6-2.

We drew several conclusions from Table 6-2:

1. The positive and negative errors are fairly well balanced. This is a consequence of the use of the multi-stage procedure in estimating the equation, insuring that the residuals are randomly distributed.

Week Ending	ACTUAL TRAFFIC Weekly	Monthly	FORECAST Weekly	Monthly	ERROR Weekly	Monthly
Jan. 2	6436		6210		- 3.5%	
9	5805		4790		-17.4%	
16	3535		4100		+15.9%	
23	3751		4110		+ 9.5%	
30	3618		3940		+ 8.9%	
TOTAL		23,145		23,150		0% (5 PAX)
Feb. 6	3451		2850		-17.4%	
13	3668		3870		+ 5.5%	
20	4017		4300		+ 7.1%	
27	4498		4250		- 5.5%	
TOTAL		15,634		15,270		-2.62% (-364 PAX)
Mar. 5	4605		4580		- .5%	
12	4736		4790		+ 1.1%	
19	5080		5100		+ .4%	
26	6917		5630		-18.6%	
TOTAL		21,338		20,100		-5.80% (-1238 PAX)
QUARTER		60,117		58,520		-2.65% (1597)

Fig. 6-2: Initial Forecast for January, February, March 1972 (using the equation and parameters estimated from 1966-1971 data)

2. The weekly forecast does follow the patter of seasonality.

3. The R^2 of the extrapolated data is almost equal to the R^2 of the original data, with a difference of only -.083.

4. When the weekly data are aggregated to monthly figures, the results become quite good with a 0% error in January, a -2.6% error in February, and a -5.8% error in March. This would lead one to conclude that such a model should be calibrated with weekly data, in order to give a better monthly forecast.

5. The monthly errors appears to increase as the forecast is extended into the future, which conforms to expectations.

6. The forecast is consistently smaller than actual traffic.

7. The prediction-realization diagram shows that the predictions are not too far from the line of perfect forecasts.

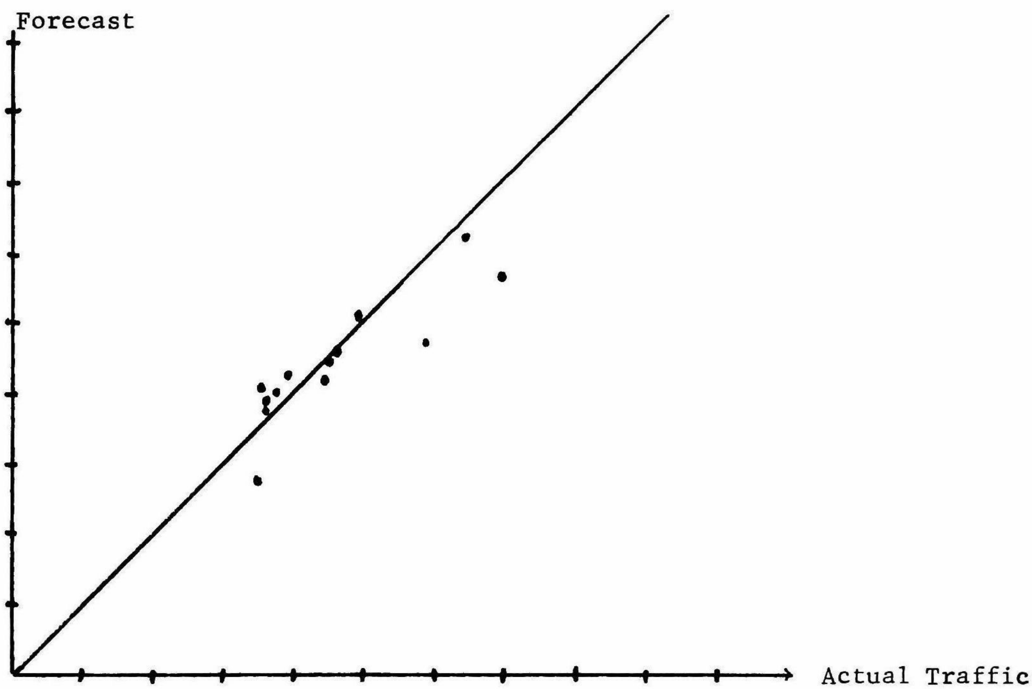


Fig 6-3: Prediction Realization Diagram.

A second forecast (cf. Fig. 6-4) was carried out in which actual traffic data for January was brought into the model which was then used to forecast February and March. As seen from the Figures from table 6-3, there was almost no changes. From this, one may conclude that any significant change imminent in March is not indicated by any trends in the figures for January.

Finally, a third forecast was made for the month of March alone, incorporating data from January and February into the model. This resulted in a modest improvement in the new March forecast. One explanation of the consistency by which the forecast of the traffic data for March is

	ACTUAL TRAFFIC	FORECAST I	FORECAST II	FORECAST III
Jan. 2	6436	6210	KNOWN	KNOWN
9	5805	4790		
16	3535	4100		
23	3751	4110		
30	3618	3940		
	-----	------		
TOTAL	23145	23150		
Feb. 6	3451	2850	2850	KNOWN
13	3668	3870	3770	
20	4017	4300	4290	
27	4498	4250	4250	
	----	----	----	
TOTAL	15636	15270	15160	
		(-2.6%)	(-3.6%)	
Mar. 5	4605	4580	4580	4680
12	4736	4790	4790	4890
19	5080	5100	5010	5100
26	6917	5630	5680	5600
	----	----	----	-----
TOTAL	21338	20100	20060	20270
		(-5.8%)	(-5.9%)	(-5%)

Fig. 6-4: Forecasts I, II & III.

below actual traffic may be in the fact that the carrier undertook a special advertising campaign in March 1971 designed to stimulate traffic on the route under study. Had it not been for this, the authors suggest that actual traffic might have been lower and therefore closer to the forecast traffic.

CHAPTER VII

CONCLUSION

As expected, this study provides additional confirmation of the importance of price, income and seasonality as important explanatory influences of the demand for coach class reserved seat air travel.

The use of yield as a measure of the price variable superior to the published full coach fare is confirmed, particularly when dealing with that historical period following the introduction of discounts and promotional fares (post 1968). The authors are of the opinion that, in time, the availability of data will improve. Eventually, it will be possible to undertake a demand analysis using fare by fare traffic data enabling the researcher to break down demand into its constituent elements.

In the exhaustive search for the best measure of income compatible with weekly traffic data, department stores sales emerged as the superior measure. Nevertheless, this measure could be improved upon. New research on other city-pairs might turn up an area for which a detailed demographic study has been performed, yielding a time series of local disposable personal income broken down by income classes of population.

Seasonality has been closely examined in this study, and this has been made possible by the weekly frequency of the data. While the authors are not excluding other possible treatment of seasonality, they suggest that the approach taken in this study, namely the use of dummy variables, offers an effective and accurate method of incorporating seasonality into an econometric model. However, the method is time consuming, and for wider applications, it might be advisable to develop a computer algorithm to determine the appropriate time periods for which dummy variables are required.

Future studies of single city-pair markets might attempt to incorporate and test a variable providing a measure for the effect of advertising expenditures. Although most trunkline carriers do not promote particular routes, prefering to emphasize the unique attraction of their entire system, the influence of advertising deserves attention. One problem will be to find an objective measure of the quantity and quality of advertising.

Other factors that may have considerable influence on the demand for non-business air travel are the cost and attractiveness of land arrangements at the destination. Passengers may be more motivated to purchase air transportation by the relative appeal and the prices of land arrangement at a destination than by the price of air travel itself. The difficulties in measuring such influences may indeed be formidable. At one stage, the authors considered attempting to gather measures of such influences, but soon realized that such a task was well beyond the scope of the study.

The results of this study confirm that demand is price elastic in this particular market. This would lead to the conclusion that revenues could be increased by means of selective fare decreases during the appropriate periods of the year.

However, in the authors' opinion, measures of price elasticities of the demand for air travel should be used with caution when making pricing and regulatory decisions. Price elasticities are not constant, and it seems clear that they vary with each population income class, time of the year, purpose of travel, destination, and degree of intermodal competition.

Lagging certain variables was found to improve correlation with the dependent variable, and in all cases, the period lagged was plausible in term of expected consumer behavior. The authors believe that their empirical investigation of lagged variables provided useful additional insights to the relationship between dependent and independent variables, and gave to the model a better forecasting capability.

The use of a multiplicative model appeared preferable to the use of an additive model. A multi-stage procedure was found necessary to avoid autocorrelation of the residuals.

The forecasting potential of the model was tested with satisfactory results. The average error over the first three month forecast was 2.65% and compares favorably with similar studies in the field.

The authors feel that this study has demonstrated the value of micro-analysis in air travel demand studies. When time series data of traffic fare by fare becomes available, it would be rewarding to refine the analysis to such a level of fare by fare demand equations. This might, for the first time, lead to price elasticities accurate enough to be used without restraint as a basis for pricing and regulatory decisions.

BIBLIOGRAPHY

Section 1: Statistical References

Anderson, T. W., The Statistical Analysis of Time Series, John Wiley & Sons, New York, 1971.

Box, George, E. P., and Jenkins, Gwilym M., Time Series Analysis, Forecasting & Control, Holden-Day, San Francisco, 1971.

Chao, Lincoln L., Statistics: Methods and Analysis, McGraw Hill, New York, 1969.

Chow, Gregory C., Tests of Equality Between Sets of Coefficients in Two Linear Regressions, Econometrica, vol. 28, no. 3, July 1960.

Christ, Carl F., Econometric Models & Methods, John Wiley & Sons, New York, 1966.

Dhrymes, Phoebus J., Distributed Lags, Problems of Estimation & Formulation, Holden-Day, San Francisco, 1971.

Draper, Norman R. and Smith, H., Applied Regression Analysis, John Wiley & Sons, New York, 1968.

Durbin, J., and Watson, G. W., Testing for Serial Correlation in Least Squares Regressions, Biometrica, vol. 38, part I & II, 1951.

Fox, Karl A., Intermediate Economic Statistics, John Wiley & Sons, New York, 1968.

Goldfeld, Stephen M., and Quandt, Richard E., Nonlinear Methods in Econometrics, North Holland Publishing Company, Amsterdam, 1972.

Hickman, Edgar P., and Hilton, James G., Probability & Statistical Analysis, Intext Educational Publishers, London, 1971.

Goldberger, A. S., Econometric Theory, John Wiley & Sons, New York, 1964.

Johnston, J., Econometric Methods, McGraw Hill, New York, 1963, 1972.

Kmenta, Jan, Elements of Econometrics, Macmillan Company, New York, 1971.

Labrousse, Christian, Statistique, vol. 1,2,3, Dunod, Paris, 1969.

Li, Jerome C., Statistical Inference, vol. 2, Edward Brothers, Ann Arbor, 1964.

Morrison, Donald F., Multivariate Statistical Methods, McGraw Hill, New York, 1967.

Rogers, Andrei, Matrix Methods in Urban and Regional Analysis, Holden-Day, San Francisco, 1971.

Senter, R. J., Analysis of Data, Scott & Foresman, Glenview, 1969.

Theil, H., and Nagor, A. L., Testing the Independence of Regression Disturbances, Journal of American Statistical Association, LVI, 1961.

Section 2: Transportation References

Asher, Norman, and others, Supersonic Transportation, National Planning Association, Washington D.C., 1971.

Bartlett, Hale C., The Demand for Passenger Air Transportation, 1947-1962, unpublished Ph.D. dissertation, University of Michigan.

Baumol, William J., Economic Theory and Operation Analysis, 3rd edition, Prentice Hall, Englewood Cliffs, N.J., 1972.

Besse, G., and Desmas, G., Forecasting for Air Transport, Methods and Results, Institut du Transport Aerien, Paris, 1966.

Blood, Dwight, M., and Lansing John B., The Changing Air Travel Market, Survey Research Center, University of Michigan, Ann Arbor, 1964.

Blood, Dwight, M., and Lansing, John B., Mode Choice in Intercity Travel: A Multivariate Statistical Analysis, Survey Research Center, University of Michigan, Ann Arbor, 1964.

Boeing Airplane Company, Mathematics and Forecasting, The Prediction of Air Traffic, Renton, Washington, 1958.

Bolt, Gordon, Market and Sales Forecasting, a Total Approach, Halsted Press, New York, 1972.

Brown, Same and Wayne S. Watkins, The Demand for Air Travel: A Regression Study of Time Series and Cross Sectional Data in the United States Domestic Market, United States Civil Aeronautics Board, Washington, D. C., 1968.

Brown, Samuel L., Forecasts of Air Travel by CAB Staff: A Method and Its Performance, paper given at the annual meeting of the American Statistical Association, Detroit, Michigan, December, 1970.

Brown, Samuel L., I Saginor, and D. B. Richards, Forecast for 1970 for the Trunk Airlines and Pan American of Scheduled Domestic Passenger Travel, unpublished CAB paper, 1969.

Brown, Samuel L., Air Travel in the Seventies, the Economic Potential, paper given at the National Transportation Engineering Meeting, ASCE, Milwaukee, Wisconsin, 1972.

Caves, Richard W., Air Transport and Its Regulators, Cambridge, Massachusetts, Harvard University Press, 1962.

Charles River Associates, Incorporated, Review of the Market for the Supersonic Transport --Methodology and Sensitivity Analysis, Prepared for Offic of Supersonic Transport Development, Federal Aviation Administration, Report No. 133-1, April, 1969, Washington, D.C.

Cherington, Paul W., Airline Price Policy: A Study of Domestic Airline Passenger Fares, Cambridge, Massachusetts, Harvard University Press, 1968.

Colbert, R.T., U.S. Domestic Trunk Passenger Traffic Forecasting Model, Renton, Washington, The Boeing Company, August, 1966.

Dogams, Rigas, "Traffic Forecasting and Gravity Model," Flight, September 29, 1966, pp. 547-549.

Douglas Aircraft Company, The Importance of Traffic Forecasting in Airline Marketing, report C1-804-2164,Long Beach, California, Douglas Aircraft Company, 1970.

Douglas, George W., "Excess Capacity, Service Quality and the Structure of Airline Fares," Proceedings of the Twelfth Annual Meeting of the Transportation Research Forum, vol. 12, no. 1, Oxford, Indiana, Richard B. Cross Company, 1971.

Eastern Air Lines, Domestic Passenger Fare Investigation, Discount Fare Phase, United States Civil Aeronatics Board Docket 21866-5, Exhibit EA 108.

Eads and Philip Eirleger, Statistical Biases in Aggregate Time Series and Cross-Section Studies of the Demand for Air Travel, paper presented at the American Statistical Association Meeting, Detroit, Michigan, December 28, 1970.

Elle, Bjorn J., The Size of Aircraft for a Fluctuating Transport Demand, Linkoping, Sweden, Saab Aktiebolag, 1967.

European Airlines Research Bureau, Comparative Examination of EARB and United States Airlines, EARB 403, Brussels, Belgium, European Airlines Research Bureau, 1970.

European Civil Aviation Conference, Forecast of Intra-European Scheduled Air Passenger Traffic, 1968-1980, ECAC, CEAC Document no. 3, Paris 1970.

Flaven, Bernard and Roger Marche, Passenger Air Travel Characteristics and Forecast of Demand in Europe, Paris, Societe d'Etudes Techniques et Economiques, Aeroport de Paris, March, 1968.

Gronau, Reuben, The Value of Time and Passenger Transportation: The Demand for Air Travel, New York, National Bureau of Economic Research, Inc., 1970.

International Civil Aviation Organization, Previsions de trafic aerien,et methodes de prevision, Montreal, 1972.

Institut du Transport Aerien, Relationship Between Fares and Demand in Air Transport: Some Approaches to the Problem, ITA Bulletin, Janvier 1965.

Institut du Transport Aerien, Demand Elasticity in Air Transport, International Symposium of Air Transport Economics, ITA, Paris, November 1964.

Institut du Transport Aerien, The Problems of Costs and Fares in the Present Air Transport Crisis in the U.S., ITA Bulletin, Mai, 1971.

Institute for Defense Analyses, "Demand Analysis for Air Travel by Supersonic Transport," Institute for Defense Analysis Report-R-118, Washington, D.C., Commerce Clearing House, 1966.

Jarvis, Jack B., Passenger Mile and Revenue Forecasting Practices,Renton, Washington, Boeing Airplane Company, Transportation Division, 1958.

Joun, Young Pyo, The Demand for Air Travel, unpublished Ph.D. Dissertation, Seattle, Washington, University of Washington, 1966.

Kessler, Davis S., Relationship Between Intercity Air Passengers and Economic and Demographic Factors: A Multiple Regression Analysis, unpublished M.S. thesis, Princeton University,1968.

Kotler, Philip, Marketing Decision Making, A Model Building Approach, Holt, Rinehart & Winston, New York, 1971.

Kraft, Gerald and Martin Wohl, "New Directions for Passenger Demand Analysis and Forecasting," Transportation Research, vol. 1, pp. 205-230.

Lansing, John B. and others, The Travel Markey Survey, 1958, 1959-1960, 1961-1962, Survey Research Center, University of Michigan, 1963 reprints.

Lansing, J.B., and Blood, D.M., "A Cross-Section Analysis of Nonbusiness Air Travel," Journal of the American Statistical Association, 53 (December, 1958), 928-947.

Lansing, and Lilienstein, E., The Travel Market 1955, Survey Research Center, The University of Michigan, Ann Arbor, Michigan, 1955.

Lansing, J.B., Liu, J., and Suits, D.B., "An Analysis of Interurban Air Travel," Quarterly Journal of Economics, 65 (February, 1961), 87-95.

Lave, Lester B., "The Demand for Intercity Passenger Transportation," Journal of Regional Science, vol. 12, no. 1, 1972.

Lockie, John A., Jr., U.S. Air Traffic Forecast, 1964-1976, Kenmore, New York, the author, December, 1964.

Locklin, Philip D., Economics of Transportation, 7th edition, Richard D. Irwin, Chicago, Illinois.

Long, Wesley, H., "The Economics of Air Travel Gravity Models," Journal of Regional Science, vol X, no. 3, pp. 353-362.

Meyer, John R. and Mahlon R. Strasszheim, "Pricing and Project Evaluation," vol. I of Techniques of Transport Planning, Washington, D.C., Brookings Institute, 1971. (Especially part 2: "Forecasting the Demand for Transportation").

Mathematica, Studies in Travel Demand, vol. I-IV, 1967-1969, Princeton, Mathematica.

Mathematics and Forecasting: The Prediction of Air Traffic, Mathematical Analysis and Operations Research Group, Boeing Airplane Company, August 11, 1958.

Mize, J. L., "An Econometric Analysis of the Demand for Airline Passenger Transportation--Domestic Routes," a Ph.D. dissertation, Georgia State College, 1968.

Official Airline Guide, Quick Reference North American Edition, Reuben H. H. Donnelley, Oak Brook, Ill., 1965-1972.

Port of New York Authority, New York's Domestic Air Passenger Market, April, 1968 -March 1964, New York, Port of New York Authority, May, 1965.

Port of New York Authority, New York's Domestic Air Passenger Market, April 1966-March 1967, New York, Port of New York Authority, May, 1969.

Pulling, Ronald W., "Intercity Travel Characteristics," Journal of the Aerospace Transportation Division, Proceedings of the American Society of Civil Engineers, April 1965.

Quandt, Richard E., The Demand for Travel - Theory & Measurement, Heath, Washington, 1970.

Quandt, Richard E., and William J. Baumol, "The Demand for Abstract Transport Models, Theory and Measurement," Journal of Regional Science, vol. VI, no. 2, 1966.

Quandt, Richard E.,"Some Perspectives on Gravity Models,"Studies in Travel Demand (I), Princeton: Mathematica.

Richmond, Samuel B., "Forecasting Air Passenger Traffic by Multiple Regression Analysis", Journal of Air Law and Commerce, Autumn, 1955.

Schultz, Randall L., The Development of a Marketing Model Through Simultaneous-Equation Multiple Regression Analysis: An Airline Study, Ph.D. in marketing, Northwestern University, 1971.

Stuart, Merril M., The Use of a Gravity Model in Interpreting Air Passenger Traffic Between Hawaii and the Conterminous United States, Columbia University, Ed D., 1968.

Straszheim, Mahlon R., The International Airline Industry, Washington,D.C., Brookings Institute, 1969.

Survey of Current Business, U. S. Department of Commerce, Washington, D.C., 1965-1972.

Systems Analysis and Research Corporation., Feasibility of Developing Dollar Values for Increments of Time Saved by Air Travelers, prepared for the Federal Aviation Agency, Washington, D.C., Systems Analysis and Research Corporation, February, 1966.

Taneja, Nawal K., A Model for Forecasting Future Air Travel Demand on the North Atlantic, MIT, 1971.

Trans World Airlines Exhibit 14, United States Civil Aeronautics Board Docket 18936, June 28, 1968

Trans World Airlines Exhibits 5001, 5002 and 5003, United States Civil Aeronautics Board Docket 21866-5, June 11, 1970.

Turner, F., Price-Elasticity and Income-Elasticity of Domestic Scheduled Air Travel in the USA, 1947-1960, SAAB, January 1, 1962.

United Air Lines Exhibit U-2, United States Civil Aeronautics Board Docket 21866-5, June 11, 1970.

United Air Lines Exhibit UI-7, United States Civil Aeronautics Board Docket 18936, June 28, 1968.

U. S. Civil Aeronautics Board, Domestic Origin-Destination Survey of Airline Passenger Traffic-Summary Information, compiled by the Civil Aeronautics Board and published by the Air Transport Association of America, yearly volumes, 1958-1972.

U. S. Civil Aeronautics Board, Forecast of Domestic Passenger Traffic for Eleven Trunkline Carriers Scheduled Service, 1968-1977, Washington, D.C., September, 1967.

U. S. Civil Aeronautics Board, Forecasts of Passenger Traffic of the Domestic Trunk Air Carriers, Domestic Operation, Scheduled Service, 1965-1975, Staff Research Report No. 5, September, 1965.

U. S. Civil Aeronautics Board, Measuring the Elasticity of Air Passenger Demand: A Study of Changes Over Time from 1953-1964, Washington, D. C., February, 1966.

U. S. Civil Aeronautics Board, A Study of the Domestic Passenger Air Fare Structure, 1968, Washington, D.C., 1968.

U. S. Civil Aeronautics Board, Competition Among Domestic Air Carriers, Civil Aeronautics Board, Washington, 1960-1969.

U. S. Civil Aeronautics Board, Handbook of Airline Statistics, Civil Aeronautics Board, Washington, 1960-1971.

U. S. Civil Aeronautics Board, Measuring Elasticities of Air Travel from New Cross Sectional Data, Civil Aeronautics Board, Washington, 1971.

Verleger, Philip K., A Point to Point Model of the Demand for Air Transportation, Ph.D., MIT, Economics, 1971.

Wallace, W. M., The Demand for Airline Travel: A Study of the Price Elasticity of Demand for Airline Travel, Renton, Washington, Boeing Company, 1964.

Watkins, Harold D., "CAB Allows More Flexibility in Discounts", Aviation Week and Space Technology (AvWk), vol. 95, no. 12, September 20, 1971, pp. 27-28.

Watkins, Harold D "Promotional Fares Aimed at Traffic Lag", Aviation Week and Space Technology (AvWk), vol. 95, no. 7, August 16, 1971, p. 31.

Watson, Peter L., Problems Associated with Time and Cost Data Used in Travel Choice Modeling and Valuation of time, HRR, 1971, Washington.

Wohl, Martin, A Methodology for Forecasting Peak and Off-peak Travel Volumes, H.R.R. #322, Washington, 1970.

VITAE

Gerard Damay was born in Lille, France, on December 27, 1948. He received a Baccalaureat in Mathematics in 1966 and in Philosophy in 1967. He was graduated from ESSEC (Ecole Superieure des Sciences Economiques et Commerciales, Paris) in 1970. During 1968 and 1969, he was responsible for the ESSEC Student Travel Service. During the summer of 1970, he prepared, for Air France, a report on the "Youth Market" in international air transportation.

From January 2, 1973, Mr. Damay will be with Air France, in Paris.

Niels de Terra was born in New York City on August 22, 1939. He received a B.Sc(Econ.) with honors from the University of London in 1962, and a Masters in International Affairs from Columbia University in 1964. From 1965 to 1967, he was an international economist with the U.S. Treasury Dept. From 1967 to 1969 he was executive Director of the U.S. National Student Travel Association, and from 1969 to 1971 he was Marketing Manager of the British Student Travel Center in London.

Mr. de Terra is presently an international transportation specialist with the U.S. Department of Transportation.