TRANSPORTATION CENTER

DEMAND FOR BUS TRANSPORTATION
IN SUBURBS AND SATELLITE CITIES
OF METROPOLITAN AREAS

Seikō Higa

RESEARCH
REPORT

NORTHWESTERN UNIVERSITY EVANSTON • ILLINOIS

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Section 1

INTRODUCTION[1]

In recent years, the rapid movement of people and places of work away from city centers to suburbs and satellite cities has occurred in many major metropolitan areas. The public transportation systems in these suburbs and satellite cities, however, are still in a state of low development. The purpose of this project has been to develop a model to predict the demand for work-trip bus transportation in and between suburbs and satellite cities. In particular, we have sought to develop a method for predicting when bus transportation in and between these cities becomes economically feasible.

Since the model that has been developed involves the prediction of both trip distribution and modal split, several methodological problems have arisen. These include:

(1) Should trip distribution and modal split be predicted simultaneously, or separately in some order?

(2) If the latter approach is adopted, how is it best to cope with the problem of simultaneity, which appears to exist in the choice-making process of trip makers?

(3) When and how should we aggregate the data in predicting the choice behavior of a group of individuals?

(4) What form should the model take so that it will be policy responsive?

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These problems have been addressed during the course of our research and will be discussed in this report.

The model developed for this study consists of two submodels and one function. The submodels include one on individual mode choice and the other on market demand for transport service. The function provides a connection between the two. Section 2 of this report evaluates existing demand modeling methods. Section 3 is concerned with the construction of a mode-choice submodel for individuals. The discussion proceeds in order of model building, source and nature of the data, and estimation of structural parameters. In Section 4, a submodel of market demand for transport service is developed. The discussion in this Section consists of a brief review of existing trip distribution models, model building, source and nature of the data, and estimation of model parameters. Section 5 deals with estimation of an aggregation function which is used to predict the mode choice of a group of individuals based on the mode choice decision made by an "average" individual in the group. In Section 6, we present methods of using the submodels and the aggregation function to predict the demand for bus service in suburbs and satellite cities. The findings of the report are summarized and future recommendations are made in Section 7.

Section 2

EVALUATION OF EXISTING DEMAND MODELLING METHODS PERTAINING TO THIS RESEARCH

The literature on demand modelling in transportation is extensive and well documented [1, 3, 5, 32, 41][2]. The discussion that follows is concerned with modelling methods that are relevant to this research.

The methods of predicting demand for transportation as currently practiced are available in the UMTA software package called Urban Transport Planning System (UTPS). UTPS contains both traditional and more recently developed approaches to predicting transportation demand. In the traditional approach, estimation is first made of the trip generation as a function of characteristics such as population and zone in which the trip originates. The result of this first step is combined with an impedence factor to estimate the flow of trip-makers between zones. The impedence factor is normally represented by the distance between zones or the time required to travel between them. During this stage, gravity models are used to estimate trip distribution.

In the next step, the results of the two previous steps are combined with mode cost, time and other variables to predict the modal split. To do this, functional relationships are derived by such means as use of graphs, tables or logit models. Finally, at the route assignment stage, the results of the modal split estimation are allocated to the route segment with least trip time until its capacity is reached. Then the overflow traffic is assigned to the segment with the next least trip time.

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[2] The numbers in brackets refer to the reference number at the end of the report.

A major drawback of this traditional approach is that it is not policy oriented. For instance, as the toll charges for an expressway connecting two zones are raised, or the bus fare is lowered, one would expect some shifts in the mode and destination choices of trip-makers. However, under the traditional approach, the effects of such changes cannot be examined since the gravity model which is supposed to predict destination choices does not contain cost variables among its explanatory variables.

There are other criticisms of the traditional approaches [5]. (1) They do not reflect behavior changes of trip-makers resulting from changes in system characteristics such as cost of the trip. (2) The decision as to what time of day to travel is not considered in the model. (3) The supply side of the transportation system is ignored except at the route assignment stage, and consideration is usually not given to equilibrium. (4) The analysis is based on data zonally aggregated. As a result, much of the information in the original data is lost.

In recent years, several studies have been done which improve upon the shortcomings of traditional modelling. The new approach estimates a model which would predict trip frequency, destination, and mode choice simultaneously or sequentially using a logit model and disaggregated data [1, 3, 5].

Domencich and McFadden use disaggregated data from Pittsburgh to estimate a sequential model for shopping trips. In their study, a mode choice logit model is first estimated. The parameter estimates are then combined with modal cost and time to estimate "inclusive prices" for time of day and for destination. These inclusive prices are then combined with other variables to estimate logit models for choice of time of day and choice of destinations. Finally, the probability of selecting alternative shopping destinations is combined with the inclusive prices of these destinations to estimate "overall price" of the shopping trip for each household. The same probability of selecting alternative shopping destinations is then combined with employment at alternative shopping centers to obtain the "overall shopping opportunity" variable. These variables are then used to estimate a logit model for shopping frequency.

Ben-Akiva, using disaggregated data from the Washington, D.C. area, estimates and compares both simultaneous destination and mode choice logit models and sequential models for shopping trips. He recommends that the simultaneous models be used to predict mode and destination choice decisions.

In predicting both choice of mode and choice of destination of commuters, however, the simultaneous logit model (or for that matter any logit model used to predict destination choices) has its limitations. When there are cross-commuters, the model tends to misclassify destination choices. For a model to be valid, one would at least expect that it would duplicate the base year destination choices of commuters whose choices were used to estimate the model. However, by construction, logit models assign higher probabilities to destinations with lower commuting cost or shorter trip time. Hence, the cross-commuters tend to be assigned to lower cost and shorter trip time destinations; i.e., those nearer to their home.

In order to test the hypothesis that simultaneous destination—mode choice models are inadequate, a small-scale simultaneous logit model for a work trip with two destination choices and two mode choices was estimated. The model was estimated from a data set that included 15 cross-commuters and 13 non-cross-commuters. When the original data were substituted back into the model, the model classified 11 of 13 non-cross-commuters correctly;

however, only 5 of 15 cross-commuters were classified correctly. That is, in 67% of the cases, the model failed to duplicate the destination choices of cross-commuters.

The phenomenon of cross-commuting arises from the aggregation of data over occupational groups. Hence, in order to make a simultaneous destination-mode choice model effective in its prediction, one is forced to provide enough job classes (perhaps even to the extent of using job seniority to differentiate job classes) just to explain away the cross-commuting. This, however, is a monumental task in terms of data collection and processing. In addition, there is a danger of losing degrees of freedom to the extent that the model might become inestimable. This arises either because the model includes a large number of parameters to be estimated or because of reduction in sample size due to data stratification. Furthermore, even if such a model could be estimated, the destination choice aspect of the model could be so constrained that it might be effective in predicting only mode choices.

Having shown the limitations of a simultaneous destination-mode choice logit model to predict the work-trip demand for bus service among suburban and satellite cities, we now turn to a discussion of the model adopted in this study. The model used in this study is behavioral and policy responsive. Whenever possible disaggregate data are used. As a work-trip study, the time of day decision was assumed not to be under the control of the trip-makers and hence it was ignored. The supply side of the transport system was also ignored to limit the scope of the study. Therefore, in order to achieve simultaneity, the model that has been developed in this study has to be used in conjunction with a supply model so that trip cost and time can be determined endogenously. The model is structured as a pure demand model in which the flow of work trips between cities may be studied in terms of the exogenously given mode time and cost which may be influenced by policy makers. The following section discusses in greater detail the individual demand submodel which was developed in the study.

Section 3

INDIVIDUAL DEMAND SUBMODEL

A. Submodel

The individual demand submodel adopted in this study consists of a disaggregate multinomial mode-choice logit model incorporating specific treatment of socioeconomic variables in either continuous or discrete form. It also incorporates dummy coefficients for mode-related variables, while dispensing with the constant term. The dummy coefficients for mode variables and socioeconomic dummy variables enable researchers to combine several model structures estimated through data stratification into one structure while retaining the individual model features.

Consider a model of the following form:

$$(1) \qquad P_{ij} = \frac{e^{X_{ij}\beta + Y_i\gamma_j}}{\sum_{j=1}^{J} e^{X_{ij}\beta + Y_i\gamma_j}} ,$$

where P_{ij}: The probability of individual i selecting mode j.

X_{ij}: A 1xK vector of mode-related variables associated with individual i and mode j. For instance, the k-th element of X_{ij} may be the commuting time facing individual i when he takes mode j. Also, X_{ij} may include a variable which is formed by interaction of two variables. We have specifically excluded the column of 1's from the matrix X.

Y_i: A 1xM vector of socioeconomic variables associated with individual i.

β : A Kx1 vector of parameters.

γ_j: A Mx1 vector of parameters specifically associated with mode j.

In equation (1), the exponential term $X_{ij}\beta + Y_i\gamma_j$ constitutes a systematic part of the indirect utility function of individual i that he derives from taking mode j; i.e.,

$$(2) \qquad V_{ij} = X_{ij}\beta + Y_i\gamma_j + u_{ij},$$

where V_{ij} is the indirect utility, and u_{ij} is a random element in his utility function.

For a case not involving socioeconomic variables as in equation (2), McFadden established that when the random term u_{ij} has Weibull distribution the probability of individual i selecting mode j takes a logit expression [22]. The validity of his Lemma 1 can be easily extended to equation (2) to obtain equation (1).

As equation (1) is specified, it is based on the assumption of separable utility [42, 43], and has the desirable properties that the odds of selecting one mode over another is independent of irrelevant alternatives and that the probabilities of choosing alternative modes sum to unity. Furthermore, even though the model tacitly starts with a notion of cardinal utility, by transformation and normalization, the objectionable aspects of the cardinality assumption such as interpersonal comparison of utilities are effectively precluded.

The advantage of expressing the socioeconomic variable as in equation (1) (rather than as a submatrix in X matrix, as Manski does [21]) is that in addition to revealing the presence of socioeconomic variables in the model, it greatly facilitates the estimation of parameters by reducing the need for storage space during computation. For instance, if there are 4 alternative modes and 5 socioeconomic variables, the required storage space for the socioeconomic variables under Manski's formulation is equivalent to that of 60 variables, while under the formulation that we have used the required space is equivalent to only 5 variables.

Before forming the likelihood function to estimate the parameters, equation (1) will have to be transformed as in equation (3), or in any similar form,to assure that its log-likelihood function is strictly concave.

$$(3a) \qquad P_{i1} = \frac{1}{1 + \sum_{j \neq 1} e^{(X_{ij} - X_{i1})b + Y_i c_j}}, \quad \text{for all } i$$

$$(3b) \qquad P_{ij} = \frac{e^{(X_{ij} - X_{i1})b + Y_i c_j}}{1 + \sum_{j \neq 1} e^{(X_{ij} - X_{i1})b + Y_i c_j}}, \quad \text{for all } i, \text{ and for all } j \neq 1.$$

where $b = \beta$ and $c_j = \gamma_j - \gamma_1$.

Since P_{ij} in equation (3) has a multinomial distribution, the likelihood function for equation (3) is given by:

$$(4) \qquad L = \prod_{i=1}^{N} \frac{m_i!}{\prod_j m_{ij}!} \prod_{j=1}^{L} P_{ij}^{m_{ij}} ,$$

where m_{ij} is the frequency with which individual i chooses mode j. Here, if we are concerned only with the choice of mode that commuters make in reaching their place of employment, then the available choice is usually made once a day, and m_i, which is the sum of m_{ij} over all alternatives, will have value one. The ratio of factorials in equation (4) then reduces to unity, and (4) becomes:

$$(5) \qquad L = \prod_{i=1}^{N} \prod_{j=1}^{L} P_{ij}^{m_{ij}} .$$

The log-likelihood function for (5), after substituting equation (3), is given by:

$$(6) \qquad \log L = \sum_{i=1}^{N} \{ \sum_{j\neq 1} m_{ij}[(X_{ij} - X_{i1})b + Y_i c_j] - \\ - (\sum_j m_{ij}) \log [1 + \sum_{j\neq 1} e^{(X_{ij} - X_{i1})b+Y_i c_j}]\} .$$

Since a log-likelihood function is a monotonically increasing transformation of a corresponding likelihood function, the parameter values which maximize (6) will also maximize (5).

The gradient vector for log-likelihood function (6) is given by:

$$(7)\quad \begin{pmatrix} \dfrac{\partial \log L}{\partial b} \\ \dfrac{\partial \log L}{\partial c_2} \\ \cdots \\ \dfrac{\partial \log L}{\partial c_L} \end{pmatrix} = \begin{pmatrix} \sum\limits_{i=1}^{N} \left[\sum\limits_{j \neq 1} m_{ij} X^{*'}_{ij} - \sum\limits_{j \neq 1} P_{ij} X^{*'}_{ij} \right] \\ \sum\limits_{i=1}^{N} (m_{i2} - P_{i2}) Y'_i \\ \cdots\cdots \\ \sum\limits_{i=1}^{N} (m_{iL} - P_{iL}) Y'_i \end{pmatrix},$$

where $X^{*}_{ij} = X_{ij} - X_{i1}$, and accent (') denotes transpose.

The Hessian matrix of (6) is given on the following page. The dimension of the Hessian matrix (8) is $K + (L-1)M$, where K is the column number of mode-related variables X_{ij}, L is the number of alternative modes, and M is the column number of socioeconomic variables Y_i. The strict concavity of the log-likelihood function may be established by showing that the Hessian matrix (8) is negative definite. Also, the fact that the log-likelihood function derived from equation (1) without transformation is not strictly concave, and that it has a singular Hessian matrix, can be established by summing the 2nd column through L-th column of the Hessian matrix of the log-likelihood function derived from (1) without transformation, and showing that the sum is zero. The mathematical proofs are rather lengthy, and use the concept of dominant negative diagonal matrix, among others.

In formulating logit models, some researchers have strong feelings against incorporating socioeconomic variables as was shown in equation (3) above. The objection is that the coefficients of socioeconomic variables are mode specific.

$$\begin{bmatrix} \frac{\partial^2 \log L}{\partial b\, \partial b'} & \frac{\partial^2 \log L}{\partial b\, \partial c_2'} & \cdots & \frac{\partial^2 \log L}{\partial b\, \partial c_L'} \\ \frac{\partial^2 \log L}{\partial c_2\, \partial b'} & \frac{\partial^2 \log L}{\partial c_2 \partial c_2'} & \cdots & \frac{\partial^2 \log L}{\partial c_2 \partial c_L'} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial^2 \log L}{\partial c_L\, \partial b'} & \frac{\partial^2 \log L}{\partial c_L \partial c_2'} & \cdots & \frac{\partial^2 \log L}{\partial c_L \partial c_L'} \end{bmatrix} \tag{8}$$

$$= \begin{bmatrix} -\sum_i \left[\sum_{j\neq 1} P_{ij} X^{*'}_{ij} X^{*}_{ij} - \left(\sum_{j\neq 1} P_{ij} X^{*'}_{ij}\right)\left(\sum_{j\neq 1} P_{ij} X^{*}_{ij}\right)\right] & -\sum_i P_{i2}\left[X^{*'}_{i2} - \sum_{j\neq 1} P_{ij} X^{*'}_{ij}\right] Y_i & \cdots & -\sum_i P_{iL}\left[X^{*'}_{iL} - \sum_{j\neq 1} P_{ij} X^{*'}_{ij}\right] Y_i \\ -\sum_i P_{i2} Y_i' \left[X^{*}_{i2} - \sum_{j\neq 1} P_{ij} X^{*}_{ij}\right] & -\sum_i P_{i2}(1 - P_{i2}) Y_i' Y_i & \cdots & \sum_i P_{i2} P_{iL} Y_i' Y_i \\ \cdots\cdots\cdots & \cdots\cdots\cdots & \cdots & \cdots\cdots \\ -\sum_i P_{iL} Y_i' \left[X^{*}_{iL} - \sum_{j\neq 1} P_{ij} X^{*}_{ij}\right] & \sum_i P_{i2} P_{iL} Y_i' Y_i & \cdots & -\sum_i P_{iL}(1 - P_{iL}) Y_i' Y_i \end{bmatrix}.$$

Stopher, therefore, recommends stratifying the data to estimate models for different socioeconomic classes, and letting the socioeconomic variables interact with the coefficient parameters [41, pp. 310-311].

However, there are at least two reasons to advocate inclusion of socioeconomic variables as shown in equation (3). For one, there are many occasions in which mode-specific models prove to be highly useful. For instance, there are situations where changes in bus fares, toll charges, and gasoline taxes occur without the accompanying introduction of a new mode. Policy makers would want to know the impact of these on the mode choice behavior of trip-makers.

Second, omitting socioeconomic variables because the resulting model will have mode-specific parameters is in itself inconsistent. For instance, most researchers include a constant term in the model. Now, if the constant term is truly independent of modes, it would drop out in the process of transforming the model from equation (1) to (3). The only way the constant term can remain in the model after the transformation is if it is different from mode to mode. That is, the constant term must be mode-specific. Furthermore, the constant term reflects the relative bias of an individual toward one mode over another which is not explained by the variables already included in the model. One cannot assume a priori that the same relative bias will apply to a new mode when it is introduced.

In this study, the constant term was excluded from the model. One reason for this pertains to the discussion in the preceding paragraph. Since the constant term is mode-specific, and reflects the relative bias of a trip-maker toward one mode over another, such a bias may well be represented by socioeconomic variables. Secondly, the constant term also incorporates the mean effects of the variables omitted from the model, as shown below.

Assume a binary mode choice situation, and that the mode choice is completely determined by two variables, X_1 and X_2, which denote the difference in characteristics of the modes, and that these two variables are independent of each other. Such a model may be expressed by:

$$(9) \qquad \log \frac{P_{i1}}{P_{i2}} = b_1 X_{i1} + b_2 X_{i2} + u_i .$$

Suppose this model is estimated by including one variable X_2, and the constant term, and by using the least squares. Then the estimate of the constant term will be given by:

$$\hat{a} = \overline{Y} - \hat{b}\,\overline{X}_2 ,$$

where $\overline{Y} = \frac{1}{N} \sum_{i=1}^{N} \log (P_{i1}/P_{i2})$, and N is the sample size. Next, substituting $\overline{Y} = b_1\overline{X}_1 + b_2\overline{X}_2 + \overline{u}$, we derive

$$\hat{a} = b_1\overline{X}_1 - (\hat{b} - b_2)\overline{X}_2 + \overline{u} \text{ , and}$$

$$(10) \qquad E\,\hat{a} = b_1\overline{X}_1 .$$

That is, the expected value of $\hat{a}$ is $b_1\overline{X}_1$, since $E\hat{b} = b_2$ and $E\,\overline{u} = 0$.

This implies that the model which has a constant term will be suitable for prediction of mode choice for the population which has the same mean value for X_1, as the sample from which the parameters are estimated. Thus, replacing the constant term with other variables such as socioeconomic variables enhances the transferrability of the model to the population which has a different mean for X_1 than the sample from which the model is estimated.

b. Source and Nature of Data

The basic data on individual mode choices and socioeconomic characteristics used to estimate the parameters of the logit model (3) were obtained from the Illinois Department of Transportation. In the summer of 1969, the Southern Transit Area Coordination Committee conducted a questionnaire survey of employees of selected firms located on the south side of Chicago and in its south suburbs [12]. The questionnaire asked the address of each employee, his choice of mode in reaching his place of work, trip time, trip cost, and socioeconomic attributes such as family income, number of cars available, number of persons over sixteen in the family, occupation, sex, and reasons for choosing a particular mode. A total of 100,300 questionnaires were sent out and approximately 9,500 that were usable were returned (9.5%).

For this study, twenty-two firms located near the southern border of Chicago and in the southern suburbs were selected from the firms which returned questionnaires. Of these, a 10% systematic random sample of automobile drivers and all questionnaire returns on bus riders, each amounting to approximately 150 observations, were obtained. This was later expanded to include those who walked to work, in order to make the model multimodal.

The reason for selecting approximately an equal number of observations for automobile and bus riders was to avoid swamping the mode choice characteristics of bus riders by those of automobile riders, which would have been in the ratio of approximately 1 to 10. One researcher criticized this approach to sampling by stating that if the sample is divided 50-50 in mode choice, the parameters estimated would have values that would always predict a 50-50 mode choice split of the population, regardless of actual choices and regardless of the variables involved. That is, the parameter estimates will all be zero. This argument is incorrect. It should be remembered that what the model predicts is the probability that an individual chooses a given mode on the basis of mode-related and socioeconomic variables.

Suppose, for simplicity, that we formulate the log-likelihood functions for a logit model that includes only mode-related variables, using two different samples: one with a 90-10 mode split, and the other with a 50-50 split. Then, as we partially differentiate the log-likelihood functions with respect to the parameters, we obtain the following first order conditions for maximization:

(11a) 90-10 mode-split sample

$$\frac{\partial \log L_1}{\partial \beta} = -\sum_{i=1}^{N_1} (P_{i2} - m_{i2}) X'_{ij} = 0 ,$$

(11b) 50-50 mode-split sample

$$\frac{\partial \log L_2}{\partial \beta} = -\sum_{i=1}^{N_2} (P_{i2} - m_{i2}) X'_{ij} = 0,$$

where N_1 and N_2 are the respective sample sizes, and P_{i2} is the probability that each individual chooses mode 2. m_{i2} is 1 if mode 2 is actually chosen by individual i, and 0 otherwise. Thus, in both cases the maximum likelihood method would set the parameters of the model (which are in P_{i2}) so that P_{i2} tends to be 1 if mode 2 was actually chosen, and it tends to be 0 if mode 1 was chosen. Therefore, the models estimated under both sampling approaches should predict the mode split equally well on the basis of the variables in the model. Once it is known that the mode-split composition of the sample used to estimate the model is not a critical issue, then it is obvious that by adopting the sample ratio, as was done, one can reduce the data processing cost without sacrificing the quality of the model.

Once the basic data were obtained, the location of firms and employees' homes were plotted on a map, and the line-haul distance of driving a car to the place of work was estimated. In this study, the access-egress distance for automobile was assumed to be zero for the home end, and the distance from parking lots for the place of employment. The latter was obtained for each firm by telephone.

In estimating the line-haul time for automobile driving, the average driving speeds within and between rings were obtained by first stratifying car drivers according to the location of their homes and the places of employment, and by taking a simple average of individual speeds (i.e., measured distance divided by reported driving time) for those in each group. The "rings" are a series of concentric areas emanating from the Chicago Loop as defined in Trip Length published by Chicago Area Transportation Study [4], and they partition the Chicago area into zones roughly equal in traffic density. The line-haul time was then obtained by dividing measured distance by the appropriate driving speed for each observation. The reason for using a simple average to obtain the zonal speed was to avoid swamping of the zonal averages by the speed of drivers with relatively longer driving distances. The average zonal speeds estimated are given in Table 1. In

TABLE 1

Average Zonal Driving Speeds
(Miles per Hour)

From		To	Ring	
		5	6	7
Ring	5	12.86	14.79	36.00
	6	15.03	12.46	21.07
	7	23.44	24.35	18.86

general, the average driving speed tends to be the slowest for within-ring driving, and becomes faster as the rings grow farther apart and a longer distance is involved.

The automobile access-egress time was estimated from access-egress distance by assuming an average walking speed of three miles per hour; the auto waiting time was assumed to be zero. Finally, the automobile driving cost was estimated by assuming a per mile driving cost of 10¢, as was indicated in the questionnaire, and to which were added toll charges and parking fees where applicable.

For bus trips, the line-haul distance was estimated from the map on the basis of the location of firms, homes and bus routes. The access-egress distance for bus trips was estimated by adding the distance from home to the nearest bus route the trip-maker was likely to take and the distance from the bus route to the firm. To this was added the distance walked in order to transfer. In estimating the line-haul time for bus trips, the average speed of buses for each bus route was first calculated from bus schedules for both peak and off-peak hours. The average bus speed for each route was then applied to the length of each segment of bus route likely to be taken by the trip maker, and the results were added. The bus fares were obtained from CTA History of Fares for those in Chicago and from the records of the Illinois Commerce Commission for suburban buses.

Finally, in order to cope with the problem of captive riders, a threshold cost of $2.50 and 20¢ per trip was added to the total driving cost of non-car-owners and those without driver's licenses, respectively. The $2.50 represents the per trip allocation of monthly payment of $100.00 including cost of the car, insurance and financing that prevailed at the time of survey taking. The 20¢ represents per trip allocation of the cost of driver training in 1969 amortized in one year. Using the same reasoning, for commuters without bus routes within normal walking distance (who were regarded as captive to the automobile),we extended access-egress distance to the nearest bus route to estimate cost and time for buses.

It has been argued that inclusion of captive riders in the data set for estimating model parameters would make the model insensitive to policy changes. Ferreri and Cherwony found that inclusion of captive bus riders into a bus demand model caused the model to become insensitive to policy changes [8]. However, the model they constructed was a linear regression model estimated by using aggregated data. Our approach here is that a good model should be able to explain or predict the mode choice behavior of a wide variety of trip makers. There is no reason to expect that the disaggregate model constructed for this study will become insensitive to policy changes, since, as shown in equation (11), the parameters of the model are estimated to reflect the mode choice behavior of individual trip makers rather than a group of individuals.

Once a decision has been made to include the data on captive riders in the model estimation, the next question that one must face is how to express the cost of the mode which is not available to the captive riders. Here, we assumed that for automobile riders, mode choice decisions are in part determined by the variable cost, or its per-trip allocation, of operating a car. The per-trip allocation of the variable cost is approximately equal to the car owner's out-of-pocket automobile costs. For those without a car, it would include the threshold cost of owning a car and the out-of-pocket costs. The implication here is that if non-car-owners were subsidized for the amount of the threshold cost, they would behave like car owners. Similarly, for those without access to buses, the threshold cost is the added access-egress distance and time to reach the closest bus stop. Notice that if in this case an extremely large number, instead of a threshold cost, is used to express the cost and time of the mode not available to a captive rider, then it amounts to saying that it is a refusal to ride, rather than the unavailability of the mode, which is governing his choice.

The data discussed above were primarily measured values, and only in one case were reported values used. This was the travel time reported by automobile drivers. We used this reported time to estimate the average driving speed within and between the rings. There are some researchers who advocate the use of perceived cost and time values for the estimation of models. Michaels [26] argues that the validity and reliability of a model will be higher if perceived cost and time values are used instead of measured values. Watson [47], on the other hand, concludes that for models to estimate the value of travel time, the perceived values are essential; but for models to predict, the distinction is not so important because the perceived values are unstable over space.

In this study, we have adopted measured values whenever possible for two reasons: (1) perceived values are often unreliable, especially for modes with which a trip-maker is unfamiliar; (2) models estimated with perceived values are not policy responsive unless the model also specifies how policy variables affect perceived values.

c. Estimation

In this study, various combinations of pertinent variables were tested for inclusion into the choice model. Among the first variables to be rejected was family income. On the basis of microeconomic theory, one would expect family income to play a prominent role on individual mode choice. However, the coefficient associated with the income variable was not statistically significant. Some consider this to be due to the unreliability of reported income data. However, there is a more fundamental reason to believe that family income, in contrast to individual income, plays a less important role in mode choice decisions. In many instances, there is more than one wage earner in a family with a higher income. The higher family income, as expected, would increase the probability of the family owning one car; however, the marginal family income due to the supplementary wage earner's earnings may not be sufficient to add a second car. Hence, the supplementary wage earner may end up riding the bus even though his family income is high. It is noted that supplementary wage earners, usually housewives, tend to take white collar jobs while the main wage earners tend to be professionals, administrators, or skilled blue collar workers. This observation seems to support the notion that the socioeconomic variables that best explain individual mode choice are occupational classifications.

Other socioeconomic variables which were rejected from inclusion in the model were age, sex, and the number of cars available to those over age 16 in the family. The first variable was rejected because of the low statistical significance of its parameter estimate, and the second because as more women enter higher paying occupations, sex becomes a less important indicator of accessibility to the automobile. The last variable was rejected for the low statistical significance of its parameter estimate and because it could not distinguish between main and supplementary wage earners.

Among the mode-related variables, those rejected from the model were access-egress distance, number of transfers, and vehicle waiting time. These were rejected because of the low statistical significance of their parameter estimates.

Thus, in this study, the following variables are included in the individual demand submodel: total trip time, total trip cost, and occupation status. Also, as an alternative to total trip cost, the ratio of total trip cost and family income was retained. Using these variables, the competing models listed below were estimated. The one with the best predictive power in terms both of "percent correctly predicted" (as defined in equation (14) below) and the "coefficient of determination in probability" (equation (12)) was adopted. Model E, however, is an exception. It was included to test the validity of replacing the constant term with socioeconomic variables.

Model	Mode-related Variable	Socioeconomic Variable
A	X_1, X_1Y_1, X_2, X_2Y_1	Y_1
B	X_1, X_1Y_1, X_2	Y_1
C	X_1, X_2	Y_1
D	X_1, X_2/Y_2	Y_1
E	X_0, X_1, X_2	(None)

where X_0: constant term

X_1: total trip time (in minutes)

X_2: total trip cost (in cents)

Y_1: occupation status

= 1, if professional, administrator, or skilled blue collar worker,

= 0, otherwise.

Y_2: family income, represented by the midpoint of income class.

X_1Y_1 and X_2Y_1 are products of total time with occupation status, and total cost with occupation status. The coefficient estimates for these variables are dummy coefficients, and in the model associated with occupation class 1, the coefficient estimates of X_1 and X_1Y_1 must be added to derive the coefficient for X_1 variable. The same applies to the X_2 and X_2Y_1 coefficients. For an explanation of the use of dummy coefficients, see Johnston [13].

Model A is the most general of the five models, and it includes all three variables as well as the dummy coefficients for both of the mode-related variables. Therefore, the model is equivalent to two separate models estimated by stratifying the data according to occupation status. Model B is similar to Model A, but is less general in that only the time variable possesses the dummy coefficient. Model C dispenses with all dummy coefficients, and contains only time and cost variables and the occupation dummy variable. As in Model C, Model D involves the cost and time variables and the occupation dummy variable, but its cost variable is deflated by family income. Thus,this model incorporates the differential impact on determining mode choice that modal cost has on families of different income levels. Some studies use wage rates to deflate the cost variable. However, in the absence of data on wage variables, family income may be viewed as its surrogate. Model E is the simplest, involving only trip time and trip cost variables in addition to a constant term.

There are three ways of comparing the efficiency of these logit models: likelihood ratio statistic, coefficient of determination in probability, and percent correctly predicted. The likelihood ratio statistic is defined by:

$$LRS = -2 \log(L(0)/L(*)) ,$$

where L(*) is the value of the likelihood function at convergence, and L(0) is the same value when the parameters take value zero. This statistic is asymptotically distributed χ^2 with the degrees of freedom equal to N(L-1)-(K+M), where N is the number of observations, L is the number of alternative modes, and (K+M) is the number of parameters estimated [14].

The coefficient of determination in probability is defined as:

$$(12) \qquad R_p^2 = 1 - \sum_{i,j} (S_{ij} - P_{ij}^*)^2 / \sum_{i,j} (S_{ij} - P_{ij}^0)^2 ,$$

where S_{ij} is the proportion in which various modes are actually chosen by the i-th observation. P_{ij}^* is the probability of choosing mode j at convergence, and P_{ij}^0 is the same probability when the parameters take zero value. This formulation is distinguished from the coefficient of determination in frequency. The latter is defined as:

$$(13) \qquad R_f^2 = 1 - \sum_{i,j} (n_{ij} - P_{ij}^* n_i)^2 / \sum_{i,j} (n_{ij} - P_{ij}^0 n_i)^2 ,$$

where n_{ij} is the number of times mode j was chosen, n_i is the sum of n_{ij} over j, and P^* and P^0 are as defined in (12). The two formulations will be identical when each observation has only one outcome; i.e., $n_i = 1$. However, the need for distinction arises when the data are aggregated, and hence n_i is greater than unity. The percent correctly predicted is defined as:

$$(14) \qquad PCP = \sum_i s_i \Big/ \sum_i n_i ,$$

where s_i is 1 if and only if P^*_{ij} has the highest value among the available alternatives and mode j was actually chosen.

These three statistics for the above five models are given in Table 2. In Table 2, Model D uses only 161 observations, while the others use 241. This is due to incomplete reporting of the family income by some respondents, and whenever this happened the particular observation was omitted from the model estimation. This means that we cannot make a direct comparison of likelihood ratio statistics (LRS), since they change with the degrees of freedom. In the case of the coefficient of determination (R_p^2), and the percent correctly predicted (PCP), they both agree on ranking among the models. In order of efficiency of prediction, Model B is the most efficient, at least in terms of the data with which the models are estimated. Next comes Model A, and then Models C, E, and D, in that order. The relation between Models A and B is somewhat unexpected since in terms of the likelihood ratio statistic, Model A is preferred, but when R_p^2 and PCP are considered, Model B is preferred. An examination of the coefficient estimate for the X_2Y_1 term in Model A reveals that it is not significantly different from zero. Hence, the higher likelihood ratio statistic may be due to the spurious effect caused by the inclusion of an irrelevant variable in the model.

The low ranking of Model D, which has the cost variable deflated by family income, is rather disappointing since one would expect that the higher the family income, the smaller would be the burden of a high cost mode; i.e., automobile. Perhaps, for the same reasons that we discussed in conjunction with inclusion of the family income as a socioeconomic variable,

TABLE 2

Comparison of Several Competing Models

Model	LRS	R_p^2	PCP	No. Obs.
A	378.965	.70946	85.892	241
B	378.644	.70968	86.307	241
C	373.198	.70057	85.477	241
D	238.942	.67675	84.472	161
E	374.174	.69665	84.647	241

the deflation of the cost data with the family income variable may be inappropriate.

Finally, the question of whether to exclude automobile passenger data from estimation of the model parameters was investigated. One would expect that the automobile costs that an automobile passenger bears would be at least lower than that of a car owner driving alone. Indeed, McFadden stated that in one of his recent studies, he divided the automobile cost for car-pool riders by the number of persons in the car, and increased the automobile trip time by five minutes. Hence, it seems plausible to treat automobile passengers separately from automobile drivers.

We decided to test the appropriateness of treating automobile passengers separately by comparing two models: one treating automobile passengers as if they drove a car, and pooling their data with those of car drivers; and the other excluding automobile passengers from the data set. The results are given below:

	LRS	$\underline{R}_p^2$	PCP	N
Auto Passengers Included	378.644	.70968	86.307	241
Auto Passengers Excluded	353.880	.70315	85.398	226

The results are rather unexpected. Even if we disregard the likelihood ratio statistic because of the difference in sample size, both the coefficient of determination and the percent correctly predicted indicate that the model performed better when automobile passenger data were pooled with those of automobile drivers, treating passengers as if they actually drove the car themselves. In addition, the t-statistics for parameter estimates were all higher when the data were pooled. Perhaps, this paradox could be explained in terms of the psychological costs that automobile passengers incur from lack of privacy, loss of flexibility in route selection, and timing conflicts. As a result, the subjective cost of trips borne by automobile passengers may well be equal to the actual outlays of car drivers. In view of the difficulty that public agencies have had in persuading automobile drivers to join car-pools during the recent energy crisis, this interpretation seems appropriate. Accordingly, the data for automobile passengers are pooled with those of automobile drivers in this study.

For the various reasons discussed above, Model B was adopted as the individual demand submodel for this study. Thus, the submodel is a disaggregate multinomial logit model having three alternative modes (automobile, bus, and walking). It incorporates total trip time, total trip cost, and occupation status as explanatory variables. The model also differentiates the coefficient of the trip time variable by occupation status.

Computation was performed on the CDC 6400 computer at Northwestern University's Vogelback Computing Center by using a general purpose multinomial logit program written by the author. A copy of the program, named ESTLOG, is attached as Appendix A. The estimates of the model parameters are given in Table 3, and the estimates of the model are given in Table 4.

As shown in Table 3, most of the parameter estimates are significant at the 5% level. One exception is the estimate of $c_{3,1}$, which is significant at the 10% level. The signs of the estimated coefficients are all correct. The coefficients b_1 and b_3 are those associated with the time and cost variables, respectively. However, b_2 is the dummy coefficient for the time variable associated with Job Class 1, which includes

TABLE 3

Parameter Estimates for Individual Demand Submodel

Variable	Logit Estimator	Stan. Error	T-Stat.
Time b_1	-.13941	.02106	-6.61850
Time b_2	.07541	.03103	2.43040
Cost b_3	-.02418	.00365	-6.63331
Job $c_{2,1}$	-.92316	.51557	-1.79054
Job $c_{3,1}$	-2.76311	1.11905	-2.46916

	(*)	(0)
Log L	-75.44350	-264.76556
PCP	86.30705	33.33333
D.F.	477	477
LRI	378.64412	
R_p^2	.70968	

(*) denotes the value at convergence, and (0) the value when parameters are all zero.

TABLE 4

Individual Demand Submodel

$$P_{i1} = \frac{1}{1 + e^{Z_{i2}} + e^{Z_{i3}}} \ ; \quad \underset{j \neq 1}{P_{ij}} = \frac{e^{Z_{ij}}}{1 + e^{Z_{i2}} + e^{Z_{i3}}}$$

(1) Exponent term for Professionals, Administrators, and Skilled Blue Collar Workers

$$Z_{i2} = \underset{(.02406)}{-.06400} (T_{i2} - T_{i1}) - \underset{(.00365)}{.02418} (C_{i2} - C_{i1})$$

$$\underset{(.51557)}{-.92316}$$

$$Z_{i3} = \underset{(.02406)}{-.06400} (T_{i3} - T_{i1}) - \underset{(.00365)}{.02418} (C_{i3} - C_{i1})$$

$$\underset{(1.11905)}{-2.76311}$$

(2) Exponent term for White Collar and Unskilled Blue Collar Workers

$$Z_{i2} = \underset{(.02106)}{-.13941} (T_{i2} - T_{i1}) - \underset{(.00365)}{.02418} (C_{i2} - C_{i1})$$

$$Z_{i3} = \underset{(.02106)}{-.13941} (T_{i3} - T_{i1}) - \underset{(.00365)}{.02418} (C_{i3} - C_{i1})$$

The number in parentheses is the standard error of estimate.

professionals, administrators, and skilled blue collar workers (PAB). Thus, when the time coefficient is being estimated in the model for PAB's, b_2 must be added to b_1. $c_{2,1}$ and $c_{3,1}$ show the relative bias of PAB's between car and bus, and between car and walking, respectively. The negative signs indicate that PAB's prefer cars over both bus and walking when trip times and costs of modes are identical.

In Table 4, the individual demand submodels are given by occupation class. Z_{i2} is the exponent term associated with mode 2 (bus), and Z_{i3} is the term associated with mode 3 (walking). The coefficient for the time variable in the model for professionals, administrators and skilled blue collar workers was derived, as mentioned previously, by adding b_1 and b_2. Its standard error was obtained by summing the respective variances and covariances, and taking the square root.

d. Summary

In this section, a new disaggregate multinomial mode-choice logit model was discussed. The model incorporates new features such as replacement of the constant term with socioeconomic variables, and inclusion of the dummy coefficients for mode-related variables. This last feature enables estimation of separate slope coefficients for different socioeconomic groups. Indeed, with an appropriate combination of dummy coefficients and dummy variables, the model has the capacity to combine into a single model several models which otherwise would have necessitated stratification of the data.

The theoretical presentation of the model was followed by a discussion of the source and nature of the data. In selecting the form of the submodel to be incorporated into the present study, we investigated the theoretical plausibility of using various model structures, tested the pertinence of variables, and compared the ability of several competing models to duplicate the base year observations. The result was to adopt a submodel which has a disaggregate multinomial logit form, and which incorporates total trip time with its dummy coefficient, total trip cost, and occupation status as explanatory variables. Upon estimation of the model parameters, it was confirmed that the selected submodel had the highest statistical significance for parameter estimates among the competitors.

Section 4

MARKET DEMAND SUBMODEL

a. Existing Trip Distribution Models

In this section, the various limitations of existing gravity and other trip distribution models are first discussed, and then the error minimizing doubly constrained gravity model is discussed. After discussion of the source and nature of data, the market demand submodel is described.

There are a large number of trip distribution models. They include traditional gravity models, entropy maximizing gravity models, intervening opportunity models, growth factor models, probabilistic distribution models, and structural models.

The traditional gravity model has the following general form:

$$(15) \qquad T_{ij} = \frac{aP_iP_j}{d_{ij}^b},$$

where T_{ij}: The number of trips made from zone i to zone j.

P_i, P_j: The population in zone i and zone j, respectively.

d_{ij}: The distance between the two zones.

a, b: The structural parameters.

There are various criticisms of this model. The criticism most pertinent to the purpose of the present study is that the forecast of trip flows made with this model does not meet the row sum and column sum conditions. That is, the sum of estimates of T_{ij} by destination may not be equal to the number of people known to have left origin i, and also the sum of T_{ij} by origins may not give the estimate equal to the number of people known to have arrived in zone j. In addition, this model (15) is nonresponsive to policy changes since it does not include system variables in its formulation.

In recent years, the traditional model has been modified in several ways. In its present form, as incorporated into the UMTA UTPS software package [32], the model assumes the following form:

$$T_{ij}^{e} = T_{i}^{e} \frac{T_{j}^{e} f_{ij}^{e}}{\sum_{j} T_{j}^{e} f_{ij}^{e}}, \tag{16}$$

where e : The trip purpose

T_i, T_j: The trip generation results.

f_{ij}: An arbitrary function of travel time.

In this formulation, some of the weaknesses have been removed by imposing the row sum condition. However, the model can still make a prediction where the number of workers arriving in a given zone exceeds the number employed there. Secondly, even though the travel time is incorporated in the model, the model essentially remains nonresponsive to policy changes, since the cost variable is not included.

In recent years, a new form of gravity model, called the entropy maximizing model, has been introduced by A.G. Wilson [52, 53, 54]. This model has the following form:

$$(17) \qquad T_{ij} = A_i B_j O_i D_j f(c_{ij}) \quad ,$$

where

$$A_i = [\sum_j B_j D_j f(c_{ij})]^{-1} \ ,$$

$$B_j = [\sum_i A_i O_i f(c_{ij})]^{-1} \ .$$

O_i: The number of trips originated in i.

D_j: The number of trips terminated in j.

$f(c_{ij})$: A generalized cost function; e.g., a linear sum of trip costs, travel time, and excess travel time.

The model (17) is subjected to three constraints. Two of these are $\sum_j T_{ij} = O_i$ and $\sum_i T_{ij} = D_j$. These constraints are imposed by means of terms A_i and B_j. The third constraint is:

$$(18) \qquad \sum_{i,j} T_{ij} c_{ij} = C \ ,$$

which states that society's budget for total travel is constant.

The parameters of the model are estimated by maximizing the objective function:

$$(19) \qquad \log \ w(T_{ij})$$

subject to the three constraints above. Here, $w(T_{ij})$ is defined as:

$$(20) \qquad w = T! / \prod_{i,j} T_{ij}! \ .$$

This model nullifies the objections raised against previous models, and can be made policy responsive by an appropriate formulation of the generalized cost function. However, it raises two new problems. One is the constraint on society's travel budget. There is no justification for such a constraint in the real world. The second problem is that when the objective function is maximized, it tends to estimate the parameters of the model in such a way that every cell of the trip distribution matrix has equal entry. That is, T_{ij} will be equal for all i and j, since only then is the objective function w maximized. This problem is partially solved by imposing the row sum and column sum constraints, but not sufficiently to duplicate the base year conditions. An example will help illustrate this.

Assume that the base year trip distribution is as given in Table 5A. Solving the maximization problem without a cost constraint will then lead to the distribution given in Table 5B. Both Tables 5A and 5B satisfy the row sum and column sum constraints. Now, the ratio of w(B) and w(A) is given by:

$$(21) \qquad \frac{w(B)}{w(A)} = \frac{\dfrac{20!}{8!4!4!4!}}{\dfrac{20!}{10!2!2!6!}} = \frac{75}{4} .$$

Since w(B) is greater than w(A), the model will estimate the distribution in Table 5B unless the cost constraint is imposed. Therefore, the fixity of society's travel budget seems to be absolutely essential to replicate the base year distributions. The problem, however, occurs when O_i and D_j increase. Assuming nonzero intrazonal travel costs, as O_i or D_j increases, we would expect more people to travel between zones. However, unless C is increased

TABLE 5

Entropy Maximizing Model and
Trip Distributions

A

From	To		Total O_i
	1	2	
1	10	2	12
2	2	6	8
Total D_j	12	8	20

B

From	To		Total O_i
	1	2	
1	8	4	12
2	4	4	8
Total D_j	12	8	20

or c_{ij} is lowered at the same time, the model will predict that less people will travel than before, because there are more people to whom the travel budget must be allocated. Thus, the levels of C and c_{ij} determine success or failure of prediciton under this model. The model, however, does not say how C and c_{ij} are determined.

In the intervening opportunity models, the number of trips made from zone i to zone j is defined as a product of the number of trip makers leaving zone i and the probability that a trip will end in zone j. In this formulation, possible destination zones are arranged in increasing order of either distance or time of travel from the origin, and this ordering plays a crucial role. No changes in relative trip time will alter the trip distribution unless there is also a change in the ordering of destination zones. Hence, these models are not policy sensitive. Furthermore, in order to maintain the internal consistency of the model when the row sum constraints are imposed, it is recommended that these models be applied to zones with populations of at least 100,000 [41].

In growth factor models, prediction of trip distribution for a future year is made by adjusting the base year trip distribution matrix with some growth factor which is estimated from the ratio of expected future year and base year zonal totals. Consequently, these models implicitly assume either that the transportation system has no influence on the trip distribution or that the transportation system remains unchanged [41]. Therefore, by construction, these models are incapable of reflecting changes in transport variables, and hence are nonresponsive to policy changes.

The structural model of trip distribution [7] uses a convex programming approach to estimate future flows of trip-makers between zones. Specifically, the method minimizes the distance between $f_{ij}d/O_iD_j$ and $f_{ij}^0d^0/O_i^0D_j^0$, while imposing the row sum condition $\sum_j(O_iD_j/d - f_{ij})=0$, and the column sum condition $\sum_i(O_iD_j/d - f_{ij})=0$. Here, f_{ij}^0 and f_{ij} are the base year and future year flow of trip-makers from zone i to zone j, and O_i^0 and O_i are the base year and future year total departures from zone i. D_j^0 and D_j are the base year and future year total arrivals in zone j, while d^0 and d are base year and future year total trips; i.e., $d^0 = \sum_i O_i^0 = \sum_j D_j^0$ and $d = \sum_i O_i = \sum_j D_j$. The model, however, does not contain any trip cost or trip time variables, and hence it is not responsive to policy changes.

Probabilistic distribtuion models [11] use a transition probability matrix to estimate the number of trips made from zone i to zone j. The model is given by

$$(22) \qquad t_{ij} = O_iP_{ij} \text{ , for all i and j,}$$

where t_{ij} is the number of trip-makers going from zone i to zone j; O_i is the total departures from zone i; and P_{ij} is the transition probability defined by the conditional probability $P(D_j|O_i)$ that a person who originates in zone i will travel to zone j. If the base year flow matrix $\{t_{ij}^0\}$ is known, then the transition probability P_{ij} may be estimated by

$$(23) \qquad P_{ij} = t_{ij}^0/O_i^0 .$$

These models also provide a means for computing the transition probabilities by incorporating various motives, as shown in (24).

$$(24) \qquad P_{ij} = P(D_j|O_i) = \sum_{k=1}^{K} P(D_j|O_i,m_k)\ P(m_k|O_i),$$

where m_k is the k-th motive. By definition of the transition matrix, this model satisfies the row sum conditions, but the column sum condition is not imposed.

Furthermore, as formulated, the model can accommodate only one class of "motives" at a time, such as one way classification by age group. As the number of classes of "motives" increases, the computation of the conditional probability becomes increasingly complex. Finally, if continuous system variables are added to the model to make it policy responsive, it becomes necessary to provide for an enormously large number of "motive" cells so that every meaningful combination of values of system variables may be assigned to a cell, and the corresponding conditional probabilities will have to be estimated. It seems that the models that are designed to use continuous variables, such as gravity models, are more suited for building policy responsive trip-distribution models.

Finally, there are a number of aggregate models which are designed to estimate simultaneously trip generation, trip distribution, and modal split. They include the abstract transport model introduced by Quandt and Baumol [33,34] and other similar models. The common characteristic of these models is that the explanatory variables enter the model in product form. As a result, it is implicitly assumed that the elasticity of demand for modes with respect to the explanatory variables is constant. This is a rather implausible assumption, since it implies that as the trip cost declines, or as the level of income rises, the demand for modes will rise without limit.

These shortcomings can be corrected by imposing row sum and column sum constraints on the trip distribution matrix. However, imposing the row sum and column sum constraints removes the trip generation aspect of the models, and they are reduced to aggregate trip distribution and modal choice models. Concerning simultaneous estimation of mode and destination choices, a preferred approach seems to be to treat these decision processes separately as has been done in this study. The rationale is to incorporate a disaggregate behavioral model into our model as a submodel, and to make full use of its high efficiency in predicting individual modal choice behavior.

The discussion above has shown that trip distribution and other pertinent aggregate models currently in existence do not adequately meet the standard desired for the present sutdy. Specifically, a trip distribution model is needed which has the following characteristics: (1) the model must contain a sufficient number of transport variables to make it policy responsive; (2) the prediction based on the model must reflect the competition among various possible origins and destinations; (3) the prediction must be consistent with total zonal departures and arrivals; and (4) the model must be capable of being interfaced with the modal choice model described in Section 3. The interfacing must reflect the simultaneity of mode choice and destination choice decisions of commuters. A discussion of the model which meets these requirements follows.

b. Error minimizing doubly constrained gravity model.

Consider a model of the following form:

$$Q_{ij} = \prod_k X_{ijk}^{a_k} \prod_m O_{im}^{b_m} \prod_n D_{jn}^{c_n} \quad , \tag{25}$$

$$O_{i1} = \sum_j Q_{ij} \quad , \tag{26a}$$

$$D_{j1} = \sum_i Q_{ij} \quad , \tag{26b}$$

where Q_{ij}: The flow of commuters from zone i to zone j during a given time period, say a day.

X_{ijk}: The k-th system variable such as the cost or time it takes to travel from i to j.

O_{im}: The m-th variable that describes a characteristic of zone i, from which the trip originates. This may include such variables as size of labor force and population.

O_{i1}: The first variable among O_{im}, specifically defined as the level of employed labor force.

D_{in}: The n-th variable describing a characteristic of zone j in which the trip terminates. This may include level of economic activities, employment, and population.

D_{i1}: The first variable among D_{in}, specifically defined as the level of employment.

The equations (25) and (26) constitute the basic model from which our market demand submodel will be derived.

In this study, we included two variables in X_{ijk}, the variables that describe the transport system. These are the composite cost of making a trip from zone i to zone j, and the composite time of doing so. The composite cost and time are the weighted averages of mode costs and mode times in traveling from i to j. The weights used are the proportion by which the total trips flow from i to j was divided among the various modes. As the descriptors of zone i, O_{im}, we initially considered the size of the employed labor force and the labor force composition, and for the descriptor of zone j, D_{jn}, we tried the industry composition and the level of employment. However, labor force composition at origin i and industry composition at destination j were later dropped to avoid multicollinearity among the variables. Hence, after changing the variable symbols for easier identification, the basic model (25) and (26) become:

$$Q_{ij} = C_{ij}^{a}\, T_{ij}^{b}\, L_{i}^{d}\, E_{j}^{e} \quad , \tag{27}$$

$$L_{i} = \sum_{j} Q_{ij} \quad , \tag{28a}$$

$$E_{j} = \sum_{i} Q_{ij} \quad , \tag{28b}$$

where Q_{ij}: The flow of commuters from zone i to zone j during a day.

C_{ij}: The composite cost of a trip from zone i to zone j.

T_{ij}: The composite time of a trip from zone i to zone j.

L_{i} : The size of the employed labor force in i.

E_{j} : The level of employment at j.

a, b, d, e are parameters.

As the model (27) is formulated, it is a market demand function for transportation in ij-th transport market, with C_{ij} and T_{ij} being "market price." L_i and E_j then act as the shift parameters that denote the size of the market. Therefore, changes in policy variables such as changes in bus fare and toll charges would affect the demand for transport service through the composite cost. Improvement of highway conditions that reduces driving time would influence the demand through the composite time variable. Therefore, the model is policy responsive.

The elasticities of trip volumes Q_{ij} for model (27) with respect to its component variables can be expressed as in equations (29a) - (29d). These relations are obtained by imposing constraint (28b) on the model as given in equation (30).

$$(29a)\qquad \frac{\partial Q_{ij}}{\partial C_{ij}}\,\frac{C_{ij}}{Q_{ij}} = (1 - P_{ij}) \left[a + \frac{P_{ij}\,(1 - P_{ij})\,L_i\,a\,d}{E_j} \right],$$

$$(29b)\qquad \frac{\partial Q_{ij}}{\partial T_{ij}}\,\frac{T_{ij}}{Q_{ij}} = (1 - P_{ij}) \left[b + \frac{P_{ij}\,(1 - P_{ij})\,L_i\,b\,d}{E_j} \right],$$

$$(29c)\qquad \frac{\partial Q_{ij}}{\partial E_j}\,\frac{E_j}{Q_{ij}} = (1 - P_{ij})\,e^2 \left[\sum_{m=1}^{N} P_{mj}(1 - P_{mj})\,L_m \right],$$

$$(29d)\qquad \frac{\partial Q_{ij}}{\partial L_i}\,\frac{L_i}{Q_{ij}} = 1\,,$$

where $P_{ij} = Q_{ij}/\sum_j Q_{ij}$. Other terms are as defined previously. Except for the labor force elasticity (29d), which has a value of one, all elasticities change with the value of the variables. Specifically, both the cost elasticity (29a) and the time elasticity (29b), which are negative, reduce to 0 as the trip cost and trip time approach zero. They approach coefficients a and b in the model, respectively, as the cost and time variables increase without limit. Employment elasticity (29c) is positive, but moves in an opposite direction to that of changes in level of employment.

Moreover, imposition of the row sum and column sum constraints (28a) and (28b) assures that the competition among various origins and destinations is reflected in the modes and eliminates the possibility of such inconsistencies as more commuters leaving for work from a given zone than there are people living in it or more commuters arriving for work in j than are employed there.

The theoretical justification for a gravity model of this type rests on the fact that there is aggregation over job markets. At the individual level, a commuter may be seen as maximizing his own welfare by selecting that destination which gives him the highest level of satisfaction in terms of resources sacrificed. Thus, if alternative destinations offer identical levels of satisfaction in all respects, but differ in the cost and time required to reach them, he would choose that destination which involves the least cost and time. Therefore, his action may well be predicted by spatial linear programming.

When welfare maximizing individuals (those whose occupations are different and who hence have different welfare maximizing work destinations) are assigned to spatially delineated zones and aggregated, then the apparent phenomenon of "cross-commuting" is observed. This cross-commuting is not due to the irrationality of trip-makers (indeed they are acting very rationally at the individual level), but it is due to aggregation over job groups. As one tries to duplicate the aggregate behavior of the commuters for the base period by applying spatial linear programming, the optimization process will completely eliminate the phenomenon of cross-commuting. The error here is application of the optimization process at the aggregated level, which implicitly assumes homogeneity of the work force, rather than at the individual level where it is appropriate. Hence, for prediction at the aggregated level, the gravity model, which is capable of duplicating base period observations, is preferred over the spatial linear programming approach.

In estimating the parameters of the model, the row sum condition was first imposed by dividing (27) by (28a) as in (30):

$$(30) \qquad \frac{Q_{ij}}{L_i} = \frac{C_{ij}^a \; T_{ij}^b \; E_j^e}{\sum_j C_{ij}^a \; T_{ij}^b \; E_j^e} ,$$

The column sum condition was also imposed by dividing (27) by (28b) as in (31):

$$(31) \qquad \frac{Q_{ij}}{E_j} = \frac{C_{ij}^{a}\, T_{ij}^{b}\, L_i^{d}}{\sum_i C_{ij}^{a}\, T_{ij}^{b}\, L_i^{d}} .$$

Equations (30) and (31) were then squared and summed over all zonal pairs to derive equations (32) and (33):

$$(32) \qquad S_0 = \sum_{i,j} \left[\frac{Q_{ij}}{L_i} - \frac{C_{ij}^{a}\, T_{ij}^{b}\, E_j^{e}}{\sum_j C_{ij}^{a}\, T_{ij}^{b}\, E_j^{e}} \right]^2 ,$$

$$(33) \qquad S_D = \sum_{i,j} \left[\frac{Q_{ij}}{E_j} - \frac{C_{ij}^{a}\, T_{ij}^{b}\, L_i^{d}}{\sum_i C_{ij}^{a}\, T_{ij}^{b}\, L_i^{d}} \right]^2 .$$

These are the objective functions used to estimate the parameters of the model. In (32), S_0 signifies that it is the objective function obtained by imposing the origin constraint (28a), while in (33), S_D denotes that it is obtained similarly by imposing the destination constraint. Since the objective functions are nonlinear in parameters, their estimation requires minimization of the functions using a nonlinear programming technique.

Once the parameters are estimated, two sets of forecasts of Q_{ij} will be estimated from:

$$(34) \qquad Q_{ij}^{0} = \frac{C_{ij}^{\hat{a}}\, T_{ij}^{\hat{b}}\, E_j^{\hat{e}}}{\sum_j C_{ij}^{\hat{a}}\, T_{ij}^{\hat{b}}\, E_j^{\hat{e}}}\, L_i ,$$

$$(35) \qquad Q^{D}_{ij} = \frac{C^{\hat{\hat{a}}}_{ij}\, T^{\hat{b}}_{ij}\, L^{\hat{d}}_{i}}{\sum\limits_{i} C^{\hat{\hat{a}}}_{ij}\, T^{\hat{b}}_{ij}\, L^{\hat{d}}_{i}}\, E_{j} \ .$$

Generally, the two sets of estimates Q^{O}_{ij} and Q^{D}_{ij} do not agree. The difference is adjusted by taking the average of equations (34) and (35) for each ij pair, and by applying the Furness iteration method to assure that the constraints are satisfied.

Another approach also was tried, which expresses the objective function in the following way:

$$(36) \qquad S = \sum_{i,j} \left[\frac{Q_{ij}}{L_{i}} - \frac{C^{a}_{ij}\, T^{b}_{ij}\, E^{e}_{j}}{\sum\limits_{j} C^{a}_{ij}\, T^{b}_{ij}\, E^{e}_{j}} \right]^{2}$$

$$+ \sum_{j} \lambda_{j} \left[E_{j} - \sum_{i} \frac{C^{a}_{ij}\, T^{b}_{ij}\, E^{e}_{j}\, L_{i}}{\sum\limits_{j} C^{a}_{ij}\, T^{b}_{ij}\, E^{e}_{j}} \right]^{2} ,$$

where λ_{j} is an externally assigned penalty value. Even though convergence was achieved after a few iterations, the resulting estimates of Q_{ij} were far from duplicating the base year values; therefore, this approach was dropped from further considerations. It was suspected that because of the manner in which the constraints are imposed in (36), the objective function lost its strict convexity, and the solution converged to a local optimum rather than to a global optimum.

c. Source and Nature of Data

The data for estimating the parameters of the market demand submodel were obtained from the 1970 Census of Population Journey to Work Report [45].

The reported data is based on a 15% sample adjusted to represent the total population. The report shows the number of workers, and their characteristics, who traveled from one subunit to another subunit in metropolitan areas with populations of 250,000 or more. In the report, cities with populations of 50,000 or more are individually identified, while those cities with populations under 50,000 are aggregated as "remainder of county." For instance, for the Chicago Metropolitan Area, which consists of 6 counties, the report identifies 17 cities of over 50,000 and 6 "remainder of county" units.

For all combinations of pairs of these subunits, the report tabulates by direction of flow the number of workers who traveled, the mode they chose, their socioeconomic attributes such as sex, age, race, family relationship, education, occupation, industry groups, and earnings. Data also exist for travel within each subunit.

In this study, fourteen cities in the Chicago Metropolitan Area were identified. Data on commuters who traveled from any one city to another among the fourteen were tabulated. Excluded from consideration was the city of Chicago proper. In a gravity model, or any aggregated model, the distance between cities is generally measured from city center to city center. However, the city of Chicago is spatially aggregated into only one large unit, and it was decided that no meaningful measure of distance between Chicago and other cities in the study area could be developed. Despite the exclusion of Chicago from the study, the validity of the study results will not be affected since the model is independent of city designations. The study results will apply equally to Chicago if the data for its subdivisions which are similar in size to suburban cities become available.

For the fourteen cities included in the study, the data were tabulated for the number of commuters by mode they selected in reaching their place of employment. Modes included private automobile (as driver); private automobile (as passenger); bus; subway, elevated train or railroad; walked to work; worked at home; and other means including taxicabs. Also tabulated were the proportion of professionals, administrators and skilled blue collar workers among the commuters who traveled between a given city pair in each direction. The stratification of commuters by such a job classification was done to assure conformity with the occupational class adopted in the individual demand submodel.

The tabulated results of commuters by mode were next given minor adjustments of the following types. (1) When the frequency for "walked to work" between two cities which are beyond normal walking distance was nonzero, it was assumed to be a reporting error, and the data were distributed among other modes according to the proportion in which commuters chose those modes. (2) Similarly allocated among other modes were those observations under "others including taxicabs," on the assumption that taxicabs were not a normal means of commuting. (3) In the case of workers who were reported to be commuting to cities other than their own, and yet classified under "worked at home," it was assumed that their normal place of work was as indicated by destination, but on the day of census taking, they worked at home. Hence, these were again distributed among other modes in the same manner. (4) In case of intracity commuters who are classified under "worked at home," it was assumed again that these observations included those who normally worked away from home, but on the day of census taking they worked at home as in

Case (3) above. We, therefore, decided to isolate these individuals from those whose place of work was actually at home. In so doing, the average "worked at home" to the total commuter ratio was computed for those who worked outside their own cities. This ratio was then multiplied to the total intracity commuters for each city, and the number for those whose normal place of work was away from home but who worked at home on the day of census taking was derived. They were subtracted from the reported frequency of "worked at home" for intracity workers, and assigned to other modes according to the proportion in which modes were used in that city.

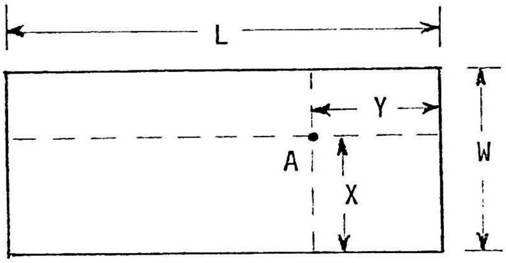
Next, the automobile driving time between cities were estimated by measuring the distance between city centers by road segments, then by applying different driving speeds to each road segment according to the traffic condition, and by aggregating the resulting driving time for all road segments. For automobile driving costs, 10¢ per mile cost was applied to the distance between city centers, and whatever toll charges that were applicable were added. Bus and rapid transit commuting time, whenever such service existed, were estimated by applying the average speed of mode on each route to the segment of commuting routes applicable, and by aggregating over the entire commuting route. Added to this were one half bus or transit headway for waiting time; access-egress time of 6 minutes computed at average walking distance of 800 ft. at both ends of the trip and at walking speed of 3 miles per hour; and walking time when transfers were involved. Bus and rapid transit speeds by route were estimated from bus and rapid transit schedules. Bus and transit fares were also obtained from the CTA History of Fares, and from the records of the Illinois Commerce Commission for suburban buses. Finally, walking time was computed at a walking speed of 3 miles per hour, and walking cost was assumed to be zero. Train time and cost were ignored because the proportion of commuters taking trains between suburban cities was very small, and hence would have a negligible influence on the composite time and cost.

In the case of intracity commuters, the average distance was estimated by assuming uniform distribution of commuters within the city limits, and by assuming that the place of employment was located at the city center. A general formula for this is given by:

$$(37) \qquad \text{Av. Dist.} = \frac{1}{3}\{[1 - 2C(1-C)]\,L + [1 - 2D(1 - D)]W\},$$

where the symbols are defined by:

A = Place of Employment
L = Length
W = Width
C = Y/L
D = X/W



In a special case, when the place of employment is located at the city center, as is assumed in this study, (37) reduces to $(L + W)/6$. Once the average commuting distance within the city was derived, the time and cost of modes were estimated in the same manner for intercity commuters.

The composite commuting time between and within cities was derived as the weighted average of mode times using the percentage of times that various modes were used by commuters as weights. The composite costs were also estimated in the same manner.

In estimating the parameters of the market demand submodel, automobile passengers were pooled with automobile drivers on the basis of the findings in Section 3. In addition, when the data for those who "worked at home" were pooled with the data for those who "walked to work," better estimation results were obtained than when they were excluded. The model estimated using the pooled data set produced the "percent correctly predicted" score of 89.9% and the coefficient of determination in frequency of .96218. On the other hand, when the data for those who "worked at home" were excluded, the resulting model had the scores of 88.7% and .95441, respectively. As such, these data were pooled and the commuting time and cost of those who "walked to work" were assigned.

d. Estimation

Estimation of the parameters for the market demand submodel (34) and (35) was performed again on the CDC 6400 computer at Northwestern's Computing Center, using the computer program written for this purpose by the author. A copy of the program called GRAVITY is attached as Appendix B.

The estimates of the parameters are presented in Table 6. These estimates were obtained using data for six cities. Cities that were excluded were located beyond the normal commuting distance in relation to each other, and the trip frequencies among them were mostly zero.

The estimates are not statistically significant at the 5% level. However, the signs of the estimates are all correct, and both the origin-constrained and destination-constrained models "predicted" 92% of the base year observations correctly, and had the coefficient of determination in frequency of .978 and .976, respectively.

In Table 7, the estimates of commuter trip frequencies among six cities for the base year are presented. These estimates were obtained as discussed

TABLE 6

Parameter Estimates for Market Demand Submodel

	Q^D_{ij}			Q^O_{ij}		
Para.	Coeff.	S.E.	T-Stat.	Coeff.	S.E.	T-Stat.
a	-1.50713	1.67730	-.89854	-1.60875	2.76648	-.58152
b	-.32653	5.46997	-.05969	-.81565	5.82389	-.14005
d	.97311	4.19361	.23205	—	—	—
e	—	—	—	1.59602	4.33886	.36784
PCP		92.076			91.725	
R^2_f		.97579			.97784	

TABLE 7

Trip Distribution
(Predicted)

		To						
		1	2	3	4	5	6	Total
From	1	5,631	1,444	10	21	217	18	7,341
	2	1,405	9,727	24	83	389	41	11,669
	3	9	18	8,165	126	30	104	8,453
	4	21	50	130	13,230	58	2,844	16,333
	5	240	390	30	39	5,403	43	6,146
	6	15	26	88	2,865	39	6,513	9,546
	Total	7,321	11,656	8,447	16,365	6,136	9,564	59,488

earlier by taking a simple average of frequencies predicted by both origin-constrained and destination-constrained models, and by six iterations of the Furness method to impose the row sum and column sum constraints. Table 7 may be compared with the actual trip frequencies observed, which are given in Table 8. From these tables, it can be observed that the column constraints are not satisfied, suggesting that it would take more than six iterations of the Furness method to achieve the desired results. Nevertheless, the model performs well in that it predicts 89.9% of the observations correctly, and has the coefficient of determination of .96218.

Finally, it is noted that in the gravity model, the commuter flow is expressed as a function of the size of the employed labor force, the level of employment, and the cost and time of commuting between cities. As such, the model is quite general in its applicability, and the parameter estimates performed well in predicting the trip frequencies whether the number of cities was more or less than the number used to estimate the model.

e. Summary

In this section, a brief survey of the existing trip-distribution models was presented and their pertinence to the present study was discussed. The error minimizing doubly constrained gravity model, which successfully solves the limitation of the existing trip-distribution models, was introduced; and the source and nature of the data were discussed. We then presented the estimates of the model parameters, and found that the model performed well in duplicating the base year conditions. The next section is concerned with interfacing the two submodels developed in Sections 3 and 4.

TABLE 8

Trip Distribution
(Actual)

To

From

	1	2	3	4	5	6	Total
1	3,494	3,092	114	14	558	49	7,321
2	1,020	9,974	86	21	432	123	11,656
3	13	70	7,537	179	35	613	8,447
4	8	28	174	14,351	46	1,758	16,365
5	212	553	116	40	5,104	111	6,136
6	0	69	227	1,598	17	7,653	9,564
Total	4,747	13,786	8,254	16,203	6,192	10,307	59,489

Section 5

AGGREGATION FUNCTION

As will be seen in Section 6, the submodels developed in this study have been used in the following sequence. First, the individual demand submodel was used to predict the modal share of commuter traffic between two cities. The estimates of mode market shares were then used to calculate the composite cost and composite time, which in turn was applied to the market demand submodel to estimate the commuter market demand for transport service. Finally, the estimated commuter market demand was allocated among various modes in proportion to commuters' modal choice prediction.

A difficulty arises, however, in predicting the modal share for the market from the individual demand submodel, which is in disaggregate form. It is known that substituting the mean value of the transport variables (such as average cost and time for the market) leads to valid prediction of mode market share only if the model is linear in the variables involved. However, as we have already seen, the disaggregate model adopted in this study is in logit form, and hence is nonlinear in the variables included. The disaggregate demand model was estimated in such a manner as to give the predicted probability close to 1 when a particular mode was chosen, and near zero otherwise. Hence, a simple substitution of mean values for transport cost and time faced by an "average" individual in the market will lead to an exaggerated prediction of market share either toward one mode or the other depending on the distribution of the variables. The objective of this section is to provide a means of solving such aggregation issues.

Koppelman, in his recent paper, discusses five aggregation procedures used in estimating the aggregate mode share from the disaggregate model. They include: enumeration, summation/integration, statistical differentiation, classification, and naive procedures [15]. The method of aggregation available to a researcher depends to a large extent upon availability and nature of the data. In this study, we have adopted a procedure which combines a modified and simplified version of the statistical differential procedure and the classification procedure.

The statistical differential method proceeds by linearizing the disaggregate model by using a Taylor series expansion, and then obtaining the weighted average or expectation over the group for which the prediction is being made. This approach requires estimation of the moments of distribution of the transport and socioeconomic variables. It is known that the series tends to be unstable when the variables are highly dispersed.

In this study, the aggregation function, still unspecified but expressed as a function of the estimates of mode share derived from the disaggregate model, was approximated by use of a Taylor series expansion about some fixed value. In this case, since the partial derivatives of all orders are evaluated at the fixed value, they reduced to constants. When such a function is rearranged and simplified, the market modal share expressed as a polynomial function of the mode share estimated from the disaggregate model is derived. The classification procedure is used in this study to weight the mode share estimated from the disaggregate model by the proportion of commuters in various occupation classes. The latter, as previously noted, were derived from the <u>1970 Population Census Journey to Work Report</u>.

Before deciding which form of aggregation function to adopt, three different aggregation functions were estimated, and the results were compared. They included: (1) The direct aggregate market share model which estimates a logit model by using the mean value of independent variables and the frequencies of modes chosen, and directly estimates the mode market share. (2) In the naive-statistical differential approach the initial mode share is estimated from the disaggregate model using the mean value of independent variables and the proportion of professionals, administrators and skilled blue collar workers (PAB) as the socioeconomic variable. The resulting probabilities were then used as independent variables of the aggregation function. In this case, separate polynomial functions of the 4th degree were estimated for each mode, and the results were normalized to assure that the mode shares would sum to unity. (3) The classification-statistical differential approach is similar to the naive-statistical differential approach, except that the initial mode shares were estimated for each job class from the disaggregate model, and the results were weighted by the job proportions before estimating the aggregation function.

On comparing the three methods of deriving the aggregation function, it was found that the third approach gave the best results in terms of the coefficient of determination in probability (R_p^2 = .952892), closely followed by the second method (R_p^2 = .952629), with the first approach being last (R_p^2 = .923886). For this reason, the third approach was used to estimate the aggregation function, which has the following form:

$$(38) \qquad S_m = \frac{S_m^*}{\sum_m S_m^*} \, ,$$

(39) $$S_m^* = a_0 + a_1P_m + a_2P_m^2 + a_3P_m^3 + a_4P_m^4 ,$$

where S_m: Final estimate of the market share of mode m.

S_m^*: Estimate of the market share of mode m, obtained from the polynomial function.

P_m: Weighted average of the market shares of mode m, derived from the disaggregate model for each occupation class.

a_0, a_1, a_2, a_3, a_4 are parameters.

The estimates of the parameters of the aggregation function are given in Table 9.

Section 6

METHOD OF PREDICTING THE DEMAND FOR BUS SERVICE IN SUBURBS AND SATELLITE CITIES OF METROPOLITAN AREAS

In the preceding three sections, the individual demand submodel predicting individual mode choice, the market demand submodel for transport service, and the aggregation function to interface the two submodels were presented. Using a few examples, this section discusses methods of estimating the work-trip demand for bus service in suburbs and satellite cities of metropolitan areas on the basis of the submodels and the aggregation function estimated. In order to facilitate discussion, the Flow Chart for the forecasting method is presented in Figure 1.

TABLE 9

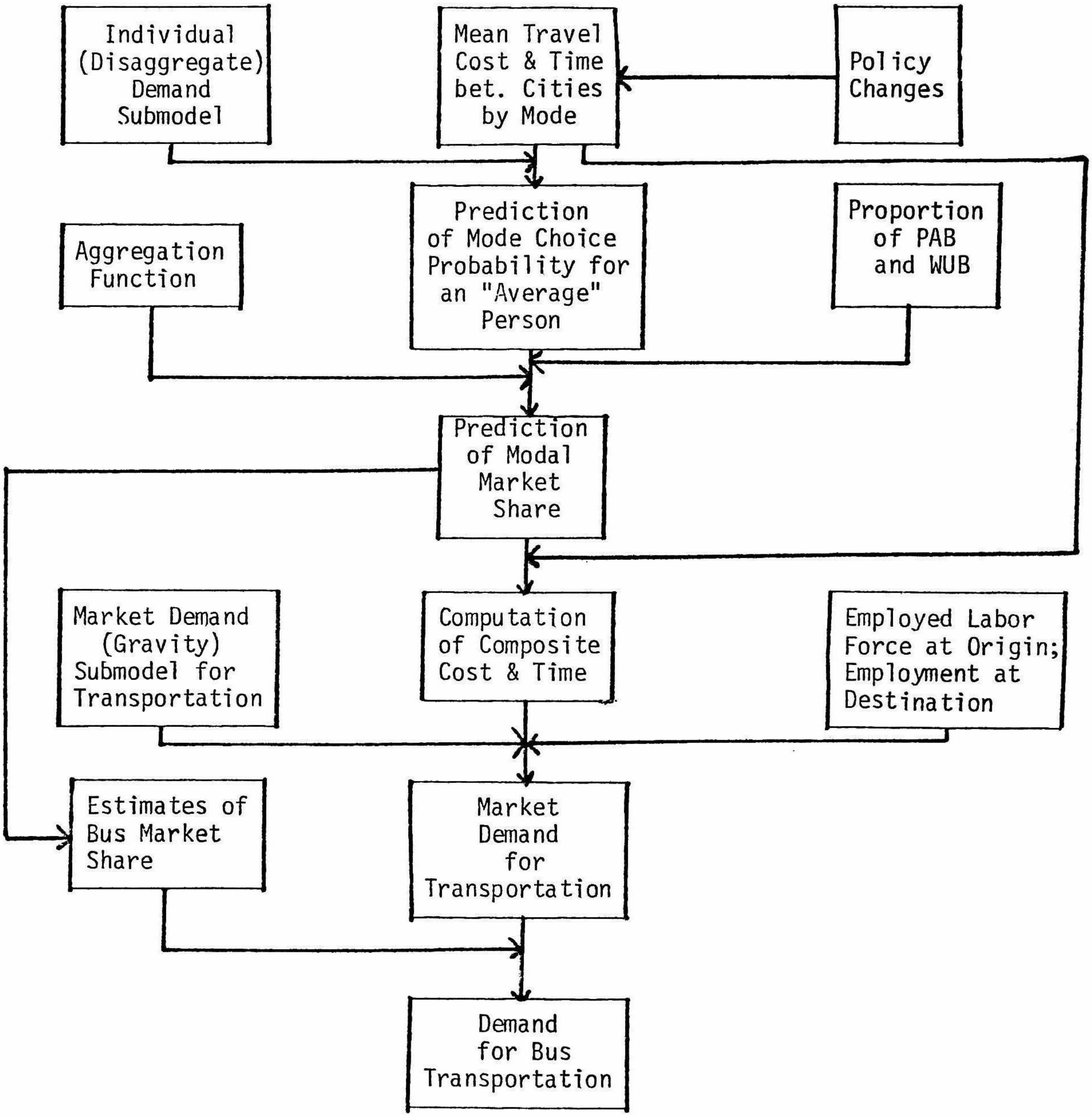
Parameter Estimates for Aggregation Function

Para.	Car	Bus	Walk
a_0	1.52855 (2.05552)	.21211 (4.56396)	.01440 (.92845)
a_1	-7.31059 (-1.02466)	-2.21352 (-2.27023)	3.21067 (.41905)
a_2	25.34278 (-1.17762)	10.74582 (1.87881)	-29.64260 (-.04366)
a_3	-33.72604 (-1.31761)	-18.67146 (-1.61553)	946.42417 (.05762)
a_4	15.02305 (1.42900)	10.55681 (1.45725)	-4272.34375 (-.03604)

The numbers in parentheses are t-statistics.

FIGURE 1

Flow Diagram for Forecasting
Work Trip Demand for Bus Transportation



PAB = professionals, administrators and skilled blue collar workers
WUB = white collar and unskilled blue collar workers

a. Estimating the Effect of Policy Changes on Demand for Bus Service

For estimating the effect of policy changes on demand for bus service, the following steps are taken:

(1) Estimate or specify the magnitude of policy changes such as the amount of changes in bus fare.

(2) Compute new average cost and time by mode for study area by incorporating the new bus fares.

(3) Compute the proportion of professionals, administrators, and skilled blue collar workers in various commuting groups. This may be derived from the previous census data.

(4) Estimate the probability with which an "average" person among professionals, administrators, and skilled blue collar workers (PAB) would select various modes by substituting into the individual demand submodel the mode costs and times derived in (2) and a dummy value of 1 for the Job variable. Repeat the same process with a dummy value of 0 for the Job variable to estimate the probability with which modes are selected by an "average" individual among white collar and unskilled blue collar workers (WUB). An option to perform these operations, including data processing and card punching, is available in the computer program ESTLOG attached as Appendix A.

(5) Derive the weighted average of disaggregate probabilities by weighting the results of (4) by the proportion of PAB and WUB obtained in (3). Repeat the process for each mode.

(6) Substitute the results of (5) into the aggregation function to estimate the mode market share for each commuting link being studied.

(7) Compute the composite mode cost and mode time by obtaining the weighted average of the mode costs and mode times derived in (2) weighted by the mode market shares estimated in (6).

(8) Obtain estimates on the level of employment and the size of the employed labor force at each community under study.

(9) Substitute the results of (7) and (8) into the market demand submodel to estimate the flow by direction of commuters between city pairs.

(10) Estimate the number of commuters taking the bus by multiplying the results of (9) by the market share for bus estimated in (6).

(11) Finally, compare the results of (10) with the results of previous studies.

b. Predicting When Installation of Bus Service Becomes Economically Feasible

In this case, the process is:

(1) Compute the minimum level of bus demand to make a given bus line economically feasible. This may be obtained by calculating the cost of running a proposed line and the average fares anticipated.

(2) Generate time series of the level of employed labor force at the origin, and the level of employment at the destination as well as the average cost and time of traveling by each mode between the cities in the study area. In this case, the average cost of modes must be deflated by the average income index or the wage index to adjust for price and income changes over time. The average income index may be obtained by dividing the estimate of the median nominal income of commuters for future years by that of the base period; and similarly for the wage index.

(3) On the basis of the data obtained in (2), estimate a time series of the demand for bus service under assumed conditions, and compare them with the minimum level needed for economic feasibility.

(4) In forecasting such as this, it is important to recognize the fact that the demand for bus service is determined not only by changes in the level of service by the bus line itself, but also by changes in service level of other modes, such as toll charges, gasoline prices, parking fees. Hence, in generating the time series data on these variables, as many alternative scenarios as can be perceived should be considered.

In the above, by means of examples, we have demonstrated how the models presented in this report may be applied. Effects of other policy changes and changes in the transportation system on demand for bus transportation may be studied by appropriately modifying the above examples.

Section 7

SUMMARY AND RECOMMENDATIONS

In this report, a method of estimating the commuter demand for bus service in suburbs and satellite cities of metropolitan areas was developed. The basic approach was to estimate the mode market share from a disaggregate model, using the average cost and time of traveling by three modes: automobile, bus, and walking. The estimates of mode choice probabilities of an "average" person were then adjusted by the aggregation function to derive the market share of modes for commuting between cities. The mode shares were used for computation of the composite cost and composite time, these in turn were used for estimation of the market demand for transportation. Mode shares were also used for allocating the market demand for commuter traffic to various modes.

The effectiveness of forecasting models such as this one depends largely upon the existing state of technical knowledge, availability and quality of data, and computational facilities. The report offers several innovations. They include:

(1) Discovery that simultaneous destination-mode-choice disaggregate logit models misclassify the destination choice of cross-commuters when they are present.

(2) Identification of a conceptual error in the entropy maximizing gravity model.

(3) Introduction of a disaggregate logit model which incorporates dummy variables and dummy coefficients. Thus, the model combines several models obtained by data stratification into a single model.

(4) Writing of the computer program to perform computation for the logit model of the above type. The program is also efficient in handling the socioeconomic variables in terms of reduced storage space and processing of data. It also has the facility to estimate the direct and cross-elasticities of demand.

(5) Introduction of the aggregation function to derive the market share of modes from the probabilities of mode choice made by an "average" person. The latter is estimated from the disaggregate model using average cost and time of alternative modes.

(6) Introduction of a gravity model which is policy responsive, and which

also reflects the competition among various origins and destinations.

(7) Writing of the computer program to perform computation for the gravity model of the above type.

Availability of data affected the structure of the present study in several ways:

(1) At the disaggregate level, lack of data on wages or individual income prevented deflating of the cost data. This, however, is a minor problem, since future data can be deflated with wage index or income index estimated by dividing the median income for the future year by that of the base period. Also, lack of data to indicate the level of comfort, such as degree of crowdedness, prevented inclusion of a comfort index into the model.

(2) At the aggregate level, absence of data on the distribution (i.e., variances and covariances) of the independent variables prevented application of other aggregation procedures to estimate market share of modes, and the error analysis of the forecast. The use of census data affected the study in several ways. Since the census data are published for cities of populations over 50,000, many small cities had to be omitted in the estimation of the model. In the case of Chicago, it was omitted because no meaningful measure of distance between Chicago and its satellite cities could be developed for its high degree of spatial aggregation. This, however, is a minor problem since the gravity model estimated in the study is still valid and transferrable to other regions. However, the high degree of spatial aggregation in the Chicago data prevented the author from answering one question. That is: how to estimate the flow of commuters taking buses between Chicago and its satellite cities. To answer this question, we need disaggregated

data of the city of Chicago so that its subunits would be of about the same size as its satellite cities.

(3) Finally, the data used for estimation of the individual demand submodel were collected in the summer of 1969, and that used to estimate the market demand submodel in April, 1970. When data for other times of the year become available, it is recommended that the model presented in this study be reestimated to examine for the presence of seasonal bias due to data.

REFERENCES

(1) Ben-Akiva, M.E., Structure of Passenger Travel Demand Models. Ph.D. Dissertation. (M.I.T.) 1973.

(2) Ben-Akiva, M.E. and F.S. Koppelman, "Multidimensional Choice Models: Alternative Structures of Travel Demand Models," Transportation Research Board Special Report. No. 149, 1974.

(3) Charles River Associates, Inc., A Disaggregated Behavioral Model of Urban Travel Demand. Final Report to Federal Highway Administration, U.S. Department of Transportation, 1973.

(4) Chicago Area Transportation Study, 1970 Travel Characteristics: Trip Length. 1975.

(5) Domencich, T.A. and D. McFadden, Urban Travel Demand: A Behavioral Analysis. North-Holland/American Elsevier, 1975.

(6) Evans, A.W., "Some Properties of Trip Distribution Method," Transportation Research. Vol.4, 1970, pp. 19-36.

(7) Ferragu, C. and M. Sakarovitch, "A Class of 'Structural' Models for Trip Distribution," Transportation Research. Vol. 4, 1970, pp. 87-92.

(8) Ferreri, M.G. and W. Cherwony, "Choice and Captive Modal-Split Model," Highway Research Record. No. 369, 1971, pp. 80-90.

(9) Halder, A., "An Alternative Approach to Trip Distribution," Transportation Research. Vol. 4, 1970, pp. 63-69.

(10) Hyman, G.M., "Trip Distribution and Modal Split by Categories of Households," Transportation Research. Vol. 4, 1970, pp. 71-76.

(11) Ickler, S.R. and G.M. Flachs, "A Probabilistic Distribution Model with Calibration Technique," Transportation Research. Vol. 6, 1972, pp. 113-117.

(12) Illinois Department of Public Works and Building, Southward Transit Area Coordination Study. 1973.

(13) Johnston, J. Econometric Methods 2nd ed., McGraw-Hill, 1972.

(14) Kendall, M.G., and A. Stuart, The Advanced Theory of Statistics. Vol. 2, Hafner, 1961.

(15) Koppelman, F.S., "Guidelines for Aggregate Travel Prediction using Disaggregate Choice Models," Paper presented at 55th Annual Meeting of Transportation Research Board, 1976.

(16) ______________, "Methodology for Analysis of Errors in Prediction with Disaggregate Choice Models," Paper presented at 55th Annual Meeting of Transportation Research Board, 1976.

(17) ______________, "Prediction with Disaggregate Models: Aggregation Issue," *Transportation Research Record*. 1975.

(18) Lisco, T.E., *The Value of Commuter's Travel Time: A Study in Urban Transportation*. Ph.D. Dissertation (Univ. of Chicago), 1967.

(19) Luce, R.D. and H. Raiffa, *Games and Decisions*. Wiley, 1957.

(20) Manski, C.F., "Maximum Score Estimation of the Stochastic Utility Model of Choice," *Journal of Econometrics* Vol. 3, 1975, pp. 205-228.

(21) ___________, "The Conditional/Polytomous Logit Program: Instruction for Use," (Mimeographed).

(22) McFadden, D., "Conditional Logit Analysis of Qualitative Choice Behavior," in Zarembka, P. (ed.), *Frontiers in Econometrics*. Academic Press, 1974.

(23) ___________, *BART-I. Travel Behavior Studies, Part III. Travel Demand Forecasting Study*, (PB-236-739) National Technical Information Service, U.S. Department of Commerce, 1973.

(24) McGillivray, R.G., "Binary Choice of Urban Transport Mode in the San Francisco Bay Region," *Econometrica*. Vol. 40, 1972, pp. 827-848.

(25) ______________, "Demand and Choice Models of Modal Split," *Journal of Transportation Economics and Policy*. Vol. 4, No. 2, May 1970, pp. 192-207.

(26) Michaels, R.M. "Behavioral Measurement: An Approach to Predicting Transport Demand," in *Transportation Research Board Special Report* No. 149, 1974, pp. 51-57.

(27) Moses, L.M. and H.F. Williamson, Jr., "Value of Time, Choice of Mode, and the Subsidy Issue in Urban Transportation," *Journal of Political Economy*. Vol. 71, 1963, pp. 247-264.

(28) Nerlove, M. and S.J. Press, *Univariate and Multivariate Log-Linear and Logistic Models*. (R-1306-EDA/NIH) Rand Corp., 1973.

(29) Neter, J. and E.S. Mayes, "On the Appropriateness of the Correlation Coefficient with a 0,1 Dependent Variable," *Journal of American Statistical Assn.* Vol. 65, 1970, pp. 501-509.

(30) Oi, W.Y. and P.W. Shuldiner, *An Analysis of Urban Travel Demand*. Northwestern Univ. Press, 1962.

(31) O'Sullivan, P. and B. Ralston, "Forecasting Intercity Commodity Transport in the U.S.A.," Regional Studies. Vol. 8, 1974, pp. 191-195.

(32) Peat, Marwick, Mitchell and Co., Software Systems Development Program: Introduction to Urban Travel Demand Forecasting. Vol. I, Demand Modeling. (PB-236-848) National Technical Information Service, U.S. Department of Commerce, 1974.

(33) Quandt, R.E., "Estimation of Modal Splits," Transportation Economics. Vol. 2, 1968, pp. 41-51.

(34) Quandt, R.E. and W.J. Baumol, "The Demand for Abstract Transport Modes: Theory and Measurement," Journal of Regional Science Vol. 1, No. 2, 1966, pp. 13-26.

(35) Quarmby, D.A., "Choice of Travel Mode for the Journey to Work: Some Findings," Journal of Transport Economics and Policies Vol. 1, No. 3, 1967, pp. 273-314.

(36) Reichman, S. and P.R. Stopher, "Disaggregate Stochastic Models of Travel-Mode Choice," Highway Research Record. No. 369, 1971, pp. 91-103.

(37) Schmidt, P. and R.P. Strauss, "Estimation of Models with Jointly Dependent Qualitative Variables: A Simultaneous Logit Approach," Econometrica. Vol. 43, No. 4, July 1975, pp. 745-755.

(38) Stopher, P.R., "A Probability Model of Travel Mode Choice for the Work Journey," Highway Research Record No. 283, 1969, pp. 57-65.

(39) Stopher, P.R. and J.O. Lavender, "Disaggregate, Behavioral Travel Demand Models: Empirical Tests of Three Hypotheses," Transportation Research Forum Proceedings. Vol. 13, No. 1, 1972, pp. 321-336.

(40) Stopher, P.R. and T.E. Lisco, "Modelling Travel Demand: A Disaggregate Behavioral Approach: Issues and Applications," Papers. Transportation Research Forum, 1970, pp. 195-214.

(41) Stopher, P.R. and A.H. Meyburg, Urban Transportation Modeling and Planning, Lexington Books, 1975.

(42) Strotz, R.H., "The Empirical Implication of a Utility Tree," Econometrica. Vol. 25, No. 2, Apr. 1957, pp. 269-280.

(43) __________, "The Utility Tree - A Correction and Further Appraisal," Econometrica Vol. 27, No. 3, July 1959, pp. 482-489.

(44) Theil, H., "A Multinomial Extension of the Linear Logit Model," International Economic Review Vol. 10, No. 3, Oct. 1969, pp. 251-259.

(45) U.S. Department of Commerce, 1970 Census of Population: Journey to Work Report. 1973.

(46) Warner, S.L., Stochastic Choice of Mode in Urban Travel: A Study in Binary Choice. Northwestern University Press. 1962.

(47) Watson, P.L., "Problems Associated with Time and Cost Data Used in Travel Choice Modeling and Valuation of Time," Highway Research Record. No. 369, 1971, pp. 148-158.

(48) ___________, The Value of Time, Behavioral Models of Modal Choice. Lexington Books, 1974.

(49) Watson, P.L. and R.B. Westin, "Transferability of Disaggregate Mode Choice Models," Regional Science and Urban Economics. Vol. 5, 1975, pp. 1-23.

(50) ___________________________, "Aggregate Predictions from Disaggregate Models of Mode Choice: Some Tests of Transferability," Paper presented at the Intersociety Conference on Transportation, 1973.

(51) Westin, R.B., "Predictions from Binary Choice Models," Journal of Econometrics. Vol. 2, No. 1, 1974, pp. 1-16.

(52) Wilson, A.G., "Advances and Problems in Distribution Modeling," Transportation Research. Vol. 4, 1970, pp. 1-18.

(53) ___________, "Further Developments of Entropy Maximizing Transport Models," Transportation Policy and Technology. Vol. 1 (1973), pp. 183-193.

(54) ___________, "The Use of Entropy Maximizing Models," Journal of Transportation Economics and Policy. Vol. 3, No. 1, Jan. 1969, pp. 108-126.

(55) Zarembka, P., (ed.), Frontiers in Econometrics. Academic Press, 1974.

APPENDIX A

ESTLOG:

Computer Program for Individual Demand Submodel

```
               PROGRAM ESTLOG (INPUT,OUTPUT,PUNCH,TAPE85=INPUT,TAPE86=OUTPUT)         LOG0
003            DIMENSION ARRAY(300,45)                                                LOG0
003            INTEGER ARRAY                                                          LOG0
003            DIMENSION B0(10),B1(10),B2(10),BZ(10),FG0(10),FG1(10),FG2(10),         LOG0
              1FG3(10),FGZ(10),FH0(10,10),FH1(10,10),FH2(10,10),FHV(10,10),           LOG0
              2DIRS(10),S4S(10),LX(10),MX(10),SE(10),TV(10),NAX(10),NBY(10)           LOG0
003            COMMON /AA1/ X(300,4, 4),Y(300,10),NOVA(300,4),P(300,4),               LOG0
              1ANAME(300,10),JSUM(300),W4(4),JFAC(100)                                LOG0
003            COMMON /AA2/ AR(100),BXR(10),CYR(4,10),VECR(10),HEXR(10,10),           LOG0
              1S1R(4),T1R(4),W1R(4),XMN(4, 4),YMN(10),XMB(4, 4),CMN(10),
              2XE(4,4, 4),YE(4,10)
003            COMMON /AA3/ NBZ(300,30),NXCD(8),NYCD(8),NRCD(8,6)                     LOG0
003            COMMON /AA4/ NAME(45),LOCECS(45),NYHOW(10),NYSO(10,10),NALTD(4),       LOG0
              1NALTX(4, 4),NIZ(35),SCLZ(35),NARG(35),NAMB(10),ITRF(10,14),            LOG0
              2ITRC(10,14),LZN(30),S5(10),NN(30),NCH1(3),NCH2(3),MFT(16)              LOG0
003            EQUIVALENCE (ARRAY(1,1),X(1,1,1))                                      LOG0
003          4 FORMAT (*1*)                                                           LOG0
003            READ (85,1) IOP,ICD,NPP,NEL,NPCH                                       LOG0
021            IF (NPCH.NE.1) GO TO 117                                               LOG0
023            WRITE (86,25)                                                          LOG0
027         25 FORMAT (*1*,4X,80HTHIS IS A PUNCHING OPERATION, PREPARING THE DATA     LOG0
              1 FOR TYPE 2 OR TYPE 3 OPERATION.)                                      LOG0
027            GO TO 116                                                              LOG0
030        117 WRITE (86,20) IOP                                                      LOA0
036         20 FORMAT (*1*,4X,13HTHIS IS TYPE ,I1,1X,10HOPERATION.)                   LOA0
036            GO TO (113,114,115), IOP                                               LOA0
045        113 WRITE (86,21)                                                          LOA0
051         21 FORMAT (//4X,86H(1)  IT ESTIMATES THE COEFFICIENTES OF A LOGIT MOD     LOA0
              1EL USING A MAXIMUM LIKELIHOOD METHOD,/9X,93HBY APPLYING THE NEWTON     LOA0
              2-HIGA ALGORITHM OF UNCONSTRAINED OPTIMIZATION IN NONLINEAR PROGRAM     LOA0
              3MING.//9X,72HTHE ESTIMATION MAY BE PERFORMED WITH OR WITHOUT SOCIO     LOA0
              4ECONOMIC VARIABLES,/9X,36HWITH OR WITHOUT DUMMY VARIABLES, AND/9X,     LOA0
              554HWITH OR WITHOUT DUMMY COEFFICIENTS FOR MODE VARIABLES.)             LOA0
051            WRITE (86,22)                                                          LOA0
055         22 FORMAT (//4X,86H(2)  IT COMPUTES THE MODE CHOICE PROBABILITIES, AN     LOA0
              1D THE DIRECT- AND CROSS-ELASTICITIES/9X,40HOF MODE CHOICE AT MEAN      LOA0
              2OF THE VARIABLES.)                                                     LOA0
055            GO TO 116                                                              LOA0
056        114 WRITE (86,21)                                                          LOA0
062            WRITE (86,23)                                                          LOA0
066         23 FORMAT (//4X,74H(2)  IT COMPUTES THE PROBABILITIES, AND THE DIRECT     LOA0
              1- AND CROSS-ELASTICITIES/9X,55HOF MODE CHOICE FOR EXTERNALLY SUPPL     LOA0
              2IED VARIABLE VALUES.)                                                  LOA0
066            GO TO 116                                                              LOA0
067        115 WRITE (86,24)                                                          LOA0
073         24 FORMAT (//4X,74H(1)  IT COMPUTES THE PROBABILITIES, AND THE DIRECT     LOA0
              1- AND CROSS-ELASTICITIES/9X,74HOF MODE CHOICE, USING EXTERNALLY SU     LOA0
              2PPLIED COEFFICIENT AND VARIABLE VALUES.)                               LOA0
073        116 IF (IOP.EQ.3) GO TO 205                                                LOA0
075            IF (ICD.EQ.2) GO TO 206                                                LOG0
077            READ (85,1) NOB,NVAR,NOFT,ND1                                          LOG0
113            GO TO 207                                                              LOG0
114        206 READ (85,1) NOB,NVAR,NOFT,(NCH1(I),I=1,3),(NCH2(I),I=1,3),ND1          LOG0
144          1 FORMAT (8I10)                                                          LOG0
144        207 READ (85,2) (MFT(I),I=1,NOFT)                                          LOG0
157          2 FORMAT (8A10)                                                          LOG0
157            READ (85,2) (NAME(I),I=1,NVAR)                                         LOG0
```

```
72            NEND=0                                                         LOG
73            DO 101 I=1,NOB                                                 LOG
75            READ (85,MFT) (ARRAY(I,J),J=1,NVAR)                            LOG
12            IF (EOF,85) 102,103                                            LOG
15        102 WRITE (86,3) I                                                 LOG
23          3 FORMAT (4X,27HNO DATA FOR OBSERVATION NO., I5)                 LOG
23            GO TO 1000                                                     LOG
24        103 IF (ICD.EQ.1) GO TO 101                                        LOG
26            K1=NCH1(1)                                                     LOG
30            L1=NCH2(1)
31            K2=NCH1(2)
33            L2=NCH2(2)
34            K3=NCH1(3)
36            L3=NCH2(3)
37            IF (ARRAY(I,K1) .NE.ARRAY(I,L1).OR.ARRAY(I,K2).NE.ARRAY(I,L2)
             1.OR.ARRAY(I,K3).GE.ARRAY(I,L3)) GO TO 104
67            GO TO 101                                                      LOGG
67        104 WRITE (86,5) I                                                 LOGG
75          5 FORMAT (4X,24HDATA FOR OBSERVATION NO., I5,2X,12HDO NOT MATCH) LOGG
75            NEND=1                                                         LOGG
76        101 CONTINUE                                                       LOGG
01            IF (NEND.EQ.1) GO TO 1000
03        105 K=1                                                            LOGG
      C       *PUT DO 117 I=17,29* HERE
04            DO 106 I=1,NVAR                                                LOGG
06            LOCECS(I)=K                                                    LOC
10        106 K=K+NOB                                                        LO
13            DO 107 I=1,NVAR                                                LO
15            CALL WRITEC (ARRAY(1,I),LOCECS(I),NOB)                         LC
21        107 CONTINUE                                                       LO
24            WRITE (86,10)                                                  L
27            WRITE (86,10)                                                  L
33            WRITE (86,6)                                                   L
37          6 FORMAT (4X,44HTHE BASIC DATA HAVE BEEN SUCCESSFULLY LOADED)    L
37            WRITE (86,18) IOP,ICD,NPP,NEL,NPCH                             L
55         18 FORMAT (//4X,4HIOP=,I5,2X,4HICD=,I5,2X,4HNPP=,I5,2X,4HNEL=,I5,2X, L
             15HNPCH=,I5)
55            IF (ICD.NE.1) GO TO 201                                        LO
57            WRITE (86,19) NOB,NVAR,NOFT,ND1                                LO
73         19 FORMAT (//4X,4HNOB=,I5,2X,5HNVAR=,I5,2X,5HNOFT=,I5,2X,4HND1=,I5) LO
73            GO TO 202                                                      LO
74        201 WRITE (86,16) NOB,NVAR,NOFT,(NCH1(I),I=1,3),(NCH2(I),I=1,3),ND1 LOC
24         16 FORMAT(//4X,4HNOB=,I5,2X,5HNVAR=,I5,2X,5HNOFT=,I5,2X,5HNCH1=,3I5,
             12X,5HNCH2=,3I5,2X,4HND1=,I5)
24        202 WRITE (86,17) (MFT(I),I=1,NOFT)                                LOG
37         17 FORMAT (//4X,16HVARIABLE FORMAT=//10X,10A10/10X,10A10)
37            WRITE (86,7) (I,I=1,10)                                        LOG
50          7 FORMAT ( // 4X,14HVARIABLE NAMES//4X,10(6X,1H(,I2,1H)))        LOGG
50            WRITE (86,8) (NAME(I),I=1,NVAR)                                LOGG
63          8 FORMAT (/10(4X,10A10//))                                       LOGG
63            IF (ND1.EQ.0) GO TO 1001
64            NDL=23                                                         LOGG
65            IF (NVAR.LE.23) NDL=NVAR                                       LOGG
71            WRITE (86,9) (I,I=1,NDL)                                       LOGG
03          9 FORMAT ('1',4X,10HBASIC DATA//11X,9(2X,1H(,I1,1H)),14(1X,1H(,I2, LOGG
             11H)))                                                          LOGG
03            N=0                                                            LOGG
04            DO 109 I=1,NOB                                                 LOGG
```

```
506         N=N+1                                                                      LOG04
510         IF (N.LE.5) GO TO 110                                                      LOG04
512         WRITE (86,10)                                                              LOG05
516      10 FORMAT (* *)                                                               LOG05
516         N=1                                                                        LOG05
517     110 WRITE (86,11) I,(ARRAY(I,J),J=1,NDL)                                       LOG05
536      11 FORMAT (4X,1H(,I3,1H),2X,23I5)                                             LOG05
536     109 CONTINUE                                                                   LOG05
541         IF (NVAR.LE.23) GO TO 1001                                                 LOG05
543         WRITE (86,12) (I,I=24,NVAR)                                                LOG05
555      12 FORMAT (*1*,4X,10HBASIC DATA//11X,23(1X,1H(,I2,1H)))                       LOG05
555         N=0                                                                        LOG05
556         DO 111 I=1,NOB                                                             LOG05
560         N=N+1                                                                      LOG06
562         IF (N.LE.5) GO TO 112                                                      LOG06
564         WRITE (86,10)                                                              LOG06
570         N=1                                                                        LOG06
571     112 WRITE (86,11) I,(ARRAY(I,J),J=24,NVAR)                                     LOG06
610     111 CONTINUE                                                                   LOG06
613    1001 MSTP=0                                                                     LOG06
614         READ (85,1) NALT,NX,NY,NZ,NOP,NDHOW,NCSTH,NBUS2,ND2,NCO                    LOG06
644         IF (EOF,85) 410,411                                                        LOG06
647     410 MSTP=1                                                                     LOG06
650         GO TO 1000                                                                 LOG07
651     411 NOBX=NOB                                                                   LOG07
653         ND=NX+(NALT-1)*NY
657         NAL=NALT-1                                                                 LOG07
660         N2=ND*ND                                                                   LOG07
 62     204 CALL PROCESS (NOB,NVAR,NALT,NX,NY,NZ,NOP,NDHOW,NCSTH,NBUS2,NT,             LOG08
           1MSTP,NCV,NAX,NBY,ND2,NCO,MX,NPCH)                                          LOG08
 05         IF (MSTP.EQ.1) GO TO 1000                                                  LOG09
 07         CALL ESTIM (NALT,NAL,NX,NY,ND,NT,MSTP,NOD,NDHOW,INIT,N2,B0,B1,B2,          LOG09
           1BZ,FG0,FG1,FG2,FG3,FGZ,FH0,FH1,FH2,FHV,DIRS,S4S,LX,MX,FL3)                 LOG09
 44         IF (MSTP.EQ.1) GO TO 1000                                                  LOG09
 46     205 CALL RESULT (NALT,NAL,NX,NY,ND,NT,MSTP,NOD,NDHOW,INIT,N2,B0,B1,            LOG09
           1FG0,FG1,FH1,FH2,FHV,LX,MX,SE,TV,NAX,NBY,FL0,IOP,NPF,NEL)                   LOG09
 02         GO TO 1001                                                                 LOG09
 03    1000 STOP                                                                       LOG10
 05         END                                                                        LOG10
```

```
             SUBROUTINE PROCESS (NOB,NVAR,NALT,NX,NY,NZ,NOP,NDHOW,NCSTH,NBUS2,
            1NT,MSTF,NOV,NAX,NBY,ND2,NCD,MX,NPCH)
26           DIMENSION NAX(10),NBY(10),MX(10)
026          COMMON /AA1/ X(300,4, 4),Y(300,10),NOVA(300,4),P(300,4),
            1ANAME(300,10),JSUM(300),W4(4),JFAC(100)
026          COMMON /AA2/ AR(100),BXR(10),CYR(4,10),VECR(10),HEXR(10,10),
            1S1R(4),T1R(4),W1R(4),XMN(4, 4),YMN(10),XMB(4, 4),CMN(10)
026          COMMON /AA3/ NBZ(300,30),NXCD(8),NYCD(8),NRCD(8,6)
026          COMMON /AA4/ NAME(45),LOCECS(45),NYHOW(10),NYSO(10,10),NALTD(4),
            1NALTX(4, 4),NIZ(35),SCLZ(35),NARG(35),NAMB(10),ITRF(10,14),
            2ITPC(10,14),LZN(30),S5(10),NN(30),NCH1(3),NCH2(3),MFT(16)
026          IF (NX.EQ.0.OR.NY.EQ.0) GO TO 701
033          READ (85,2) NOV,(NAX(I),I=1,NX), (NBY(I),I=1,NY)
070          GO TO 702
074      701 IF (NX.EQ.0) GO TO 703
075          READ (85,2) NOV,(NAX(I),I=1,NX)
117          GO TO 702
123      703 READ (85,2) NOV,(NBY(I),I=1,NY)
146      702 IF (NY.EQ. 0) GO TO 704
153          READ (85,1) (NYHOW(I),I=1,NY)
2            DO 301 I=1,NY
7            IF (NYHOW(I).EQ.0) GO TO 301
1            READ (85,1) (NYSO(I,J),J=1,8)
3        301 CONTINUE
1        704 IF (NX.EQ. 0) GO TO 705
2            DO 302 I=1,NALT
4        302 READ (85,2) NALTD(I),(NALTX(I,J),J=1,NX)
57           GO TO 706
         705 READ (85,2) (NALTD(J),J=1,NALT)
         706 READ (85,2) (NIZ(I),I=1,NZ)
2            READ (85,3) (SCLZ(I),I=1,NZ)
5            READ (85,1) (NARG(I),I=1,NZ)
0            IF (NOP.EQ.0) GO TO 303
5            DO 304 I=1,NOP
77           READ (85,4) NAMB(I),(ITRF(I,J),J=1,14)
413      304 READ (85,5) (ITRC(I,J),J=1,14)
435      303 IF (NCD.EQ.0) GO TO 206
437          DO 362 I=1,NCD
440      362 READ (85,35) NXCD(I),NYCD(I),(NRCD(I,J),J=1,6)
465       35 FORMAT (2A10,6I10)
465      206 LZ=NX+NY+11
467          DO 300 I=1,LZ
471      300 LZN(I)=1
475        1 FORMAT (8I10)
475        2 FORMAT (8A10)
475        3 FORMAT (8F10.0)
475        4 FORMAT (A10,14I5)
475        5 FORMAT (10X,14F5.0)
475          NX1=NX
476          WRITE(86,17) NALT,NX,NY,NZ,NOP,NDHOW,NCSTH,NBUS2,ND2,NCD
525       17 FORMAT (*1*,4X,13HCONTROL CODES,/4X,6X,4HNALT,8X,2HNX,8X,2HNY,8X,
            12HNZ,7X,3HNOP,5X,5HNDHOW,5X,5HNCSTH,5X,5HNBUS2,7X,3HND2,7X,3HNCD//
            24X,11I10)
525          IF (NX.EQ.0 .OR. NY.EQ.0) GO TO 605
536          WRITE (86,18) NOV,(NAX(I),I=1,NX),(NBY(I),I=1,NY)
573       18 FORMAT (//4X,12A10)
573          GO TO 607
```

```
67    605 IF (NX.EQ.0) GO TO 605                                                PRO
1600      WRITE (86,18) NOV,(NAX(I),I=1,NX)                                     PRO
1622      GO TO 607                                                             PRO
1626  606 WRITE (86,18) NOV,(NBY(I),I=1,NY)                                     PRO
1651  607 IF (NY.EQ.0) GO TO 608                                                PRO
1656      WRITE (86,19) (NYHOW(I),I=1,NY)                                       PRO
1675   19 FORMAT (//4X,5HNYHOW,//4X,12I10)                                      PRO
1675      DO 609 I=1,NY                                                         PRO
1702      IF (NYHOW(I).EQ.0) GO TO 609                                          PRO
1704      WRITE (86,20) I,(NYSO(I,J),J=1,10)                                    PRO
1720  609 CONTINUE                                                              PRO
1726   20 FORMAT (//4X,5HNYSO(,I2,1H),/4X,12I10)                                PRO2
726   608 IF (NX.EQ.0) GO TO 610                                                PRO2
727       WRITE (86,21)                                                         PRO2
733    21 FORMAT (//4X,5X,5HNALTO,5X,5HNALTX)                                   PRO2
733       DO 611 I=1,NALT                                                       PRO2
740   611 WRITE (86,18) NALTO(I),(NALTX(I,J),J=1,NX1)                           PRO2
763   610 WRITE (86,22)                                                         PRO2
767    22 FORMAT (//4X,3HNIZ)                                                   PRO2
767       WRITE (86,18) (NIZ(I),I=1,NZ)                                         PRO2
012       WRITE (86,23)                                                         PRO2
016    23 FORMAT (//4X,4HSCLZ)                                                  PRO2
016       WRITE (86,24) (SCLZ(I),I=1,NZ)                                        PRO2
041    24 FORMAT (//4X,12F10.3)                                                 PRO2
041       WRITE (86,25)                                                         PRO2
045    25 FORMAT (//4X,4HNARG)                                                  PRO2
045       WRITE (86,26) (NARG(I),I=1,NZ)                                        PRO2
070    26 FORMAT (//4X,12I10)                                                   PRO2
070       IF (NOP.EQ.0) GO TO 612                                               PRO2
075       WRITE (86,27)                                                         PRO2
101    27 FORMAT (//4X,6X,4HNAMB,6X,4HITRF/20X,4HITRC)                          PRO2
101       DO 613 I=1,NOP                                                        PRO2
106       WRITE (86,28) NAMB(I),(ITRF(I,J),J=1,14)                              PRO2
122   613 WRITE (86,29) (ITRC(I,J),J=1,14)                                      PRO2
144    28 FORMAT (//4X,A10,14I5)                                                PRO2
144    29 FORMAT (//4X,10X,14F5.1)                                              PRO2
144   612 IF (NCO.EQ.0) GO TO 378                                               PRO2
146       WRITE (86,36) (I,I=1,6)                                               PRO2
156    36 FORMAT (//4X,6X,4HNXCO,6X,4HNYCO,6(3X,5HNRCO(,I1,1H))/4X)             PRO2
156       DO 379 I=1,NCO                                                        PRO2
163   379 WRITE (86,37) NXCO(I),NYCO(I),(NRCO(I,J),J=1,6)                       PRO2
210    37 FORMAT (4X,2A10,6I10)                                                 PRO2
210   378 DO 305 I=1,NZ                                                         PRO0
212       DO 306 J=1,NVAR                                                       PRO0
213       IF (NIZ(I).NE.NAME(J)) GO TO 306                                      PRO0
216       CALL READEC (NBZ(1,I),LOCECS(J),NOB)                                  PRO0
226       GO TO 305                                                             PRO0
232   306 CONTINUE                                                              PRO0
235   305 CONTINUE                                                              PRO0
240       DATA NMA /10H     DSTRP/                                              PRO0
240       DATA NMB /10H      NBAV/                                              PRO0
240       DATA NMC /10H     MODCH/                                              PRO0
240       DATA NMD /10H      ZERO/                                              PRO0
240       DATA NME /10H      NAAV/
240       DATA NMF /10H       ONE/                                              PRO0
240       N=0                                                                   PRO0
241       DO 307 I=1,NOB                                                        PRO0
242       DO 308 J=1,NZ                                                         PRO0
```

```
243               IF (NIZ(J).EQ. NMA) GO TO 309
246               IF (NBZ(I,J).EQ.-9.OR.NBZ(I,J).EQ.-8) GO TO 307
257           309 IF (NBUS2.EQ.0 .AND. NIZ(J).EQ.NMB .AND. NBZ(I,J).EQ.2) GO TO 307
275               IF (NARG(J).EQ.1 .AND. NBZ(I,J).EQ.0) GO TO 307
305               IF (NARG(J).EQ.2 .AND. NBZ(I,J).LE.0) GO TO 307
320               IF (NDHOW.EQ.2 .AND. NIZ(J).EQ.NMC .AND.NBZ(I,J).EQ.12) GO TO 307
336               IF (NCSTH.EQ.1 .AND. NIZ(J).EQ.NME .AND. NBZ(I,J).NE.10) GO TO 307
354               IF (NCSTH.EQ.1 .AND. NIZ(J).EQ.NM6 .AND. NBZ(I,J).NE.10) GO TO 307
372           308 CONTINUE
374               N=N+1
375               DO 1111 J=1,NZ
377          1111 NBZ(N,J)=NBZ(I,J)
+12           307 CONTINUE
+15               NT=N
+16               IF (NY.EQ.0) GO TO 720
+17               DO 401 J=1,NY
+21               IF (NYHOW(J).EQ.0 .OR. NYHOW(J).EQ.3) GO TO 401
+30               DO 402 K=1,NZ
+31               IF (NBY(J).EQ.NIZ(K)) GO TO 403
+35           402 CONTINUE
+37               GO TO 401
+37           403 KX=NYHOW(J)
+41               GO TO (411,412,401,411), KX
+51           411 DO 404 I=1,NT
+53               K1=NBZ(I,K)
+57               NBZ(I,K)=NYSO(J,K1)
64            404 CONTINUE
67                GO TO 401
67            412 DO 405 I=1,NT
71                IF (NBZ(I,K).GE.NYSO(J,1).AND.NBZ(I,K).LT.NYSO(J,2)) GO TO 406
5                 NBZ(I,K)=0
0                 GO TO 405
1             406 NBZ(I,K)=1
5             405 CONTINUE
              401 CONTINUE
              720 IF (NOP.EQ.0) GO TO 310
                  DO 311 I=1,NOP
                  KX=ITRF(I,1)
0                 IF (KX.GE.4) GO TO 315
32                GO TO (312,313,314), KX
41            312 DO 316 J=1,NT
43                K1=ITRF(I,3)
45                S1=NBZ(J,K1)*SCLZ(K1)
52                KX=ITRF(I,2)
54                GO TO (201,202,203,204,205), KX
65            201 ANAME(J,I)=S1**2
71                GO TO 316
72            202 ANAME(J,I)=SQRT(S1)
00                GO TO 316
04            203 ANAME(J,I)=1.0/S1
11                GO TO 316
11            204 IF (S1.EQ.0.0) S1=1.0E-10
13                ANAME(J,I)=ALOG(S1)
21                GO TO 316
25            205 ANAME(J,I)=EXP(S1)
33            316 CONTINUE
42                GO TO 311
42            313 DO 317 J=1,NT
```

```
4           K1=ITRF(I,2)
6           S1=NBZ(J,K1)*SCLZ(K1)*ITRC(I,2)
5           IF (ITRF(I,3).EQ.6) GO TO 217
0           K2=ITRF(I,4)
1           S2=NBZ(J,K2)*SCLZ(K2)*ITRC(I,4)
0       217 KX=ITRF(I,3)
2           GO TO (211,212,213,214,215,216),KX
4       211 ANAME(J,I)=S1+S2
1           GO TO 655
2       212 ANAME(J,I)=S1-S2
7           GO TO 655
0       213 ANAME(J,I)=S1*S2
5           GO TO 655
5       216 K1=ITRF(I,4)
7           K2=NBZ(J,K1)
3           K3=K2+5
5           S2=ITRF(I,K3)*SCLZ(K1)*ITRC(I,4)
4       214 ANAME(J,I)=S1/S2
1           GO TO 655
1       215 ANAME(J,I)=S1**S2
0       655 IF (ITRF(I,5).NE.1) GO TO 317
3           IF (ANAME(J,I).EQ.0.0) ANAME(J,I)=1.0E-10
2           ANAME(J,I)=ALOG(ANAME(J,I))
1       317 CONTINUE
0           GO TO 311
0       314 DO 318 J=1,NT
2           K1=ITRF(I,2)
4           K2=ITRF(I,4)
6           K3=ITRF(I,6)
7           S1=NBZ(J,K1)*SCLZ(K1)*ITRC(I,2)
6           S2=NBZ(J,K2)*SCLZ(K2)*ITRC(I,4)
6           S3=NBZ(J,K3)*SCLZ(K3)*ITRC(I,6)
5           KX=ITRF(I,3)
7           GO TO (221,222,223,224,225), KX
7       221 S4=S1+S2
1           GO TO 319
2       222 S4=S1-S2
4           GO TO 319
5       223 S4=S1*S2
7           GO TO 319
0       224 S4=S1/S2
2           GO TO 319
3       225 S4=S1**S2
7       319 KX=ITRF(I,5)
1           GO TO (231,232,233,234,235), KX
2       231 ANAME(J,I)=S4+S3
7           GO TO 318
0       232 ANAME(J,I)=S4-S3
5           GO TO 318
6       233 ANAME(J,I)=S4*S3
3           GO TO 318
3       234 ANAME(J,I)=S4/S3
0           GO TO 318
0       235 ANAME(J,I)=S4**S3
7       318 CONTINUE
2           GO TO 311
2       315 N=ITRF(I,1)
4           DO 320 J=1,NT
```

```
[illegible]       DO 321 K=1,N                                                   PR
157               K1=ITRF(I,K+2)                                                 PR
163         321 S5(K)=NBZ(J,K1)*SCLZ(K1)*ITRC(I,K+2)                              PR
200               KX=ITRF(I,2)                                                   -
202               GO TC (241,242), KX
207         241 ANAME(J,I)=0.0                                                    PRO
213               DO 322 K=1,N                                                    PRO
214         322 ANAME(J,I)=ANAME(J,I)+S5(K)                                       PRO1
224               GO TC 320                                                       PRO1
225         242 ANAME(J,I)=1.0                                                    PRO
231               DO 323 K=1,N                                                    PFO
233         323 ANAME(J,I)=ANAME(J,I)*S5(K)                                       PRO
243         320 CONTINUE                                                          PRO
246         311 CONTINUE                                                          PRO
251         310 IF (NX.EQ.0) GO TO 601                                            PR
252               DO 329 K=1,NALT                                                 PRO
254               DO 330 L=1,NX                                                   PRO
255               DO 331 I=1,NZ                                                   PRO
256               IF (NALTX(K,L).EQ.NIZ(I)) GO TO 332                             PRO
262         331 CONTINUE                                                          PR
264               DO 333 I=1,NOP                                                  PRO
265               IF (NALTX(K,L).EQ.NAMB(I)) GO TO 334                            PRO
271         333 CONTINUE                                                          PRO
274               IF (NALTX(K,L).EQ.NMF) GC TC 1113                               PRO
277               IF (NALTX(K,L).EQ.NMD) GC TC 332                                PRO
302               MSTP=1                                                          PRO
303               WRITE (86,16) NALTX(K,L)                                        PRO
313          16 FORMAT (//4X,8HVARIABLE,A10,14HDOES NOT EXIST)                    PRO
313               GO TC 359                                                       PRO
17         1113 DO 108 J=1,NT
 1          108 X(J,K,L)=1.0
 2                GC TC 330
332         332 DO 335 J=1,NT                                                     P
334               IF (NALTX(K,L) .NE. NMD) GO TO 1112                             P
540               X(J,K,L)=0.0                                                    P
44                GO TC 335                                                       P
345        1112 X(J,K,L)=N9Z(J,I)*SCLZ(I)                                         P
357         335 CONTINUE                                                          PF
362               GO TO 330                                                       PF
62          334 DO 336 J=1,NT                                                     P
364         336 X(J,K,L)=ANAME(J,I)                                               PF
01          330 CONTINUE                                                          PR
04          329 CONTINUE                                                          PR
06                IF (NCO.EQ.0) GO TO 709                                         PR
07                DO 363 I=1,NCO                                                  PR
11                DO 364 J=1,NZ                                                   PR
12                IF (NYCO(I).EQ.NIZ(J)) GO TO 365                                PRO
15          364 CONTINUE                                                          PRO
17                WRITE (86,16) NYCO(I)                                           PRO
24                MSTP=1                                                          PRO
26                GO TO 363                                                       PRO
32          365 DO 366 K=1,NT                                                     PRO
34                KX=NEZ(K,J)                                                     PRO1
40          366 NDVA(K,I)=NRCO(I,KX)                                              PRO1
47          363 CONTINUE                                                          PRO1
52                IF (MSTP.EQ.1) GO TO 359                                        PRO1
54                MN=0                                                            PRO1
55                DO 367 I=1,NCO                                                  PRO1
```

```
 56            NMAX=NOVA(1,I)                                       PRO19
461            DO 368 J=1,NT                                        PRO19
463            IF (NOVA(J,I).LE.NMAX) GO TO 368                     PRO19
470            NMAX=NOVA(J,I)                                       PRO19
473       368 CONTINUE                                              PRO19
476            MX(I)=NMAX                                           PRO19
501            MN=MN+NMAX-1                                         PRO19
503       367 CONTINUE                                              PRO19
505            DO 369 K=1,NX                                        PRO19
506            NF=NX-K+1                                            PRO19
510            DO 369 J=1,NALT                                      PRO19
512            DO 369 I=1,NT                                        PRO19
513       369 X(I,J,NF+MN)=X(I,J,NF)                                PRO19
536            N=0                                                  PRO19
537            DO 370 K=1,NX                                        PRO19
540            DO 371 M=1,NCO                                       PRO19
541            IF (NAX(K).EQ.NXCO(M)) GO TO 372                     PRO19
545       371 CONTINUE                                              PRO19
550            JSUM(K+N)=NAX(K)                                     PRO19
554            DO 373 J=1,NALT                                      PRO19
555            DO 373 I=1,NT                                        PRO19
556       373 X(I,J,K+N)=X(I,J,K+MN)                                PRO19
 00            GO TO 370                                            PRO19
 00       372 NL=MX(M)                                              PRO1
 03            LZN(K)=MX(M)                                         PRO1
 06            DO 374 L=1,NL                                        PRO1
 07       374 JSUM(K+N+L-1)=NAX(K)                                  PRO1
 16            DO 375 I=1,NT                                        PRO1
 20            LA=NOVA(I,M)                                         PRO1
 24            DO 375 J=1,NALT                                      PRO1
 25            S=X(I,J,K+MN)                                        PRO1
 34            DO 377 L=1,NL                                        PRO1
 35       377 X(I,J,K+N+L-1)=0.0                                    PRO1
 47            X(I,J,K+N)=S                                         PRO1
 56            IF (NPCH.NE.1) GO TO 102
 61            IF (LA.NE.MX(M)) X(I,J,K+N+LA)=1.0
 74            GO TO 375
 75       102 IF (LA.NE.MX(M)) X(I,J,K+N+LA)=S
 11       375 CONTINUE                                              PRO1
 17            N=N+MX(M)-1                                          PRO1
 23       370 CONTINUE                                              PRO1
 26            NX=NX+MN                                             PRO1
 27            DO 376 I=1,NX                                        PRO1
 31       376 NAX(I)=JSUM(I)                                        PRO1
 36       709 S=0.0                                                 PRO19
 37            DO 731 K=1,NX                                        RES22
 41            DO 731 J=1,NALT                                      RES2
 42            S=0.0                                                RES22
 43            N=0                                                  RES22
 44            DO 732 I=1,NT                                        RES2
 45            S=S+X(I,J,K)                                         RES2
 53       732 N=N+1                                                 RES2
 60       731 XMN(J,K)=S/N                                          RES2
 70       601 M2=NY                                                 PRO1
 71            IF (NY.EQ.0) GO TO 602                               PRO2
 72            M1=0                                                 PRO20
 73            M3=0                                                 PRO20
 74            DO 337 L=1,NY                                        PRO2
```

```
 75            DO 338 I=1,NZ
776            IF (NBY(L).EQ.NIZ(I)) GO TO 339
002        338 CONTINUE
004            DO 340 I=1,NOP
005            IF (NBY(L).EQ.NAMB(I)) GO TO 341
011        340 CONTINUE
014            IF (NBY(L).EQ.NMF) GO TO 339
016            MSTP=1
020            WRITE (86,16) NBY(L)
026            GO TO 359
032        339 IF (NYHOW(L).EQ.3.OR.NYHOW(L).EQ.4) GO TO 342
042            M2=L+M1
044            DO 343 J=1,NT
045            Y(J,M2)=0.0
050            IF (NBY(L).NE.NMF) GO TO 380
054            Y(J,M2)=1.0
060            GO TO 343
060        380 Y(J,M2)=NBZ(J,I)*SCLZ(I)
070        343 CONTINUE
073            GO TO 337
 73        342 M2=L+M1
               DO 344 J=1,NT
077            DO 345 K=1,5
100        345 Y(J,M2+K-1)=0.0
107            IF (NBZ(J,I).GT.5) GO TO 346
115            N=NBZ(J,I)
120            Y(J,M2+N-1)=1.0
124        346 IF (NBZ(J,I).GT.M3) M3=NBZ(J,I)
 35        344 CONTINUE
 40            M1=M1+M3-2
142            LZN(NX+1+L)=M3-1
145            IF (L.EQ.NY) M2=L+M1
  0            GO TO 337
  1        341 M2=L+M1
  3            DO 347 J=1,NT
  5            Y(J,M2)=0.0
           347 Y(J,M2)=ANAME(J,I)
           337 CONTINUE
           602 GO TO (251,251,252,252), NOHOW
  3        251 DO 324 J=1,NALT
 05            DO 324 I=1,NT
206        324 NOVA(I,J)=0
217            DO 325 K=1,NZ
220            IF (NIZ(K).EQ.NOV) GO TO 326
223        325 CONTINUE
225            MSTP=1
226            WRITE (86,16) NOV
234            GO TO 359
240        326 DO 327 I=1,NT
242            L=NBZ(I,K)/10
250        327 NOVA(I,L)=1
256            GO TO 328
256        252 DO 256 J=1,NALT
260            DO 256 I=1,NT
261        256 NOVA(I,J)=0
272            DO 257 J=1,NALT
273            DO 258 K=1,NZ
274            IF (NALTD(J).EQ.NIZ(K)) GO TO 259
```

```
27        258 CONTINUE                                                    PRO01
301           MSTP=1                                                      PRO01
302           WRITE (86,16) NALTO(J)                                      PRO01
310           GO TO 257                                                   PRO01
314       259 DO 260 I=1,NT                                               PRO01
316       260 NOVA(I,J)=NBZ(I,K)                                          PRO01
331       257 CONTINUE                                                    PRO01
334       328 IF (MSTP.EQ.1) GO TO 359                                    PRO01
337           GO TO (351,352,351,352),NOHOW                               PRO02
347       351 WRITE (86,6) NDV                                            PRO02
355         6 FORMAT (*1*,4X,14HPROCESSED DATA//4X,21HDEPENDENT VARIABLE IS,A10, PRO02
             15X,26H(INCLUDES AUTO PASSENGERS))                           PRO02
355           GO TO 353                                                   PRO02
361       352 WRITE (86,7) NDV                                            PRO02
367         7 FORMAT (*1*,4X,14HPROCESSED DATA//4X,21HDEPENDENT VARIABLE IS,A10, PRO02
             15X,26H(EXCLUDES AUTO PASSENGERS))                           PRO02
367       353 IF (NX.EQ.0) GO TO 603                                      PRO02
374           WRITE (86,8) (NAX(I),I=1,NX)                                PRO02
413         8 FORMAT (//4X,38HMODE RELATED INDEPENDENT VARIABLES ARE,//14X, PRO02
             1 9(A10,3X),//14X,9(A10,3X))                                 PRO02
413       603 IF (NY.EQ.0) GO TO 604                                      PRO02
420           WRITE (86,9) ((NBY(I),LZN(I+NX+1)),I=1,NY)                  PRO02
446         9 FORMAT (//4X,40HSOCIO-ECONOMIC INDEPENDENT VARIABLES ARE,//14X, PRO02
             1 9(A10,1H(,I1,1H)),//14X, 9(A10,1H(,I1,1H)))                PRO02
446           N=0
447           DO 650 I=1,NY
454       650 N=N+LZN(NX+I+1)
461           IF (N.EQ.NY) GO TO 651
462           DO 652 I=1,NY
463           J=NY-I+1
465           M1=LZN(NX+J+1)
470           DO 653 M=1,M1
471           NBY(N)=NBY(J)
475           N=N-1
477       653 CONTINUE
01        652 CONTINUE
04        651 S=0.0
05        604 NY=M2                                                       PRO2
06            NXY=NX+NY                                                   PRO2
10            IF (NY.EQ.0) GO TO 101                                      PRO2
11            DO 733 K=1,NY                                               RES2
12            S=0.0                                                       RES2
13            N=0                                                         RES2
14            DO 734 I=1,NT                                               RES2
15            N=N+1                                                       RES2
17        734 S=S+Y(I,K)                                                  RES2
25        733 YMN(K)=S/N                                                  RES2
32        101 DO 354 I=1,30                                               PRO2
34        354 NN(I)=I                                                     PRO2
37            IF (NPCH.NE.1) GO TO 103
41            DO 104 I=1,NT
42            IF (NX.EQ.0) GO TO 105
43            DO 106 J=1,NALT
44        106 PUNCH 38, I,J,(X(I,J,K),K=1,NX)
00        105 IF (NY.EQ.0) GO TO 104
01            N=NALT+1
03            PUNCH 38, I,N,(Y(I,K),K=1,NY)
30        104 CONTINUE
```

```
537      38 FORMAT (I3,I2,5X,7F10.3)
637         MSTP=1
640     103 N=0
641         IF (ND2.EQ.0) GO TO 359                                                  PR
643         IF (NX.EQ.0.OR.NY.EQ.0) GO TO 614                                        P
650         IF (NXY.LE. 7) GO TO 355                                                 PR
652         WRITE (86,10) (NN(I),I=1, 7)                                             PRO
664      10 FORMAT (///9X,8X,2HDV,7(10X,2HX(,I2,1H))//4X)                            PR
664         DO 356 I=1,NT                                                            PR
671         WRITE (86,11) I,NOVA(I,1),(X(I,1,K),K=1,NX)                              PR
721      11 FORMAT (4X,1H(,I3,1H),I10, 7F15.5)                                       PR
721         DO 357 J=2,NALT                                                          P
726     357 WRITE (86,12) NOVA(I,J),(X(I,J,K),K=1,NX)                                PR
770      12 FORMAT (9X,I10, 7F15.5)                                                  P
770     356 CONTINUE                                                                 P
773         WRITE (86,13) (NN(I),I=1,11)                                             PR
004      13 FORMAT (///9X,7(10X,2HY(,I2,1H))/4X)                                     PR
004         DO 358 I=1,NT                                                            PR
011     358 WRITE (86,14) I,(Y(I,K),K=1,NY)                                          PR
043      14 FORMAT (4X/4X,1H(,I3,1H), 7F15.5)                                        PR
043         GO TO 359                                                                PR
043     355 WRITE (86,15) (NN(I),I=1, 7)                                             PR
055      15 FORMAT (///9X,8X,2HDV,7(11X,1H(,I2,1H))//4X)                             PR
055         DO 360 I=1,NT                                                            PR
062         WRITE (86,11) I,NOVA(I,1),(X(I,1,K),K=1,NX),(Y(I,K),K=1,NY)              P
127         DO 361 J=2,NALT                                                          PR
134     361 WRITE (86,12) NOVA(I,J),(X(I,J,K),K=1,NX)                                PR
176     360 CONTINUE                                                                 PR
            GO TO 359                                                                PR
        614 IF (NX.EQ.0) GO TO 615                                                   PR
12          WRITE (86,10) (NN(I),I=1,NX)                                             PR
221         DO 616 I=1,NT                                                            PR
226         WRITE (86,11) I,NOVA(I,1),(X(I,1,K),K=1,NX)                              PR
256         DO 617 J=2,NALT                                                          PR
263     617 WRITE (86,12) NOVA(I,J),(X(I,J,K),K=1,NX)                                P
325     616 CONTINUE                                                                 P
330         GO TO 359                                                                P
330     615 WRITE (86,30) (NN(I),I=1,NY)                                             P
350      30 FORMAT (///9X,8X,2HDV,7(10X,2HY(,I2,1H))//4X)                            PR
350         DO 618 I=1,NT                                                            PR
355         WRITE (86,11) I,NOVA(I,1),(Y(I,K),K=1,NY)                                PR
406         DO 619 J=2,NALT                                                          PR
413     619 WRITE (86,12) NOVA(I,J)                                                  PR
430     618 CONTINUE                                                                 PR
433     359 RETURN                                                                   PRO
434         END                                                                      PR
```

```
            SUBROUTINE LOGIT (NALT,NAL,NX,NY,ND,NT,MSTP,NDHOW,INIT,NH,A,GRAD,   LGT
           1HESSN,ALGL,NG)                                                       LGT0
022         DIMENSION A(10),GRAD(10),HESSN(10,10)                                LGT0
022         COMMON /AA1/ X(300,4,4),Y(300,10),NOVA(300,4),P(300,4),              LGT0
           1ANAME(300,10),JSUM(300),S1(4),JFAC(100)                              LGT0
022         COMMON /AA2/ AR(100),BX(10),CY(4,10),VEC(10),HEX(10,10),S1R(4),      LGT0
           1T1(4),W1(4)                                                          LGT0
022         IFT=1                                                                LGT0
023         IF (INIT.NE.0) GO TO 402                                             LGT0
024         IFT=0                                                                LGT0
025         IF (NX.EQ.0) GO TO 403                                               LGT0
026         READ (85,1) (BX(I),I=1,NX)                                           LGT0
045     403 IF (NY.EQ.0) GO TO 500                                               LGT0
052         DO 401 J=1,NAL                                                       LGT0
053     401 READ (85,1) (CY(J,I),I=1,NY)                                         LGT0
100       1 FORMAT (8F10.5)                                                      LGT0
100     500 WRITE (86,2)                                                         LGT0
104       2 FORMAT (*1*,4X,17HLOG OF ITERATIONS)                                 LGT0
104         WRITE (86,3)                                                         LGT0
110       3 FORMAT (//4X,28HINITIAL PARAMETER VALUES ARE/4X)                     LGT0
110         IF (NX.EQ.0) GO TO 404                                               LGT0
115         WRITE (86,4) ((I,BX(I)),I=1,NX)                                      LGT0
136       4 FORMAT (/9X,7(5X,2HB(,I2,2H)=,F6.3)//9X,7(5X,2HB(,I2,2H)=,F6.3)/)    LGT0
136     404 IF (NY.EQ.0) GO TO 405                                               LGT0
143         DO 406 I=1,NAL                                                       LGT0
144     406 WRITE (86,5) ((I,J,CY(I,J)),J=1,NY)                                  LGT0
175       5 FORMAT (/9X,7(3X,2HC(,I1,1H,,I2,2H)=,F6.3)//9X,7(3X,2HC(,I1,1H,,     LGT0
           1I2,2H)=,F6.3)/4X)                                                    LGT0
175     405 N=0                                                                  LGT0
176         IF (NX.EQ.0) GO TO 703                                               LGT0
177         DO 701 I=1,NX                                                        LGT0
200         N=N+1                                                                LGT0
202     701 A(N)=BX(I)                                                           LGT0
206     703 IF (NY.EQ.0) GO TO 704                                               LGT0
207         DO 702 I=1,NAL                                                       LGT0
211         DO 702 J=1,NY                                                        LGT0
212         N=N+1                                                                LGT0
214     702 A(N)=CY(I,J)                                                         LGT0
224     704 S=0.0                                                                LGT0
225     407 INIT=1                                                               LGT0
227         GO TO 408                                                            LGT0
227     402 N=0                                                                  LGT0
230         IF (NX.EQ.0) GO TO 409                                               LGT0
231         DO 410 I=1,NX                                                        LGT0
232         N=N+1                                                                LGT0
234     410 BX(I)=A(N)                                                           LGT0
241     409 IF (NY.EQ.0) GO TO 408                                               LGT0
242         DO 411 J=1,NAL                                                       LGT0
244         DO 411 I=1,NY                                                        LGT0
245         N=N+1                                                                LGT0
247     411 CY(J,I)=A(N)                                                         LGT0
257     408 S=0.0                                                                LGT0
260     601 DO 503 I=1,NT
262         DO 412 J=1,NALT                                                      LGT0
263         S1(J)=0.0                                                            LGT0
264     412 T1(J)=0.0                                                            LGT0
267         DO 505 J=1,NAL                                                       LGT0
```

```
1270          IF (NX.EQ.0) GO TO 551
1271          DO 506 K=1,NX
1272      506 S1(J)=S1(J)+X(I,J,K)*BX(K)
1306      551 IF (NY.EQ.0) GO TO 505
1307          DO 507 K=1,NY
1311      507 T1(J)=T1(J)+Y(I,K)*CY(J,K)
1325      505 CONTINUE
1330      552 U=0.0
1331          DO 508 J=1,NAL
1333          W1(J)=S1(J)+T1(J)
1336          W1(J)=EXP(W1(J))
1342      508 U=U+W1(J)
1352          P(I,1)=1.0/(1.0+U)
1356          DO 509 J=2,NALT
1357      509 P(I,J)=W1(J-1)/(1.0+U)
1373      503 CONTINUE
1376          ALGL=0.0
1377          DO 510 I=1,NT
1401          N=0
1402          S=0.0
1403          DO 511 J=1,NALT
1404          N=N+NOVA(I,J)
1410          U=ALOG(P(I,J))
1417      511 S=S+NOVA(I,J)*U
1433          JSUM(I)=N
1435      510 ALGL=ALGL+S
1441          IF (NOHOW.LE.2) GO TO 512
443           DO 513 I=1,NT
445           IF (IFT.EQ.1) GO TO 513
447           N=JSUM(I)
451           S=0.0
452           DO 515 L=1,N
453           U=L
454       515 S=S+ALOG(U)
466           T=0.0
467           DO 516 J=1,NALT
470           T1(J)=0.0
471           N=NOVA(I,J)
475           DO 517 L=1,N
476           V=L
477       517 T1(J)=T1(J)+ALOG(V)
512       516 T=T+T1(J)
516           JFAC(I)=S-T
522       513 ALGL=ALGL+JFAC(I)
527       512 IF (NG.EQ.0) GO TO 1000
531           IF (NX.EQ.0) GO TO 553
532           DO 518 K=1,NX
533           GRAD(K)=0.0
535           DO 519 I=1,NT
537           S2=0.0
540           S3=0.0
541           DO 520 J=1,NAL
542           S2=S2+NOVA(I,J+1)*X(I,J,K)
554       520 S3=S3+P(I,J+1)*X(I,J,K)
565       519 GRAD(K)=GRAD(K)+(S2-JSUM(I)*S3)
576       518 CONTINUE
600       553 L=NX
601           IF (NY.EQ.0) GO TO 554
```

```
              DO 521 J=1,NAL                                                  LGT1
604           DO 522 K=1,NY                                                   LGT1
605           L=L+1                                                           LGT1
607           GRAD(L)=0.0                                                     LGT1
611           DO 523 I=1,NT                                                   LGT1
612       523 GRAD(L)=GRAD(L)+(NDVA(I,J+1)*Y(I,K)-JSUM(I)*P(I,J+1)*Y(I,K))    LGT1
644       522 CONTINUE                                                        LGT1
647       521 CONTINUE                                                        LGT1
651       554 ND=L                                                            LGT1
652       606 IF (NH.EQ.0) GO TO 1000                                         LGT1
654           DO 524 I=1,ND                                                   LGT1
655           DO 524 J=1,ND                                                   LGT1
656       524 HESSN(I,J)=0.0                                                  LGT1
666           DO 525 I=1,NT                                                   LGT1
667           IF (NX.EQ.0) GO TO 555                                          LGT1
670           DO 526 K=1,NX                                                   LGT1
671           VEC(K)=0.0                                                      LGT1
672           DO 526 L=1,NX                                                   LGT1
674       526 HEX(K,L)=0.0                                                    LGT1
704           DO 527 K=1,NX                                                   LGT1
705           S=0.0                                                           LGT1
706           DO 528 L=1,NAL                                                  LGT1
710       528 S=S+P(I,L+1)*X(I,L,K)                                           LGT1
725       527 VEC(K)=S                                                        LGT1
730           DO 529 M=1,NX                                                   LGT1
732           DO 529 N=1,NX                                                   LGT1
733           DO 530 L=1,NAL                                                  LGT1
734       530 HEX(M,N)=HEX(M,N)+P(I,L+1)*X(I,L,M)*X(I,L,N)                    LGT1
762       529 HESSN(M,N)=HESSN(M,N)-JSUM(I)*(HEX(M,N)-VEC(M)*VEC(N))          LGT1
005           IF (NY.EQ.0) GO TO 525                                          LGT1
006           K1=NX                                                           LGT1
007           L1=NX                                                           LGT1
010           DO 531 J=1,NAL                                                  LGT1
011           DO 532 L=1,NY                                                   LGT1
012           L1=L1+1                                                         LGT1
014           DO 533 K=1,NX                                                   LGT1
015       533 HESSN(K,L1)=HESSN(K,L1)-JSUM(I)*(P(I,J+1)*(X(I,J,K)-VEC(K))*Y(I,L) LGT1
             1)                                                               LGT1
054       532 CONTINUE                                                        LGT1
057       531 CONTINUE                                                        LGT1
061           DO 534 J=1,NAL                                                  LGT1
062           DO 534 K=1,NY                                                   LGT1
063           K1=K1+1                                                         LGT1
065           DO 535 L=1,NX                                                   LGT1
066       535 HESSN(K1,L)=HESSN(K1,L)-JSUM(I)*(P(I,J+1)*Y(I,K)*(X(I,J,L)-VEC(L)) LGT1
             1)                                                               LGT1
122       534 CONTINUE                                                        LGT1
127       555 M1=NX                                                           LGT1
130           N1=NX                                                           LGT1
131           DO 537 J=1,NAL                                                  LGT1
133           DO 537 K=1,NAL                                                  LGT1
134           M1=NX+(J-1)*NY                                                  LGT1
140           DO 538 M=1,NY                                                   LGT1
142           M1=M1+1                                                         LGT1
144           N1=NX+(K-1)*NY                                                  LGT1
147           DO 538 N=1,NY                                                   LGT1
151           N1=N1+1                                                         LGT1
153           IF (J.EQ.K) GO TO 539                                           LGT1
```

```
              HESSN(M1,N1)=HESSN(M1,N1)+JSUM(I)*(P(I,J+1)*P(I,K+1)*Y(I,M)*Y(I,N) L
             1)                                                                  LG
202           GO TO 538                                                          LG
202       539 HESSN(M1,N1)=HESSN(M1,N1)-JSUM(I)*(P(I,J+1)-P(I,J+1)**2)*Y(I,M)*Y( LG
             1I,N)                                                               LG
230       538 CONTINUE                                                           LG
235       537 CONTINUE                                                           LG
241       525 CONTINUE                                                           LG
244      1000 RETURN                                                             LG
245           END                                                                LG
```

```
               SUBROUTINE RESULT (NALT,NAL,NX,NY,NO,NT,MSTP,NOD,NOHCW,INIT,N2,B0, RESO
              1B1,FG0,FG1,FH1,FH2,FHV,LX,MX,SE,TV,MAX,NBY,FLO,ICP,NPP,NEL)        RESO
37             DIMENSION B0(10),B1(10),FG0(10),FG1(10),FH1(10,10),FH2(10,10),    RESO
              1FHV(10,10),LX(10),MX(10),SE(10),TV(10),NAX(10),NBY(10)            RESO
37             COMMON /AA1/ X(300,4, 4),Y(300,10),NOVA(300,4),P(300,4),          RESO
 .            1ANAME(300,10),JSUM(300),W4(4),JFAC(100)                            RESO
37             COMMON /AA2/ AR(100),BX(10),CY(4,10),VECR(10),HEXR(10,10),S1R(4),  RESO
 .            1         T1R(4),W1R(4),XMN(4, 4),YMN(10),XMB(4, 4),CMN(10),
 .            2XE(4,4, 4),YE(4,10)
37             COMMON /AA4/ NAME(45),LOCECS(45),NYHOW(10),NYSO(10,10),NALTO(4),   LOGO
              1NALTX(4, 4),NIZ(35),SCLZ(35),NARG(35),NAMB(10),ITRF(10,14),        LOGO
              2ITRC(10,14),LZN(30),S5(10),NN(30),NCH1(3),NCH2(3),MET(16)          LOGO
37             NOD=NO
40             IF (IOP.NE.3) GO TO 331                                            RESO
42             INIT=1                                                             RESO
43             READ (85,26) NE,NALT,NX,NY                                         RESO
57          26 FORMAT (8I10)                                                      RESO
57             NAL=NALT-1                                                         RESO
64             IF (NX.EQ.0) GO TO 343                                             RESO
65             READ (85,27) (BX(I),I=1,NX)                                        RESO
05          27 FORMAT (8F10.0)                                                    RESO
05         343 IF (NY.EQ.0) GO TO 107                                             LOAD
12             DO 332 J=1,NAL                                                     RESO
13         332 READ (85,27) (CY(J,I),I=1,NY)                                      RESO
40         107 WRITE (86,35) NE,NALT,NX,NY                                        LOAD
54          35 FORMAT (//4X,8X,2HNE,6X,4HNALT,8X,2HNX,8X,2HNY//4X,4I10)           LOAD
54         110 WRITE (86,32)                                                      LOAD
60          32 FORMAT (///4X,36HEXTERNALLY SUPPLIED COEFFICIENTS ARE)             LOAD
60             IF (NX.EQ.0) GO TO 113                                             LOAD
65             WRITE (86,33) ((I,BX(I)),I=1,NX)                                   LOAD
06          33 FORMAT (//8X,6(2HB(,I1,2H)=,F12.5,3X))                             LOAD
06             IF (NY.EQ.0) GO TO 114                                             LOAD
13         113 DO 112 J=1,NAL                                                     LOAD
15         112 WRITE (86,34) ((J,I,CY(J,I)),I=1,NY)                               LOAD
46          34 FORMAT (//8X,6(2HC(,I1,1H,,I1,2H)=,F10.5,3X))                      LOAD
46             GO TO 114                                                          LOAD
46         331 IF (NOD.EQ.3.OR.NOD.EQ.4) GO TO 601                                RESO
56             WRITE (86,1)                                                       RESO
61           1 FORMAT (///4X,24HCONVERGENCE NOT ACHIEVED)                         RESO
61         602 WRITE (86,2)                                                       RESO
65           2  FORMAT (//4X,59HTHE FOLLOWING ARE THE ESTIMATES OBTAINED BEFORE T RESO
              1ERMINATION/4X)                                                     RESO
65             IF (NX.EQ.0) GO TO 603                                             RESO
72             N=0                                                                RESO
73             DO 606 I=1,NX                                                      RESO
74             N=N+1                                                              RESO
76         606 BX(I)=B0(N)                                                        RESO
03             WRITE (86,4) ((I,BX(I)),I=1,NX)                                    RESO
24           4 FORMAT (/9X,5(3X,2HB(,I2,2H)=,F15.6)//9X,5(3X,2HB(,I2,2H)=,F15.6)/ RESO
              14X)                                                                RESO
24         603 IF (NY.EQ.0) GO TO 604                                             RESO
31             DO 607 J=1,NAL                                                     RESO
32             DO 607 I=1,NY                                                      RESO
33             N=N+1                                                              RESO
35         607 CY(J,I)=B0(N)                                                      RESO
45             DO 605 I=1,NAL                                                     RESO
46         605 WRITE (86,5) ((I,J,CY(I,J)),J=1,NY)                                RESO
```

```
77          5 FORMAT (/9X,5(2X,2HG(,I1,1H,,I1,2H)=,F15.6)//9X,5(2X,2HG(,I1,1H,,
             1I1,2H)=,F15.6)/4X)
1377      604 GO TO 1000
1400      601 NH=1
1401          NG=1
1402          CALL LOGIT (NALT,NAL,NX,NY,ND,NT,MSTP,NDHOW,INIT,NH,B0,FG0,FH2,
             1FL0,NG)
1421          IF (NPP.EQ.0) GO TO 333
1426          DO 152 I=1,NT
1430          S=0.0
1431          DO 153 J=1,NALT
1433      153 S=S+NOVA(I,J)
1443          DO 154 J=1,NALT
1445      154 ANAME(I,J)=NOVA(I,J)/S
1461      152 CONTINUE
1463          WRITE (86,77) (I,I=1,4),(J,J=1,4)
1500       77 FORMAT(*1*,20X,47HCOMPARISON OF ACTUAL AND PREDICTED MODE CHOICES/
             1/4X,13HACTUAL CHOICE,37X,16HPREDICTED CHOICE//4X,4HMODE,1X,
             24(8X,1H(,I1,1H)),11X,4(8X,1H(,I1,1H)))
1500          WRITE (86,78)
504           N=0
505           DO 347 I=1,NT
512           N=N+1
514           IF (N.NE.6) GO TO 348
516           WRITE (86,78)
521        78 FORMAT (* *)
521           N=1
522       348 GO TO (102,102,103,104),NALT
535       102 WRITE (86,80) I,(ANAME(I,J),J=1,NALT),(P(I,J),J=1,NALT)
575        80 FORMAT (4X,1H(,I3,1H),2F11.5,33X,2F11.5)
575           GO TO 347
601       103 WRITE (86,81) I,(ANAME(I,J),J=1,NALT),(P(I,J),J=1,NALT)
641        81 FORMAT (4X,1H(,I3,1H),3F11.5,22X,3F11.5)
641           GO TO 347
645       104 WRITE (86,79) I,(ANAME(I,J),J=1,NALT),(P(I,J),J=1,NALT)
705        79 FORMAT (4X,1H(,I3,1H),4F11.5,11X,4F11.5)
705       347 CONTINUE
713           S=0.0
714           T=0.0
715           DO 160 I=1,NT
716           DO 160 J=1,NALT
717           S=S+(ANAME(I,J)-P(I,J))**2
726       160 T=T+(ANAME(I,J)-1.0/NALT)**2
740           R2P=1.0-S/T
743           WRITE (86,47) R2P
750        47 FORMAT (///4X,45HCOEFFICIENT OF DETERMINATION IN PROBABILITY =,
             1F15.6)
750       333 S=0.0
751           DO 507 I=1,ND
756           DO 507 J=1,ND
757       507 FHV(I,J)=-FH2(I,J)
775           CALL MINV (FHV,ND,D,LX,MX,N2)
002           IF (D.NE.0.0) GO TO 701
007           WRITE (86,6)
012         6 FORMAT (/8X,69HHESSIAN SINGULAR AT CONVERGENCE, INDICATING EXTREME
             1 MULTICOLLINEARITY)
012           GO TO 602
016       701 T=0.0
```

```
017           NH=0                                                                  RES0
020           MSTP=0                                                                 ESC
021           NG=1                                                                  RES0
022           CALL LOGIT (NALT,NAL,NX,NY,ND,NT,MSTP,NDHOW,INIT,NH,80,FGG,FH2,       RES0
             1FLG,NG)                                                               RES0
040           KO=0                                                                  RES0
041       608 T=0.0                                                                 RES0
042           U=0.0                                                                 RES0
043           V=0.0                                                                 RES0
044           DO 609 I=1,NT                                                         RESC
051           S=0.0                                                                 RES0
052           DO 610 J=1,NALT                                                       RES0
054       610 S=S+NDVA(I,J)                                                         RES0
064           JSUM(I)=S                                                             RES0
067           DO 611 J=1,NALT                                                       RES0
070           V=V+(NOVA(I,J)-P(I,J)*JSUM(I))**2                                     RES0
102       611 T=T+(NOVA(I,J)-P(I,J)*JSUM(I))**2/(P(I,J)*JSUM(I))                    RES0
117           U=U+NAL*JSUM(I)                                                       RES0
123       609 CONTINUE                                                              RES0
125           IF (KO.EQ.1) GO TO 621                                                RES0
127           SRX2=T                                                                RES0
130           CF=1.0                                                                RES0
132           DF=U-ND                                                               RES0
134           SR2=V                                                                 RES0
135           SR2M=V/DF                                                             RES0
137           GO TO 622                                                             RES0
137       621 SRX20=T                                                               RES0
141           SR20=V                                                                RES0
142           SR2M0=V/(U-ND)                                                        RES0
145       622 S=0.0                                                                 RES0
146           T=0.0                                                                 RES0
147           DO 612 I=1,NT                                                         RES0
150           T=T+JSUM(I)                                                           RES07
153           IF (NDHOW.GT.2) GO TO 613                                             RES07
156           PC=P(I,1)                                                             RES07
160           U=NOVA(I,1)                                                           RES07
162           N=1                                                                   RES07
163           DO 614 J=2,NALT                                                       RES07
164           IF (P(I,J)-PC) 614,616,617                                            RES07
172       616 U=U+NOVA(I,J)                                                         RES07
177           N=N+1                                                                 RES07
201           GO TO 614                                                             RES07
201       617 PC=P(I,J)                                                             RES08
205           N=1                                                                   RES08
206           U=NOVA(I,J)                                                           RES08
212       614 CONTINUE                                                              RES08
215           S=S+U/N                                                               RES08
220           GO TO 612                                                             RES08
221       613 DO 618 J=1,NALT                                                       RES08
223           U=NOVA(I,J)-P(I,J)*JSUM(I)                                            RES08
233       618 S=S+ABS(U)                                                            RES08
240       612 CONTINUE                                                              RES08
243           IF (KO.EQ.1) GO TO 623                                                RES09
245           PCP=S/T*100.0                                                         RES09
247           GO TO 624                                                             RES09
250       623 PCP0=S/T*100.0                                                        RES09
253           DO 625 I=1,ND                                                         RES09
254           B1(I)=0.0                                                             RES09
```

```
256          DO 625 J=1,ND
260      625 B1(I)=B1(I)+FH1(I,J)*FG1(J)
277          GO TO 626
277      624 DO 619 I=1,ND
301          S=FHV(I,I)*CF
306          IF (S.GT.0.0) GO TO 303
310          S=-S
310      303 SE(I)=SQRT(S)
315      619 TV(I)=B0(I)/SE(I)
327          DO 620 I=1,ND
331      620 B1(I)=0.0
335          NH=1
336          NG=1
337          CALL LOGIT (NALT,NAL,NX,NY,ND,NT,MSTP,NDHOW,INIT,NH,B1,FG1,FH1,
            1FL1,NG)
355          CALL MINV (FH1,ND,D,LX,MX,N2)
366          KO=1
367          GO TO 608
373      626 WRITE (86,7)
377        7 FORMAT (*1*,4X,18HESTIMATION RESULTS//4X,19HPARAMETER ESTIMATES//
            111X,8HVARIABLE,7X,8HVARIABLE,10X,5HLOGIT,7X,8HSTANDARD,13X,2HT-,
            29X,6HLINEAR/13X,6HNUMBER,11X,4HNAME,6X,9HESTIMATOR,10X,5HERROR,6X,
            39HSTATISTIC,6X,9HPROB.EST./4X)
377          N=0
400          IF (NX.EQ.0) GO TO 627
405          DO 628 I=1,NX
406          N=N+1
410      628 WRITE (86,8) I,NAX(I),B0(N),SE(N),TV(N),B1(N)
.42        8 FORMAT (14X,2HB(,I2,1H),5X,A10,4F15.5)
.42          WRITE (86,9)
45         9 FORMAT (* *)
445      627 IF (NY.EQ.0) GO TO 629
452          DO 630 J=1,NAL
.53          DO 630 I=1,NY
54           N=N+1
6            KZ=J+1
         630 WRITE (86,10) KZ,I,NBY(I),B0(N),SE(N),TV(N),B1(N)
          10 FORMAT (11X,2HC(,I2,1H,,I2,1H),5X,A10,4F15.5)
16       629 WRITE (86,31)
22        31 FORMAT (//4X,64HWHEN THE NAME OF A MODE-RELATED VARIABLE APPEARS M
            1ORE THAN ONCE,/4X,36HTHE VARIABLE HAS DUMMY COEFFICIENTS./4X,46HCH
            2ECK NXCD NYCD AND NRCD FOR MORE INFORMATION.//4X,69HTO FIND IF A S
            3OCIOECONOMIC VARIABLE IS A DUMMY, CHECK NYHOW AND NYSO.)
             IF (NDHOW.LE.2) GO TO 730
             PCP=100.0-PCP
             PCP0=100.0-PCP0
534      730 WRITE (86,25)
540       25 FORMAT (//4X,20HAUXILIARY STATISTICS/36X,14HAT CONVERGENCE,13X,
            17HAT ZERO/ 4X)
540          WRITE (86,11) FL0,FL1
550       11 FORMAT (9X,14HLOG LIKELIHOOD, 12X,F15.5,5X,F15.5/4X)
550          WRITE (86,12) SRX2,SRX20
560       12 FORMAT (9X,19HRESIDUAL CHI-SQUARE,7X,F15.5,5X,F15.5/4X)
560          WRITE (86,13) SR2,SR20
570       13 FORMAT (9X,24HSUM OF SQUARED RESIDUALS,2X,F15.5,5X,F15.5/4X)
570          WRITE (86,14) DF,DF
600       14 FORMAT (9X,18HDEGREES OF FREEDOM,8X,F15.5,5X,F15.5/4X)
600          WRITE (86,15) PCP,PCP0
```

```
10         15 FORMAT (9X,27HPERCENT CORRECTLY PREDICTED,4X,F10.5,10X,E10.5/4X)      RES1
610           RI=1.0-FL0/FL1                                                          RES1
614           RS=-2.0*(FL1-FL0)                                                       RES1
616           R2=1.0-SR2/SR20                                                         RES1
521           WRITE (86,16)                                                           RES1
524        16 FORMAT (//4X,26HGOCDNESS OF FIT STATISTICS,35X,10HABOUT ZERO/4X)        RES1
624           WRITE (86,17) RI                                                        RES1
632        17 FORMAT (9X,45HLIKELIHOOD RATIO INDEX (= 1.0-LGL(*)/LGL(0) ),8X,         RES1
             1F15.5/4X)                                                               RES1
632           WRITE (86,18) RS                                                        RES1
640        18 FORMAT (9X,50HLIKELIHOOD RATIO STATISTIC (= -2*(LGL(0)-LGL(*)) ),       RES1
             11X,F15.5/4X)                                                            RES1
540           WRITE (86,19) R2                                                        RES1
546        19 FORMAT (9X,28HCOEFFICIENT CF DETERMINATION,23X,F15.5/4X)                RES1
546           WRITE (86,20) (I,I=1,ND)                                                RES1
565        20 FORMAT (*1*,4X,13HMOMENT MATRIX//9X,7(11X,1H(,I2,1H))/9X,7(11X,         RES1
             11H(,I2,1H)))                                                            RES1
565           DO 631 I=1,ND                                                           RES1
572           DO 632 J=1,ND                                                           RES1
573       632 FH1(I,J)=-FH2(I,J)                                                      RES1
707       631 WRITE (86,21) I,(FH1(I,J),J=1,ND)                                       RES1
741        21 FORMAT (/4X,1H(,I2,1H), 7E15.6,/8X, 7E15.6)                             RES1
741           WRITE (86,22) (I,I=1,ND)                                                RES1
754        22 FORMAT (*1*,4X,17HCOVARIANCE MATRIX//9X,7(11X,1H(,I2,1H))/9X,7(11X RES1
             1,1H(,I2,1H)))                                                           RES1
754           DO 633 I=1,ND                                                           RES1
761           DO 634 J=1,ND                                                           RES1
762       634 FHV(I,J)=CF*FHV(I,J)                                                    RES1
771       633 WRITE (86,21) I,(FHV(I,J),J=1,ND)                                       RES1
123       114 IF (IOP.EQ.3) GO TO 172
126           N=0
127           IF (NX.EQ.0) GO TO 173
130           DO 174 K=1,NX
131           N=N+1
133       174 BX(K)=B0(N)
140       173 IF (NY.EQ.0) GO TO 172
141           DO 175 J=1,NALT
143           DO 175 K=1,NY
144           N=N+1
146       175 CY(J,K)=B0(N)
156       172 IF (IOP.EQ.1) GO TO 334                                                 LOA0
161           IF (IOP.EQ.3) GO TO 115                                                 LOA0
163           READ (85,26) NE                                                         LOA0
170       115 IF (NX.EQ.0.OR.NY.EQ.0) GO TO 116                                       LOA0
.01           READ (85,36) (NAX(I),I=1,NX),(NBY(I),I=1,NY)                            LOA0
.35           GO TO 117                                                               LOA0
.41        36 FORMAT (8A10)                                                           LOA0
.41       116 IF (NX.EQ.0) GO TO 118                                                  LOA0
.42           READ (85,36) (NAX(I),I=1,NX)                                            LOA0
163           GO TO 117                                                               LOA0
167       118 READ (85,36) (NBY(I),I=1,NY)                                            LOA0
207       117 DO 119 I=1,NE                                                           LOA0
214           IF (NX.EQ.0) GO TO 120                                                  LOA0
215           DO 121 J=1,NALT                                                         LOA0
216       121 READ (85,28) (X(I,J,K),K=1,NX)
247       120 IF (NY.EQ.0) GO TO 119                                                  LOA0
250           READ (85,28) (Y(I,K),K=1,NY)
271        28 FORMAT (10X,7F10.0)
```

```
271      119 CONTINUE                                                        LO
277          WRITE (86,37)                                                   LOA
303       37 FORMAT (//4X,33HEXTERNALLY SUPPLIED VARIABLES ARE)              LOA
303          IF (NX.EQ.0) GO TO 122                                          LOA
310          WRITE (86,38) (NAX(I), I=1,NX)                                  LOA
330       38 FORMAT (/6X,24HMODE RELATED VARIABLES =,6(5X,A10))              LO
330      122 IF (NY.EQ.0) GO TO 123                                          LO
335          WRITE (86,39) (NBY(I),I=1,NY)                                   LO
354       39 FORMAT (/6X,24HSOCIOECONOMIC VARIABLES=,6(5X,A10))              LO
354      123 WRITE (86,40)                                                   LO
360       40 FORMAT (/6X,59H(WHEN NAME OF MODE RELATED VARIABLE APPEARS MORE TH LO
            1AN ONCE,/7X,37HTHAT VARIABLE HAS DUMMY COEFFICIENTS./7X,64HFOR A S LO
            2OCIOECONOMIC VARIABLE, IF ITS VALUE IS EITHER 1.0 OR 0.0,/7X,24HTH LO
            3E VARIABLE IS A DUMMY.)                                         L
360          WRITE (86,41)                                                   L
364       41 FORMAT (*1*,4X,39HEXTERNALLY SUPPLIED VARIABLE VALUES ARE)      L
364          IF (NX.EQ.0 .OR. NY.EQ.0) GO TO 146                             LO
375          WRITE (86,42) (NAX(I),I=1,NX),(NBY(I),I=1,NY)                   LO
431       42 FORMAT (//4X,5X,7(5X,A10))                                      L
431          GO TO 125                                                       L
435      146 IF (NX.EQ.0) GO TO 124                                          L
436          WRITE (86,42) (NAX(I),I=1,NX)                                   LO
457          GO TO 125                                                       LO
463      124 WRITE (86,42) (NBY(I),I=1,NY)                                   L
503      125 N=0                                                             L
504          WRITE (86,78)                                                   L
510          DO 126 I=1,NE                                                   L
515          N=N+1                                                           L
517          IF (N.NE.6) GO TO 127                                           L
521          WRITE (86,78)                                                   L
524          N=1                                                             L
525      127 IF (NX.EQ.0 .OR. NY.EQ.0) GO TO 128                             L
536          WRITE (86,43) I,(X(I,1,K),K=1,NX),(Y(I,K),K=1,NY)               L
576       43 FORMAT (4X,1H(,I3,1H),7F15.5)                                   L
576          DO 129 J=2,NALT                                                 L
603      129 WRITE (86,44) (X(I,J,K),K=1,NX)                                 L
634       44 FORMAT (9X,7F15.5)                                              L
634          GO TO 126                                                       LO
634      128 IF (NY.EQ.0) GO TO 130                                          LO
635          WRITE (86,43) I,(Y(I,K),K=1,NY)                                 LO
660          GO TO 126                                                       LO
664      130 WRITE (86,43)  I,(X(I,1,K),K=1,NX)                              LO
710          DO 131 J=2,NALT                                                 LO
715      131 WRITE (86,44) (X(I,J,K),K=1,NX)                                 LO
746      126 CONTINUE                                                        LOA
751          GO TO 873                                                       LO
751      334 IF (NEL.EQ.0) GO TO 1000                                        RES
753          NS=1
754          IF (NY.EQ.0) GO TO 873
755          DO 840 K=1,NY                                                   RLT8
756          LX(K)=0                                                         R  8
760          M=0                                                             PR 8
761          N=0                                                             R  8
762          DO 841 I=1,NT                                                   R  8
763          IF (Y(I,K).EQ.1.0) M=M+1                                        R  8
771          IF (Y(I,K).EQ.1.0.OR.Y(I,K).EQ.0.0) N=N+1                       RLT8
003      841 CONTINUE                                                        R  8
006          IF (M.EQ.NT) GO TO 840                                          R  8
```

```
07              IF (N.NE.NT) GO TO 840
011             LX(K)=1
013         840 CONTINUE
015             DO 842 K=1,NY
017             B1(K)=0.0
021             IF (LX(K).EQ.0) B1(K)=YMN(K)
026         842 CONTINUE
031             DO 843 I=1,NY
032             DO 860 J=1,NY
033         860 FH1(I,J)=0.0
041             IF (LX(I).NE.1) GO TO 843
044             DO 844 J=1,NY
046             IF (NBY(I).EQ.NBY(J)) FH1(I,J)=1.0
056         844 CONTINUE
061         843 CONTINUE
064             DO 845 J=1,NY
065             N=0
066             DO 846 I=1,NY
070             IF (N.EQ.1) FH1(I,J)=0.0
076             IF (FH1(I,J).EQ.1.0) N=1
105         846 CONTINUE
110         845 CONTINUE
112             N=0
113             DO 847 I=1,NY
114             IF (FH1(I,I).NE.1.0) GO TO 847
121             S=0.0
122             N=N+1
123             DO 848 J=I,NY
25          848 S=S+FH1(I,J)
34              LX(N)=I
37              MX(N)=S
42          847 CONTINUE
45              NJ=N
46              DO 159 I=1,NY
50          159 FH2(1,I)=0.0
56              IF (NJ.EQ.0) GO TO 873
57              S=1.0
60              DO 850 I=1,NJ
62          850 S=S*(MX(I)+1.0)
70              NS=S
72              DO 851 I=1,NS
73              DO 851 J=1,NY
74          851 FH2(I,J)=0.0
04              J1=1
05              K1=0
06              L1=0
07              M1=0
10              KL=MX(1)+1
12              LL=MX(2)+1
14              ML=MX(3)+1
15              DO 849 K=1,KL
17              K1=LX(1)+K-1
22              IF (K.EQ.KL) GO TO 855
24              FH2(J1,K1) =1.0
30          855 IF (NJ.EQ.1) GO TO 852
32              DO 853 L=1,LL
34              L1=LX(2)+L-1
37              IF (L.EQ.1) GO TO 863
```

```
240              LT=LX(2)-1                                                R
242              DO 861 L2=1,LT                                            R
243        861   FH2(J1,L2)=FH2(J1-1,L2)                                   R
256        863   IF (L.EQ.LL) GO TO 856                                    R
260              FH2(J1,L1)=1.0                                            R
265        856   IF (NJ.EQ.2) GO TO 854                                    R
267              DO 872 M=1,ML                                             R
271              M1=LX(3)+M-1                                              R
274              IF (M.EQ.1) GO TO 864                                     R
275              MT=LX(3)-1                                                R
277              DO 862 M2=1,MT                                            R
300        862   FH2(J1,M2)=FH2(J1-1,M2)                                   R
313        864   IF (M.EQ.ML) GO TO 857                                    RLT
315              FH2(J1,M1) =1.0                                           RLT
322        857   J1=J1+1                                                   RL
324        872   CONTINUE                                                  RL
326              GO TO 853                                                 RLT
327        854   J1=J1+1                                                   R
331        853   CONTINUE                                                  R
334              GO TO 849                                                 RL
334        852   J1=J1+1                                                   R
336        849   CONTINUE                                                  RL
341        873   IF (NX.EQ.0) GO TO 874                                    RL
342              DO 304 I=1,NX                                             RE
344              DO 304 J=1,NX                                             RE
345              FHV(I,J)=0.0                                              RE
351              IF (NAX(I).EQ.NAX(J)) FHV(I,J)=1.0                        RE
360        304   CONTINUE                                                  RE
365              DO 305 J=1,NX                                             RE
366              N=0                                                       RE
367              DO 305 I=1,NX                                             RE
371              IF (FHV(I,J).EQ.0.0) GO TO 305                            RE
375              IF (N.EQ.1) GO TO 306                                     RE
377              N=1                                                       RE
400              GO TO 305                                                 RE
400        306   FHV(I,J)=0.0                                              RE
405        305   CONTINUE                                                  RE
412              DO 307 I=1,NX                                             RES
413              N=0                                                       RES
414              DO 308 J=1,NX                                             RES
416        308   N=N+FHV(I,J)                                              RES
427        307   NIZ(I)=N                                                  RES
432              IF (IOP.NE.1) GO TO 874                                   LOA
434              N=0                                                       RES
435              DO 309 I=1,NX                                             RES
436              IF (NIZ(I).LE.1) GO TO 309                                RES
441              N=N+1                                                     RES6
442              LX(N)=I                                                   RES6
445              MX(N)=NIZ(I)                                              RES6
450        309   CONTINUE                                                  RES6
453              MN=N                                                      RES6
454              IF (MN.EQ.0) GO TO 874                                    RES6
455              S=1.0                                                     RES6
456              DO 310 I=1,MN                                             RES6
460        310   S=S*MX(I)                                                 RES6
465              MS=S                                                      RES6
467              DO 311 I=1,MS                                             RES6
470              DO 311 J=1,NX                                             RES6
```

```
471       311 FH1(I,J)=0.0                                                   RES6
501           J1=1                                                           RES6
502           K1=0                                                           RES6
503           L1=0                                                           RES6
504           M1=0                                                           RES6
505           KL=MX(1)                                                       RES6
507           LL=MX(2)                                                       RES6
510           ML=MX(3)                                                       RES6
512           DO 312 K=1,KL                                                  RES6
513           L2=LX(1)                                                       RES6
515           FH1(J1,L2)=1.0                                                 RES6
522           IF (K.EQ.KL) GO TO 313                                         RES6
524           FH1(J1,L2+K)=1.0                                               RES6
532       313 IF (MN.EQ.1) GO TO 314                                         RES6
534           L3=LX(2)                                                       RES6
536           DO 315 L=1,LL                                                  RES6
540           IF (L.EQ.1) GO TO 316                                          RES6
542           LT=LX(2)-1                                                     RES6
544           DO 317 L2=1,LT                                                 RES6
545       317 FH1(J1,L2)=FH1(J1-1,L2)                                        RES6
560       316 FH1(J1,L3)=1.0                                                 RES6
565           IF (L.EQ.LL) GO TO 318                                         RES6
567           FH1(J1,L3+L) =1.0                                              RES6
575       318 IF (MN.EQ.2) GO TO 319                                         RES6
577           L4=LX(3)                                                       RES6
601           DO 320 M=1,ML                                                  RES6
603           IF (M.EQ.1) GO TO 321                                          RES6
605           MT=LX(3)-1                                                     RES6
607           DO 322 M2=1,MT                                                 RES6
610       322 FH1(J1,M2)=FH1(J1-1,M2)                                        RES6
623       321 FH1(J1,L4)=1.0                                                 RES6
630           IF (M.EQ.ML) GO TO 323                                         RES6
632           FH1(J1,L4+M)=1.0                                               RES6
640       323 J1=J1+1                                                        RES6
642       320 CONTINUE                                                       RES6
644           GO TO 315                                                      RES6
645       319 J1=J1+1                                                        RES6
647       315 CONTINUE                                                       RES6
652           GO TO 312                                                      RES6
652       314 J1=J1+1                                                        RES6
654       312 CONTINUE                                                       RES6
657           DO 324 J=1,NX                                                  RES6
660           IF (NIZ(J).NE.1) GO TO 324                                     RES6
663           DO 325 I=1,MS                                                  RES6
664       325 FH1(I,J)=1.0                                                   RES6
673       324 CONTINUE                                                       RES6
676           DO 326 I=1,MS                                                  RES6
677           DO 326 K=1,NX                                                  RES6
700       326 FH1(I,K)=B0(K)*FH1(I,K)                                        RES6
713           DO 350 I=1,MS                                                  RES6
715           N=0                                                            RES6
716           DO 351 K=1,NX                                                  RES6
720           IF (NIZ(K).EQ.0) GO TO 351                                     RES6
722           N=N+1                                                          RES6
723           M=NIZ(K)                                                       RES6
725           S=0.0                                                          RES6
726           DO 352 J=1,M                                                   RES6
727       352 S=S+FH1(I,K+J-1)                                               RES6
```

```
      EH1(I,N)=S
  351 CONTINUE
  350 CONTINUE
      N=0
      DO 105 K=1,NX
      IF (NIZ(K).EQ.0) GO TO 105
      N=N+1
      DO 106 J=1,NALT
  106 XMN(J,N)=XMN(J,K)
  105 CONTINUE
  874 IX=1
      IF (IOP.NE.1) GO TO 132
      WRITE (86,71)
   71 FORMAT (*1*,4X,53HPROBABILITIES AND ELASTICITIES OF MODE CHOICE AT
     1 MEAN)
      N=0
      IF (NY.EQ.0) GO TO 171
      DO 161 J=1,NAL
      DO 161 K=1,NY
      N=N+1
  161 FG1(N)=CY(J,K)
  171 GO TO 133
  132 WRITE (86,45)
   45 FORMAT (*1*,4X,45HPROBABILITIES AND ELASTICITIES OF MODE CHOICE)
      IF (NX.EQ.0) GO TO 151
      DO 149 K=1,NX
  149 SE(K)=BX(K)
  151 IF (NY.EQ.0) GO TO 133
      DO 150 J=1,NAL
      DO 150 K=1,NY
  150 ANAME(J,K)=CY(J,K)
  133 NX2=NX
      N=0
      IF (NX.EQ.0) GO TO 702
      DO 134 K=1,NX
      IF (NIZ(K).EQ.0) GO TO 134
      N=N+1
      NAX(N)=NAX(K)
  134 CONTINUE
      NX=N
  145 IF (NY.EQ.0) GO TO 703
      WRITE (86,72) (NAX(I),I=1,NX),(NBY(I),I=1,NY)
   72 FORMAT (//4X,9HVARIABLES,10(1X,A10))
      GO TO 871
  702 WRITE (86,72) (NBY(I),I=1,NY)
      GO TO 871
  703 WRITE (86,72) (NAX(I),I=1,NX)
   76 FORMAT(/4X,75H(IGNORE THE ELASTICITIES UNDER A SOCIOECONOMIC VARIA
     1BLE WHEN IT IS A DUMMY./5X,94HALSO IGNORE THE CASE WHERE DUMMY COE
     2FFICIENT AND DUMMY VARIABLE COMBINATIONS ARE INCONSISTENT./5X,57HI
     3N TYPE 1 OPERATION, CHECK NYHOW NYSO AND NRCD FOR THIS.))
  871 IZ=1
      IEL=1
  341 IF (IOP.EQ.1) GO TO 330
      N=0
      IF (NX.EQ.0) GO TO 139
      DO 135 I=1,NX2
      IF (NIZ(I).EQ.0) GO TO 135
```

```
233           N=N+1                                                         LOA1
234           IF (NIZ(I).EQ.1) GO TO 136                                    LOA1
236           S=0.0                                                         LOA1
237           NL=NIZ(I)-1                                                   LOA1
240           DO 137 K=1,NL                                                 LOA1
242       137 S=S+SE(I+K)*X(IEL,1,I+K)                                      LOA1
256           FG1(N)=SE(I)+S                                                LOA1
263           GO TO 135                                                     LOA1
264       136 FG1(N)=SE(I)                                                  LOA1
271       135 CONTINUE                                                      LOA1
274       139 IF (NY.EQ.0) GO TO 140                                        LOA1
275           DO 138 J=1,NAL                                                LOA1
277           DO 138 K=1,NY                                                 LOA1
300           N=N+1                                                         LOA1
302       138 FG1(N)=ANAME(J,K)                                             LOA1
313       140 IF (NX.EQ.0) GO TO 144                                        LOA1
314           N=0                                                           LOA1
315           DO 141 K=1,NX2                                                LOA1
316           IF (NIZ(K).EQ.0) GO TO 141                                    LOA1
320           N=N+1                                                         LOA1
321           DO 142 J=1,NALT                                               LOA1
323       142 X(IEL,J,N)=X(IEL,J,K)                                         LOA1
342       141 CONTINUE                                                      LOA1
345           DO 143 J=1,NALT                                               LOA1
346           DO 143 K=1,NX                                                 LOA1
347       143 XMN(J,K)=X(IEL,J,K)                                           LOA1
365       144 IF (NY.EQ.0) GO TO 147                                        LOA1
366           DO 148 K=1,NY                                                 LOA1
370       148 Y(1,K)=Y(IEL,K)                                               LOA1
402       147 ND=NX+(NALT-1)*NY                                             LOA1
406           GO TO 705                                                     LOA1
406       330 IF (NX.EQ.0) GO TO 704                                        REL6
407           IF (MN.EQ.0) GO TO 156                                        REL6
410           DO 353 I=1,NX                                                 REL6
412       353 FG1(I)=FH1(IZ,I)                                              REL6
424           GO TO 158                                                     REL6
424       156 DO 157 I=1,NX                                                 REL6
426       157 FG1(I)=B0(I)                                                  REL6
434       158 N1=NAL*NY                                                     REL6
436           N=NOD-N1
440           DO 354 I=1,N1
441       354 FG1(NX+I)=B0(N+I)
451           ND=NX+NAL*NY
453       705 DO 801 K=1,NX                                                 REL6
455           S=XMN(1,K)                                                    REL6
460           DO 801 J=2,NALT                                               REL6
461       801 X(1,J-1,K)=XMN(J,K)-S                                         REL6
477       704 IF (NY.EQ.0) GO TO 815                                        REL6
500           IF (IOF.NE.1) GO TO 815                                       LOA1
503           DO 802 K=1,NY                                                 REL7
504       802 Y(1,K)=B1(K)+FH2(IX,K)                                        REL7
523       815 NT1=1                                                         REL7
524           NH=0                                                          REL7
525           NG=1                                                          REL7
526           CALL LOGIT (NALT,NAL,NX,NY,ND,NT1,MSTP,NCHOW,INIT,NH,FG1,FG0,FHV, REL7
             1FL0,NG)                                                       REL7
545           IF (NX.EQ.0) GO TO 706                                        REL7
552           DO 803 J=1,NALT                                               REL7
```

```
553        DO 803 K=1,NX
554    803 XMB(J,K)=XMN(J,K)*FG1(K)
574    706 IF (NY.EQ.0) GO TO 814
575        N=NX
576        DO 813 J=1,NAL
600        DO 813 K=1,NY
601        N=N+1
603    813 CY(J,K)=FG1(N)
613        DO 804 K=1,NY
614        S=0.0
615        DO 805 J=1,NAL
617    805 S=S+P(1,J+1)*CY(J,K)
631    804 CMN(K)=S
634    814 DO 806 I=1,NALT
636        IF (NX.EQ.0) GO TO 707
637        DO 807 J=1,NALT
640        DO 807 K=1,NX
641        IF (I.EQ.J) GO TO 808
643        XE(I,J,K)=-P(1,J)*XMB(J,K)
653        GO TO 807
654    808 XE(I,J,K)=(1.0-P(1,J))*XMB(J,K)
667    807 CONTINUE
674    707 IF (NY.EQ.0) GO TO 806
675        DO 809 K=1,NY
677        IF (I.EQ.1) GO TO 810
701        YE(I,K)=Y(1,K)*(CY(I-1,K)-CMN(K))
713        GO TO 809
 13    810 YE(I,K)=-Y(1,K)*CMN(K)
 22    809 CONTINUE
 25    806 CONTINUE
 30        N=0
 31        IF (IOP.EQ.1) GO TO 346
733        WRITE (86,75) IEL
 40     75 FORMAT (//4X,37HELASTICITIES FOR EXTERNAL DATA NUMBER,I5,1H.)
 40    346 IF (NX.EQ.0) GO TO 708
  5        IF (NY.EQ.0) GO TO 709
  6        IF (IOP.NE.1) GO TO 355
  0        WRITE (86,74)
 53     74 FORMAT (/13X,24HEVALUATED AT MEAN VALUES/4X)
 53        GO TO 356
757    355 WRITE (86,82)
763     82 FORMAT (/13X,28HEVALUATED AT VARIABLE VALUES/4X)
763    356 DO 357 J=1,NALT
770    357 WRITE (86,83) J,(XMN(J,K),K=1,NX),(Y(1,K),K=1,NY)
035     83 FORMAT ( 6X,5HMODE(,I1,1H),10F11.5)
035        DO 811 J=1,NALT
036        WRITE (86,84) J,(FG1(K),K=1,ND)
050     84 FORMAT (/13X,32HBY CHANGING THE VALUES FOR MODE(, I1,2H) ,45HAND U
          1SING THE COMPACTED COEFFICIENT VALUES OF/13X,88H(COEFFICIENTS ARE
          2ARRANGED IN ORDER OF B(1)...B(NX) C(ALT2,1)...C(ALT2,NY)...C(NALT,
          3NY))//13X,10F11.5)
060        WRITE (86,46)
064     46 FORMAT (/13X,44HELASTICITIES AND PROBABILITIES (LAST COLUMN))
064        DO 811 I=1,NALT
071    811 WRITE (86,73) I,(XE(I,J,K),K=1,NX),(YE(I,K),K=1,NY),P(1,I)
150     73 FORMAT (/3X,8HE FOR M(,I1,1H),10F11.5)
150        GO TO 710
150    708 IF (IOP.NE.1) GO TO 358
```

```
155          WRITE (86,74)                                                   REL7
156          GO TO 359                                                       REL7
162      358 WRITE (86,82)                                                   REL7
166      359 WRITE (86,85) (Y(1,K),K=1,NY)                                   REL7
213       85 FORMAT (13X,10F11.5)                                            REL7
213          WRITE (86,86) (FG1(K),K=1,ND)                                   REL7
237       86 FORMAT (/13X,44HBY USING THE COMPACTED COEFFICIENT VALUES OF/13X, LOA1
            188H(COEFFICIENTS ARE ARRANGED IN ORDER OF B(1)...B(NX) C(ALT2,1).. LOA1
            2.C(ALT2,NY)...C(NALT,NY))//13X,10F11.5)                         LOA1
237          WRITE (86,46)                                                   LOA1
243          DO 711 I=1,NALT                                                 REL7
250      711 WRITE (86,73) I,(YE(I,K),K=1,NY),P(1,I)                         REL7
306          GO TO 710                                                       REL7
306      709 IF (ICP.NE.1) GO TO 380                                         REL7
311          WRITE (86,74)                                                   REL7
314          GO TO 381                                                       REL7
320      380 WRITE (86,82)                                                   REL7
324      381 DO 382 J=1,NALT                                                 REL7
331      382 WRITE (86,83) J,(XMN(J,K),K=1,NX)                               REL7
361          DO 712 J=1,NALT                                                 REL7
362          WRITE (86,84) J,(FG1(K),K=1,ND)                                 REL7
404          WRITE (86,46)                                                   REL7
410          DO 712 I=1,NALT                                                 REL7
415      712 WRITE (86,73) I,(XE(I,J,K),K=1,NX),P(1,I)                       REL7
460      710 IF (ICP.EQ.1) GO TO 342                                         REL7
463          IEL=IEL+1                                                       REL7
464          IF (IEL.LE.NE) GO TO 341                                        REL7
466          GO TO 1000                                                      REL7
467      342 IZ=IZ+1                                                         REL7
471          IF (IZ.LE.MS) GO TO 330                                         REL7
473          IX=IX+1                                                         REL7
474          IF (IX.GT.NS) GO TO 1000                                        REL7
477          GO TO 871                                                       REL7
477     1000 RETURN                                                          REL7
500          END                                                             REL7
```

```
            SUBROUTINE MINV (AH,N,D,L,M,N2)                                       I
011         DIMENSION AH(10,10),L(10),M(10)                                       IN
011         COMMON /AA2/ A(100)                                                   IN
011         K=0                                                                   IN
012         DO 201 I=1,N                                                          IN
013         DO 201 J=1,N                                                          IN
014         K=K+1                                                                 INV
016     201 A(K)=AH(J,I)                                                          IN
026         D=1.0                                                                 IN
027         NK=-N                                                                 IN
030         DO 80 K=1,N                                                           IN
031         NK=NK+N                                                               IN
033         L(K)=K                                                                I
034         M(K)=K                                                                I
035         KK=NK+K                                                               IN
036         BIGA=A(KK)                                                            IN
040         DO 20 J=K,N                                                           INV
041         IZ=N*(J-1)                                                            IN
044         DO 20 I=K,N                                                           I
046         IJ=IZ+I                                                               IN
050         S=ABS(A(IJ))                                                          IN
052         T=ABS(BIGA)                                                           IN
054      10 IF (T-S) 15,20,20                                                     IN
057      15 BIGA=A(IJ)                                                            I
061         L(K)=I                                                                IN
064         M(K)=J                                                                IN
065      20 CONTINUE                                                              IN
072         J=L(K)                                                                IN
074         IF (J-K) 35,35,25                                                     IN
075      25 KI=K-N                                                                INV
077         DO 30 I=1,N                                                           INV
100         KI=KI+N                                                               IN
102         HOLD=-A(KI)                                                           IN
103         JI=KI-K+J                                                             INV
105         A(KI)=A(JI)                                                           INV
107      30 A(JI)=HOLD                                                            INV
113      35 I=M(K)                                                                INV
115         IF (I-K) 45,45,38                                                     INV
117      38 JP=N*(I-1)                                                            INV
122         DO 40 J=1,N                                                           INV
124         JK=NK+J                                                               INV
126         JI=JP+J                                                               INV
127         HOLD=-A(JK)                                                           INV
131         A(JK)=A(JI)                                                           INV
133      40 A(JI)=HOLD                                                            INV
137         T=ABS(BIGA)                                                           INV
141      45 IF (T-1.0E-20) 46,46,48                                               INV
144      46 D=0.0                                                                 INV
145         RETURN                                                                INV
145      48 DO 55 I=1,N                                                           INV
147         IF (I-K) 50,55,50                                                     INV
151      50 IK=NK+I                                                               INV
153         A(IK)=A(IK)/(-BIGA)                                                   INV
156      55 CONTINUE                                                              INV
161         DO 65 I=1,N                                                           INV
162         IK=NK+I                                                               INV
164         HOLD = A(IK)                                                          INV
```

```
 56            IJ=I-N                                                                   INV0
167            DO 65 J=1,N                                                              INV0
171            IJ =IJ+N                                                                 INV0
173            IF (I-K) 60,65,60                                                        INV0
175        60  IF (J-K) 62,65,62                                                        INV0
177        62  KJ=IJ-I+K                                                                INV0
202            A(IJ)=HOLD*A(KJ)+A(IJ)                                                   INV0
205        65  CONTINUE                                                                 INV0
212            KJ=K-N                                                                   INV0
213            DO 75 J=1,N                                                              INV0
215            KJ=KJ+N                                                                  INV0
217            IF (J-K) 70,75,70                                                        INV0
220        70  A(KJ)=A(KJ)/BIGA                                                         INV0
223        75  CONTINUE                                                                 INV0
226            D=D*BIGA                                                                 INV0
227            A(KK)=1.0/BIGA                                                           INV0
231        80  CONTINUE                                                                 INV07
233            K=N                                                                      INV07
234       100  K=(K-1)                                                                  INV07
236            IF (K) 150,150,105                                                       INV07
237       105  I=L(K)                                                                   INV07
241            IF (I-K) 120,120,108                                                     INV07
243       108  JO=N*(K-1)                                                               INV07
246            JR=N*(I-1)                                                               INV07
252            DO 110 J=1,N                                                             INV07
253            JK=JG+J                                                                  INV07
255            HOLD=A(JK)                                                               INV08
257            JI=JR+J                                                                  INV08
260            A(JK)=-A(JI)                                                             INV08
  2       110  A(JI)=HOLD                                                               INV08
  5       120  J=M(K)                                                                   INV08
270            IF (J-K) 100,100,125                                                     INV08
272       125  KI=K-N                                                                   INV08
274            DO 130 I=1,N                                                             INV08
275            KI=KI+N                                                                  INV08
277            HOLD = A(KI)                                                             INV08
300            JI=KI-K+J                                                                INV09
302            A(KI)=-A(JI)                                                             INV09
304       130  A(JI)=HOLD                                                               INV09
310            GO TO 100                                                                INV09
310       150  K=0                                                                      INV09
311            DO 202 I=1,N                                                             INV09
313            DO 202 J=1,N                                                             INV09
314            K=K+1                                                                    INV09
316       202  AH(J,I)=A(K)                                                             INV09
326            RETURN                                                                   INV09
326            END                                                                      INV10
```

```
            SUBROUTINE ESTIM (NALT,NAL,NX,NY,ND,NT,MSTP,NOD,NDHOW,INIT,N2,B0,   ES
           1B1,B2,BZ,FG0,FG1,FG2,FG3,FGZ,FH0,FH1,FH2,FHV,DIRS,S4S,LX,MX,FL0)   ES
040         DIMENSION B0(10),B1(10),B2(10),BZ(10),FG0(10),FG1(10),FG2(10),     ES
           1FG3(10),FGZ(10),FH0(10,10),FH1(10,10),FH2(10,10),FHV(10,10),       ES
           2DIRS(10),S4S(10),LX(10),MX(10)                                     ES
040         COMMON /AA1/ X(300,4,4),Y(300,10),NDVA(300,4),P(300,4),            ES
           1ANAME(300,10),JSUM(300),W4(4),JFAC(100)                            EST
040         COMMON /AA2/ AR(100),BXR(10),CYR(4,10),VECR(10),HEXR(10,10),       EST
           1S1R(4),T1R(4),W1R(4)                                               EST
040         NH=1                                                               EST
041         NQ=0                                                               EST
042         NOD=2                                                              ES
043         NAL=NALT-1                                                         ES
044         INIT=0                                                             ES
045         ND=NX+NAL*NY                                                       EC
050         IF (NX.EQ.0) GO TO 201                                             ES
051         DO 202 I=1,NT                                                      ES
052         DO 203 K=1,NX                                                      ES
053         S=X(I,1,K)                                                         ES
057         DO 203 J=2,NALT                                                    ES
060     203 X(I,J-1,K)=X(I,J,K)-S                                              ES
101     202 CONTINUE                                                           ES
103     201 S=0.0                                                              ES
104         NG=1                                                               ES
105         CALL LOGIT (NALT,NAL,NX,NY,ND,NT,MSTP,NDHOW,INIT,NH,B0,FG0,FH0,    ES
           1FL0,NG)                                                            ES
124         DO 204 I=1,ND                                                      ES
131         DO 204 J=1,ND                                                      ES
132     204 FH1(I,J)=FH0(I,J)                                                  ES
150         CALL MINV (FH0,ND,D,LX,MX,N2)                                      ES
155         IF (D.NE.0.0) GO TO 501                                            ES
162         WRITE (86,10)                                                      ES
165      10 FORMAT (//4X,96HHESSIAN IS SINGULAR AT INITIAL PARAMETER VALUES.    .
           1MAKE SURE THAT THE DATA ARE PROPERLY ARRANGED.)
165         MSTP=1
167         IF (MSTP.EQ.1) GO TO 407
174         DO 205 I=1,ND                                                      ES
175         DO 205 J=1,ND                                                      ES
176         FHV(I,J)=0.0                                                       ES
202         IF (I.EQ.J) FHV(I,J)=1.0/FH1(I,J)                                  ES
212     205 CONTINUE                                                           ES
217         GO TO 206                                                          ES
217     501 DO 401 J=1,ND                                                      ES
221         DO 401 I=1,ND                                                      EST
222     401 FHV(I,J)=FH0(I,J)                                                  ES
240     206 S=0.0                                                              EST
241         ICONV=1                                                            EST
242         ITR=0                                                              EST
243         NR=0                                                               EST
244         JK=0                                                               EST
245         JS=0                                                               EST
246         JT=0                                                               EST
247         GM=0                                                               EST
250     500 ITR=ITR+1                                                          EST
252       1 FORMAT (///4X,21HSUMMARY OF ITERATIONS)                            EST
252       2 FORMAT (//4X,4X,26HCONVERGENCE ACHIEVED AFTER,I5,13H  ITERATIONS., EST
           15X,30HROOT MEAN SQUARE OF GRADIENT =,F15.7)                        EST
```

```
252          3 FORMAT (//8X,57HTERMINATION DUE TO UNDERFLOW IN CALCULATION OF LIK ESTC
              1ELIHOOD)                                                           ESTC
252          4 FORMAT (//8X,27HCONVERGENCE NOT ACHIEVED IN,I5,12H  ITERATIONS)    ESTC
252       1003 JK=0                                                               ESTC
253            MOD=2                                                              ESTC
254            CALL NLMAXH (ND,ITR,ICONV,NP,NALT,NAL,NX,NY,NT,MSTP,NDHOW,INIT,NH, ESTC
              1FL0,B0,B1,B2,BZ,FG0,FG1,FG2,FG3,FGZ,FH1,FH2,FHV,DIRS,S4S,LX,MX,     ESTC
              2JK,JS,JT,GM,NOD)                                                   ESTC
333            IF (JS.EQ.1) GO TO 610                                             ESTC
341            IF (JK.EQ.1) GO TO 610                                             ESTC
342            IF (ITR.LE.19) GO TO 614                                           ESTC
345            NOD=4                                                              ESTC
346            GO TO 610                                                          ESTC
347        614 IF (JT.NE.1) GO TO 500                                             ESTC
351            WRITE (86,1)                                                       ESTC
355        610 IF (JS.EQ.1) GO TO 611                                             ESTC
363            IF (NOD.EQ.4) GO TO 613                                            ESTC
365            WRITE (86,2) ITR,GM                                                ESTC
374            GO TO 407                                                          ESTC
400        611 WRITE (86,6)                                                       ESTC
404          6 FORMAT(//4X,43HTERMINATION DUE TO ABSENCE OF MAXIMUM POINT)        ESTC
404            MSTP=1                                                             ESTC
406            GO TO 407                                                          ESTC
412        613 WRITE (86,4) ITR                                                   ESTC
420        407 RETURN                                                             ESTC
421            END                                                                ESTC
```

```
            SUBROUTINE NLMAXH (ND,IT,ICONV,NR,NALT,NAL,NX,NY,NT,MSTP,NDHOW,
          1INIT,NH,FL0,B0,B1,B2,BZ,FG0,FG1,FG2,FG3,FGZ,FH1,FH2,FHV,DIR,S4,
          2LX,MX,JK,JS,JI,GM,NOD)
046         DIMENSION B0(10),B1(10),B2(10),BZ(10),FG0(10),FG1(10),FG2(10),
          1FG3(10),FGZ(10),FH1(10,10),FH2(10,10),FHV(10,10),DIR(10),S4(10),
          2LX(10),MX(10)
046         COMMON /AA1/ X(300,4, 4),Y(300,10),NOVA(300,4),P(300,4),
          1ANAME(300,10),JSUM(300),W4(4),JFAC(100)
046         COMMON /AA2/ AR(100),BXR(10),CYR(4,10),VECR(10),HEXR(10,10),
          1S1R(4),T1R(4),W1R(4)
046         IA=1
047         JV=0
050         DO 501 I=1,ND
051         FGZ(I)=FG0(I)
055     501 BZ(I)=B0(I)
062         FLZ=FL0
063         DO 502 I=1,ND
065         S=0.0
066         DO 503 J=1,ND
070     503 S=S+FHV(I,J)*FG0(J)
103     502 DIR(I)=S
107         NH=0
110         DO 201 I=1,ND
112     201 FG1(I)=B0(I)
120         S1R(1)=FL0
121         JJ=1
122         ST1=1.0
124     317 ST2=ST1+ST1
126         DO 202 I=1,ND
127     202 FG2(I)=FG1(I)+ST1*DIR(I)
140         NG=0
141         CALL LOGIT (NALT,NAL,NX,NY,ND,NT,MSTP,NDHOW,INIT,NH,FG2,S4,FH1,
          1FL1,NG)
164         S1R(2)=FL1
166         IF (FL1-FL0) 101,102,103
174     101 ST1=-ST1
175         DO 204 I=1,ND
177     204 FG3(I)=FG1(I)+ST1*DIR(I)
210         CALL LOGIT (NALT,NAL,NX,NY,ND,NT,MSTP,NDHOW,INIT,NH,FG3,S4,FH1,FL2
          1,NG)
233         IF (FL2.GT.FL0) GO TO 311
243         ST3=ST1
244         ST2=ST1+ST1
245         DO 312 I=1,ND
247         B0(I)=FG2(I)
253         B1(I)=FG1(I)
256     312 B2(I)=FG3(I)
264         FL3=FL2
265         FL2=FL0
267         GO TO 120
267     311 DO 313 I=1,ND
271     313 FG2(I)=FG3(I)
277         FL1=FL2
300         S1R(2)=FL2
301         ST2=ST1+ST1
302         GO TO 103
303     102 ST3=ST1/2.0
```

```
305            ST2=ST1                                                          MA
306            DO 216 I=1,ND                                                    MA
307        216 FG0(I)=FG1(I)+ST3*DIR(I)                                         MA
320            CALL LOGIT (NALT,NAL,NX,NY,ND,NT,MSTP,NDHOW,INIT,NH,FG0,S4,FH1,  MA
              1FL2,NG)                                                          MA
343            IF (FL2.GE.FL0) GO TO 217                                        MA
352            WRITE (86,10) IT,(FG0(I),I=1,ND)                                 MA
373         10 FORMAT (//4X,47HTHE FUNCTION IS NOT CONCAVE AT ITERATION NUMBER, MA
              1I5,3HAND//4X,2HB=,10F10.5)                                       MA
373            DO 218 I=1,ND                                                    MA
400        218 B0(I)=FG2(I)                                                     MA
406            GO TO 1002                                                       MA
406        217 DO 219 I=1,ND                                                    MA
410            B0(I)=FG1(I)                                                     MA
414            B1(I)=FG0(I)                                                     MA
417        219 B2(I)=FG2(I)                                                     MA
425            FL1=S1R(1)                                                       MA
426            FL3=S1R(2)                                                       MA
430            GO TO 120                                                        MA
430        103 DO 205 I=1,ND                                                    MA
432        205 FG3(I)=FG2(I)+ST2*DIR(I)                                         MA
443            CALL LOGIT (NALT,NAL,NX,NY,ND,NT,MSTP,NDHOW,INIT,NH,FG3,S4,FH1,  MA
              1FL2,NG)                                                          MA
466            S1R(4)=FL2                                                       MA
470        215 S1=S1R(2)                                                        MA
472            S2=S1R(4)                                                        MA
473            IF (S2-S1) 111,111,113                                           MA
501        111 ST3=ST2/2.0                                                      MA
503            DO 206 I=1,ND                                                    MA
505        206 FG0(I)=FG2(I)+ST3*DIR(I)                                         MA
516            CALL LOGIT (NALT,NAL,NX,NY,ND,NT,MSTP,NDHOW,INIT,NH,FG0,S4,FH1,  MA
              1FL3,NG)                                                          MA
541            S1R(3)=FL3                                                       MA
543            IF (S1R(2).LT.S1R(3)) GO TO 208                                  MA
551            DO 209 I=1,ND                                                    MA
552            B0(I)=FG1(I)                                                     MA
556            B1(I)=FG2(I)                                                     MA
561        209 B2(I)=FG0(I)                                                     MA
567            FL1=S1R(1)                                                       MA
570            FL2=S1R(2)                                                       MA
572            FL3=S1R(3)                                                       MA
573            GO TO 120                                                        MA
574        208 DO 210 I=1,ND                                                    MA
576            B0(I)=FG2(I)                                                     MA
602            B1(I)=FG0(I)                                                     MA
605        210 B2(I)=FG3(I)                                                     MA
613            FL1=S1R(2)                                                       MA
614            FL2=S1R(3)                                                       MA
616            FL3=S1R(4)                                                       MA
617            GO TO 120                                                        MA
620        113 DO 211 I=1,ND                                                    MA
622            FG1(I)=FG2(I)                                                    MA
626        211 FG2(I)=FG3(I)                                                    MA
633            S1R(1)=S1R(2)                                                    MA
634            S1R(2)=S1R(4)                                                    MA
635            FL0=S1R(1)                                                       MA
637            FL1=S1R(2)                                                       MA
640            ST2=ST2+ST2                                                      MA
```

```
42          DO 212 I=1,ND
43      212 FG3(I)=FG2(I)+ST2*DIR(I)
54          CALL LOGIT (NALT,NAL,NX,NY,ND,NT,MSTP,NDHCW,INIT,NH,FG3,S4,FH1,
           1FL2,NG)
77          S1R(4)=FL2
01          JJ=JJ+1
02          IF (JJ.LE.6) GO TO 215
10          JS=1
12          GO TO 1000
12      120 IA=IA+1
14          IF (IA.GE.3) GO TO 315
16          ST1=ST3/10.0
20          DO 316 I=1,ND
22      316 FG1(I)=B1(I)
30          FL0=FL2
31          S1R(1)=FL0
32          GO TO 317
33      315 R1=(FL1-FL2)/(-ST3)
37          R2=(FL1-FL3)/(-ST2)
41          ST5=ST3
43          WRITE (86,21) FL1,FL2,FL3,ST2,ST3
60       21 FORMAT (///4X,20HFL1,FL2,FL3,ST2,ST3=/9X,5E15.6)
60          IF (R1.NE.R2) GO TO 301
66          IF (R1.NE.0.0) GO TO 121
67          ST4=ST3*1.0E-5
71          ST3=ST3+ST4
72          DO 302 I=1,ND
74      302 FG1(I)=B1(I)+ST4*DIR(I)
05          CALL LOGIT (NALT,NAL,NX,NY,ND,NT,MSTP,NDHOW,INIT,NH,FG1,FG0,FHV,
           1S,NG)
30          R1=(FL1-S)/(-ST3)
34          R2=(FL1-FL3)/(-ST2)
36          ST5=ST2/2.0-ST4
41      301 S=0.0
42          IF (R1.EQ.R2.OR.R1.EQ.0.0) GO TO 122
54          SC1=(R1*(-ST5))/(2.0*(R1-R2))
57          DO 220 I=1,ND
61      220 B0(I)=(B0(I)+B1(I))/2.0-SC1*DIR(I)
73          GO TO 1002
73      122 DO 123 I=1,ND
75      123 B0(I)=B2(I)
03     1002 NG=1
04          NH=1
06          CALL LOGIT (NALT,NAL,NX,NY,ND,NT,MSTP,NDHOW,INIT,NH,B0,FG0,FHV,
           1FL0,NG)
31          IF (FL0.GE.FL2) GO TO 404
40      121 DO 405 I=1,ND
42      405 B0(I)=B1(I)
50          NG=1
51          NH=1
52          CALL LOGIT (NALT,NAL,NX,NY,ND,NT,MSTP,NDHCW,INIT,NH,B0,FG0,FHV,
           1FL0,NG)
75      404 S=0.0
76          DO 513 I=1,ND
03      513 S=S+FG0(I)*FG0(I)
10          S=S/ND
12          IF (S.EQ.0.0) GO TO 514
12          S=SQRT(S)
```

```
 [illegible]5   514 GM=S                                                     MA
217             S=0.0
220             DO 230 I=1,ND                                                MA
225       230 S=S+(B0(I)-BZ(I))**2                                           MA
235             S=S/ND                                                       MA
236             IF (S.EQ.0.0) GO TO 221                                      MA
237             S=SQRT(S)                                                    MA
242       221 STZ=S                                                          MA
244             BMAX=ABS(BZ(1)-B0(1))                                        MA
250             DO 231 I=1,ND                                                MA
255             S=ABS(BZ(I)-B0(I))                                           MA
262             IF (S.LE.BMAX) GO TO 231                                     MA
265             BMAX=S                                                       MA
265       231 CONTINUE                                                       MA
270             FLCH=(FLZ-FL0)*100.0/FL0                                     MA
273             DO 232 I=1,ND                                                MA
275             DO 232 J=1,ND                                                MA
276       232 FH1(I,J)=FHV(I,J)                                              MA
314             CALL MINV (FHV,ND,D,LX,MX,N2)                                MA
321             IF (D.NE.0.0) GO TO 233                                      MA
326             WRITE (86,12)                                                MA
331        12 FORMAT (//4X,90HHESSIAN IS SINGULAR. THE DIAGONAL ELEMENTS ARE USE MA
             1D FOR ITS INVERSE AT THE NEXT ITERATION.)                      MA
331             DO 234 I=1,ND                                                MA
336             DO 234 J=1,ND                                                MA
337             FHV(I,J)=0.0                                                 MA
343             IF (I.EQ.J) FHV(I,J)=1.0/FH1(I,J)                            MA
353       234 CONTINUE                                                       MA
360       233 S=0.0                                                          MA
361             IF (GM.LE.1.0E-15) GO TO 1005
364             IF (ABS((FLZ-FL0)/FL0).GT.1.0E-6) GO TO 516                  MA
372             IF (BMAX.GE.1.0E-6) GO TO 516                                MA
374      1005 NOD=3
376             JT=1                                                         MA
377       516 WRITE (86,11) IT,STZ,BMAX,GM,FL0,FLCH                          MA
417        11 FORMAT (//4X,17HITERATION NUMBER ,I3,2X,16HNEWTON-HIGA MODE/8X,  MA
             150HROOT MEAN SQUARE OF CHANGE IN PARAMETER ESTIMATES=,G15.6/8X,  ES
             219HMAXIMUM ADJUSTMENT=,G15.6/8X,29HROOT MEAN SQUARE OF GRADIENT=, MA
             3G15.6/8X,15HLOG LIKELIHOOD=,G15.8/8X,27HLOG LIKELIHOOD INCREASED B MA
             4Y,G10.3,2X,9H PERCENT.)                                        MA
417      1000 RETURN                                                         MA
420             END                                                          MA
```

APPENDIX B

GRAVITY:

Computer Program for
Market Demand Submodel

```
            PROGRAM GRAVITY (INPUT,OUTPUT)
03          DIMENSION ARRAY(250,32)
03          INTEGER ARRAY
03          DIMENSION B0(10),B1(10),B2(10),BZ(10),FG0(10),FG1(10),FG2(10),
           1FG3(10),FGZ(10),FH0(10,10),FH1(10,10),FH2(10,10),FHV(10,10),
           2DIRS(10),S4S(10),LX(10),MX(10),SE(10),TV(10),ZSCL(30)
03          DIMENSION NAME(35),LOCECS(35),NRD(14),NYON(10),NYDN(10),NZN(30),
           1NZTP(30),MTFX(5,5),NXN(5),NXM(5),MTFYO(10,5),NYOM(10),MTFYD(10,5),
           2NYDM(10),NYN(10),Z(14,14,30),NSUM(14,14),WT(14),Z1(14,14,30)
03          COMMON /AA1/ X(14,14,5),Y(14,10),P(14,14),QX(14,14),QD(14),GO(14),
           1BR(10),FHX(10,10),YO(14,10),YD(14,10)
03          COMMON /AA2/ AR(100),SIR(4),NQX(14,14),NQD(14),QXP(14,14),
           1NQXP(14,14),NQOP(14),NQDP(14),NQO(14),NQXP1(14,14),BT1(10),BT2(10)
03          COMMON /AA3/ BX(5),CY(10),XM(14,5),YM(14,10),XXM(14,5,5),XYM(14,5
           1,10),YYM(14,10,10),DOP(14,14),DOP(14),DXM(14,14,5),DYM(14,14,10),
           2DSX(14,5),DSY(14,10),NDW
03          EQUIVALENCE (ARRAY(1,1),BX(1))
03          IX=0
04          READ 1, NT,NV,NRO,ND1,NDT
22          READ 2, (NAME(I),I=1,NV)
35          IF (NDT.NE.1) GO TO 121
37          DO 101 I=1,NT
41      101 READ 3,(ARRAY(I,J),J=1,NV)
60        1 FORMAT (8I10)
60        2 FORMAT (8A10)
60        3 FORMAT (I3,I1,2I2,12I6/I3,I1,2I2,8I6,3I7,I3)
60        7 FORMAT (8F10.0)
60          NEND=0
61          PRINT 10
64          DO 102 I=1,NT
66          IF (ARRAY(I,1).NE.ARRAY(I,17)) GO TO 103
70          IF (ARRAY(I,2).GE.ARRAY(I,18)) GO TO 103
73          IF (ARRAY(I,3).NE.ARRAY(I,19)) GO TO 103
75          IF (ARRAY(I,4).NE.ARRAY(I,20)) GO TO 103
77          GO TO 102
77       10 FORMAT (*1*)
77      103 PRINT 4, I
05        4 FORMAT (//4X,12HDATA NUMBER ,I3,2X,12HDO NOT MATCH)
05          NEND=1
06      102 CONTINUE
11          IF (NEND.EQ.1) GO TO 1000
13          GO TO 122
13      121 DO 123 I=1,NT
15      123 READ 8, (ARRAY(I,J),J=1,NV)
34        8 FORMAT (3X,2I3,14I5)
34      122 N=1
35          N=1
36          DO 105 I=1,NV
40          LOCECS(I)=N
42          CALL WRITEC (ARRAY(1,I),LOCECS(I),NT)
46      105 N=N+NT
52          PRINT 5
56        5 FORMAT (///4X,44HTHE BASIC DATA HAVE BEEN SUCCESSFULLY LOADED)
56          READ 1, NR,NX,NYO,NYD,NZ,ND2,NDW,NIT
02          READ 7, (WT(I),I=1,NR)
15          READ 1, (NRD(I),I=1,NR)
30          READ 2, NDVN,(NXN(I),I=1,NX),(NYON(J),J=1,NYO),(NYDN(J),J=1,NYD)
```

```
      READ 2, (NZN(I),I=1,NZ)
      READ 1, (NZTP(I),I=1,NZ)
      READ 7, (ZSCL(I),I=1,NZ)
      DO 107 I=1,NX
  107 READ 6, (MTFX(I,J),J=1,5),NXM(I)
    6 FORMAT (5I10,A10)
      DO 109 I=1,NYO
  109 READ 6, (MTFYO(I,J),J=1,5),NYOM(I)
      DO 110 I=1,NYD
  110 READ 6, (MTFYD(I,J),J=1,5),NYDM(I)
      PRINT 11, NT,NV,NRO,ND1,NDT
   11 FORMAT (///4X,13HCONTROL CODES//9X,8X,2HNT,8X,2HNV,7X,3HNRO,7X,
     13HND1,7X,3HNDT/9X,5I10)
      PRINT 12, (NAME(I),I=1,NV)
   12 FORMAT (//4X,14HVARIABLE NAMES//9X,10A10/9X,10A10/9X,10A10/9X,
     110A10)
      IF (ND1.NE.1) GO TO 401
      PRINT 10
      MA=16
      IF (NV.LE.16) MA=NV
      PRINT 13, (I,I=1,MA)
   13 FORMAT (//4X,10HBASIC DATA//9X,2(1X,1H(,I1,1H)),14(4X,1H(,I2,1H)))
      N=0
      DO 402 I=1,NT
      N=N+1
      IF (N.NE.6) GO TO 402
      PRINT 14
   14 FORMAT (* *)
      N=1
  402 PRINT 15, I,(ARRAY(I,J),J=1,MA)
   15 FORMAT (4X,1H(,I3,1H),2I4,14I8)
      IF (NV.LE.16) GO TO 401
      PRINT 10
      PRINT 13, (I,I=17,NV)
      N=0
      DO 403 I=1,NT
      N=N+1
      IF (N.NE.6) GO TO 403
      PRINT 14
      N=1
  403 PRINT 15, I,(ARRAY(I,J),J=17,NV)
  401 PRINT 10
      IF (IX.EQ.1) PRINT 61
   61 FORMAT (4X,37HPREDICTION TEST WITH INDEPENDENT DATA//5X)
      PRINT 16, NR,NX,NYO,NYD,NZ,ND2
   16 FORMAT (4X,16HESTIMATION CODES//9X,8X,2HNR,8X,2HNX,7X,3HNYO,7X,
     13HNYD,8X,2HNZ,7X,3HND2,7X,3HNDW,7X,3HNIT,//9X,8I10)
      PRINT 17, (WT(I),I=1,NR)
   17 FORMAT (//4X,4HWT =,1X,10F10.3/9X,10F10.3)
      PRINT 18,(NRD(I),I=1,NR)
   18 FORMAT (//4X,5HNRD =,10I10/9X,10I10)
      PRINT 19, NDVN,(NXN(I),I=1,NX)
   19 FORMAT (//4X,5HNDVN=,A10,15X,5HNXN =,5A10)
      PRINT 20, (NYON(J),J=1,NYO)
   20 FORMAT (//4X,5HNYON=,10A10)
      PRINT 21, (NYDN(J),J=1,NYD)
   21 FORMAT (//4X,5HNYDN=,10A10)
      PRINT 22, (NZN(I),I=1,NZ)
```

```
   22 FORMAT (//4X,5HNZN =,10A10//9X,10A10)
      PRINT 23, (NZTP(I),I=1,NZ)
      PRINT 37, (ZSCL(I),I=1,NZ)
   37 FORMAT (//4X,5HZSCL=,10F10.4//9X,10F10.4)
   23 FORMAT (//4X,5HNZTP=, 10I10//9X,10I10)
      PRINT 24
   24 FORMAT (//4X,5X,6X,4HMTFX,40X,7X,3HNXM)
      DO 412 I=1,NX
  412 PRINT 25, I,(MTFX(I,J),J=1,5),NXM(I)
   25 FORMAT (5X,1H(,I2,1H),5I10,A10)
      PRINT 26
   26 FORMAT (//4X,10X,5HMTFYO,40X,6X,4HNYOM)
      DO 404 I=1,NYO
  404 PRINT 25, I, (MTFYO(I,J),J=1,5),NYOM(I)
      PRINT 27
   27 FORMAT ( //4X,10X,5HMTFYD,40X,6X,4HNYDM)
      DO 405 I=1,NYD
  405 PRINT 25, I,(MTFYD(I,J),J=1,5),NYDM(I)
      DO 111 I=1,NZ
      DO 112 J=1,NV
      IF (NZN(I).NE.NAME(J)) GO TO 112
      CALL READEC (ARRAY(1,I),LOCECS(J),NT)
      GO TO 111
  112 CONTINUE
      PRINT 52, I,NZN(I)
   52 FORMAT (//4X,4HNZN(,I2,2H)=,A10,2X,14HDOES NOT EXIST)
      NEND=1
  111 CONTINUE
      NRP=NR+1
      DO 113 K=1,NZ
      DO 113 I=1,NR
      DO 113 J=1,NR
  113 Z(I,J,K)=0.0
      DO 114 I=1,NR
      DO 114 J=1,NR
  114 NSUM(I,J)=0
      DO 115 I=1,NT
      DO 116 K=1,NR
      IF (ARRAY(I,1).EQ.NRD(K)) GO TO 117
  116 CONTINUE
      GO TO 115
  117 DO 118 L=1,NR
      IF (ARRAY(I,2).EQ.NRD(L)) GO TO 119
  118 CONTINUE
      GO TO 115
  119 DO 120 J=1,NZ
  120 Z(K,L,J)=ARRAY(I,J)*ZSCL(J)
  115 CONTINUE
      DO 203 I=1,NX
      DO 204 J=1,NZ
      IF (NXN(I).EQ.NZN(J)) GO TO 205
  204 CONTINUE
      PRINT 51, I, NXN(I)
   51 FORMAT (//4X,4HNXN(,I2,2H)=,A10,2X,14HDOES NOT EXIST)
      NEND=1
      GO TO 203
  205 IF (MTFX(I,3).EQ.MTFX(I,4)) GO TO 206
      NO=MTFX(I,2)
```

```
      N1=MTFX(I,3)
      N2=MTFX(I,4)
      DO 207 J=1,NR
      DO 208 K=1,NR
      S=0.0
      DO 209 L=N1,N2
  209 S=S+Z(J.K,L)
      X(J,K,I)=Z(J,K,N0)/S
      MT=MTFX(I,5)
      IF (X(J,K,I).EQ.0.0) X(J,K,I)=1.0E-10
      GO TO (211,212), MT
  211 X(J,K,I)=ALOG(X(J,K,I))
  212 NXN(I)=NXM(I)
  208 CONTINUE
  207 CONTINUE
      GO TO 203
  206 DO 213 J=1,NR
      DO 213 K=1,NR
      N0=MTFX(I,2)
      X(J,K,I)=Z(J,K,N0)
      MT=MTFX(I,5)
      IF (X(J,K,I).EQ.0.0) X(J,K,I)=1.0E-10
      GO TO (231,232), MT
  231 X(J.K,I)=ALOG(X(J,K,I))
  232 NXN(I)=NXM(I)
  213 CONTINUE
  203 CONTINUE
      DATA NMC / 10H       A/D /
      DATA NMD / 10H      COST /
      DO 710 I=1,NR
      DO 710 J=1,NR
      DO 710 K=1,NZ
  710 Z1(I,J,K)=Z(I,J,K)
      DO 711 I=1,NYO
      DO 712 J=1,NZ
      IF (NYON(I).EQ.NZN(J))  GO TO 713
  712 CONTINUE
      PRINT 53, I, NYON(I)
   53 FORMAT (//4X,5HNYON(,I2,2H)=,A10,2X,14HDOES NOT EXIST)
      NEND=1
      GO TO 711
  713 IF (NDT.NE.2) GO TO 124
      DO 125 K=1,NR
      DO 125 L=1,NR
      S=0.0
      ML=7
      IF (NXN(I).EQ.NMD) ML=8
      DO 126 M=3,ML
  126 S=S+Z(K,L,M)
      QX(K,L)=S
  125 Z(K,L,3)=S
  124 N0=MTFYO(I,2)
      N1=MTFYO(I,3)
      N2=MTFYO(I,4)
      MT=MTFYO(I,5)
      DO 714 M=N1,N2
      DO 714 K=1,NR
      S=0.0
```

```
      DO 715 L=1,NR
  715 S=S+Z(K,L,M)
  714 Z(K,1,M)=S
      DO 717 J=1,NR
      IF (N1.EQ.N2) GO TO 716
      S=0.0
      DO 718 L=N1,N2
  718 S=S+Z(J,1,L)
      YO(J,I)=Z(J,1,N0)/S
      GO TO 719
  716 YO(J,I)=Z(J,1,N0)
  719 IF (YO(J,I).EQ.0.0) YO(J,I)=1.0E-10
      GO TO (720,721),MT
  720 YO(J,I)=ALOG(YO(J,I))
  721 NYON(I)=NYOM(I)
  717 CONTINUE
      DO 722 L=1,NR
      DO 722 M=1,NR
      DO 722 N=1,NZ
  722 Z(L,M,N)=Z1(L,M,N)
  711 CONTINUE
      DO 731 I=1,NYD
      DO 732 J=1,NZ
      IF (NYDN(I).EQ.NZN(J)) GO TO 733
  732 CONTINUE
      PRINT 54, I, NYDN(I)
   54 FORMAT (//4X,5HNYDN(,I2,2H)=,A10,2X,14HDOES NOT EXIST)
      NEND=1
      GO TO 731
  733 IF (NDT.NE.2) GO TO 127
      DO 128 K=1,NR
      DO 128 L=1,NR
  128 Z(K,L,3)=QX(K,L)
  127 N0=MTFYD(I,2)
      N1=MTFYD(I,3)
      N2=MTFYD(I,4)
      MT=MTFYD(I,5)
      DO 734 M=N1,N2
      DO 734 K=1,NR
      S=0.0
      DO 735 L=1,NR
  735 S=S+Z(L,K,M)
  734 Z(1,K,M)=S
      DO 737 J=1,NR
      IF (N1.EQ.N2) GO TO 736
      S=0.0
      DO 738 L=N1,N2
  738 S=S+Z(1,J,L)
      YD(J,I)=Z(1,J,N0)/S
      GO TO 739
  736 YD(J,I)=Z(1,J,N0)
  739 IF (YD(J,I).EQ.0.0) YD(J,I)=1.0E-10
      GO TO (740,741), MT
  740 YD(J,I)=ALOG(YD(J,I))
  741 NYDN(I)=NYDM(I)
  737 CONTINUE
      DO 742 L=1,NR
      DO 742 M=1,NR
```

```
      DO 742 N=1,NZ
  742 Z(L,M,N)=Z1(L,M,N)
  731 CONTINUE
      IF (NDT.EQ.2) GO TO 129
      DO 272 I=1,NZ
      IF (NDVN.EQ.NZN(I)) GO TO 291
  272 CONTINUE
      PRINT 55, NDVN
   55 FORMAT (//4X,9HDEP VAR =,A10,2X,14HDOES NOT EXIST)
      NEND=1
      IF (NEND.EQ.1) GO TO 1000
  291 DO 273 K=1,NR
      DO 273 L=1,NR
  273 QX(K,L)=Z(K,L,I)
  129 DO 274 K=1,NR
      S=0.0
      DO 275 L=1,NR
  275 S=S+QX(K,L)
  274 QO(K)=S
      DO 276 L=1,NR
      S=0.0
      DO 277 K=1,NR
  277 S=S+QX(K,L)
  276 QD(L)=S
      IF (ND2.NE.1) GO TO 406
      PRINT 10
      DO 407 I=1,NX
      PRINT 28, I, (J,J=1,NR)
   28 FORMAT (//4X,2HX(,I1,1H)/9X,10(8X,1H(,I2,1H))/9X,10(8X,1H(,I2,1H))
     1)
      DO 408 J=1,NR
  408 PRINT 29, J,(X(J,K,I),K=1,NR)
   29 FORMAT  (/5X,1H(,I2,1H),10E12.4/9X,10E12.4)
  407 CONTINUE
      PRINT 30, (J,J=1,NR)
   30 FORMAT (///4X,5X,10(6X,3HYO(,I2,1H))/9X,10(6X,3HYO(,I2,1H))/4X)
      DO 409 I=1,NYO
  409 PRINT 29, I, (YO(J,I),J=1,NR)
      PRINT 31, (J,J=1,NR)
   31 FORMAT (///4X,5X,10(6X,3HYD(,I2,1H))/9X,10(6X,3HYD(,I2,1H))/4X)
      DO 410 I=1,NYD
  410 PRINT 29, I,(YD(J,I),J=1,NR)
  406 PRINT 10
      PRINT 32, (I,I=1,NR)
   32 FORMAT (4X,4HQX =//9X,10(8X,1H(,I2,1H))/9X,10(8X,1H(,I2,1H)))
      DO 411 I=1,NR
  411 PRINT 33, I,(QX(I,J),J=1,NR)
   33 FORMAT (/5X,1H(,I2,1H),10F12.5/9X,10F12.5)
      PRINT 34, (I,I=1,NR)
   34 FORMAT (///5X,4HQO =,10(8X,1H(,I2,1H))/9X,10(8X,1H(,I2,1H)))
      PRINT 35, (QO(I),I=1,NR)
   35 FORMAT (/9X,10F12.5/9X,10F12.5)
      PRINT 36, (I,I=1,NR)
   36 FORMAT (///5X,4HQD =,10(8X,1H(,I2,1H))/9X,10(8X,1H(,I2,1H)))
      PRINT 35, (QD(I),I=1,NR)
      KTN=0
 1001 DO 280 I=1,NX
  280 BX(I)=0.0
```

```
      MSTP=0
      IF (KTN.EQ.1) GO TO 281
      DO 282 I=1,NYO
      NYN(I)=NYON(I)
  282 CY(I)=0.0
      DO 283 J=1,NR
      DO 283 K=1,NYO
  283 Y(J,K)=YO(J,K)
      NY=NYO
      IF (IX.EQ.1) GO TO 1003
      GO TO 284
  281 DO 285 I=1,NYD
      NYN(I)=NYDN(I)
  285 CY(I)=0.0
      DO 286 J=1,NR
      S=QD(J)
      QD(J)=QO(J)
      QO(J)=S
      DO 286 K=1,NYD
  286 Y(J,K)=YD(J,K)
      NY=NYD
      DO 289 M=1,NX
      DO 701 I=1,NR
      N=I+1
      IF (N.GT.NR) GO TO 701
      DO 702 J=N,NR
      S=X(I,J,M)
      X(I,J,M)=X(J,I,M)
  702 X(J,I,M)=S
  701 CONTINUE
  289 CONTINUE
      DO 290 I=1,NR
      N=I+1
      IF (N.GT.NR) GO TO 290
      DO 703 J=N,NR
      S=QX(I,J)
      QX(I,J)=QX(J,I)
  703 QX(J,I)=S
  290 CONTINUE
      IF (IX.EQ.1) GO TO 1003
  284 NU1=1
  501 CALL ESTIM (NX,NY,ND,MSTP,NOD,INIT,B0,B1,B2,BZ,FG0,FG1,FG2,FG3,
     1FGZ,FH0,FH1,FH2,FHV,DIRS,S4S,LX,MX,FL0,WT,NXN,NYN,NR,NU1,NU2)
      IF (MSTP.EQ.1) GO TO 1000
      N2=ND*ND
 1003 CALL RESULT (NX,NY,ND,MSTP,NOD,INIT,N2,B0,B1,FG0,FG1,FH1,FH2,FHV,
     1LX,MX,SE,TV,FL0,KTN,WT,NXN,NYN,NR,NU1,NU2,IX)
      IF (MSTP.EQ.1) GO TO 1000
      IF (KTN.EQ.0) GO TO 1002
      IF (NRO.NE.1) GO TO 1000
      IF (NIT.EQ.0) GO TO 1000
      IF (IX.EQ.1) GO TO 1000
      READ 1, NP,(NRD(I),I=1,NR)
      READ 2, NDVN,(NXN(I),I=1,NX),(NYON(I),I=1,NYO),(NYDN(I),I=1,NYD)
      IX=1
      GO TO 401
 1002 KTN=1
      GO TO 1001
```

```
 1000 STOP                                                          GRA
      END                                                           GRA
```

```
            SUBROUTINE ESTIM (NX,NY,ND,MSTP,NOD,INIT,B0,B1,B2,BZ,FG0,FG1,FG2,  ETM00
           1FG3,FGZ,FH0,FH1,FH2,FHV,DIRS,S4S,LX,MX,FL0,WT,NXN,NYN,NR,NU1,NU2)  ETM00
 1          DIMENSION B0(10),B1(10),B2(10),BZ(10),FG0(10),FG1(10),FG2(10),     ETM00
           1FG3(10),FGZ(10),FH0(10,10),FH1(10,10),FH2(10,10),FHV(10,10),       ETM00
           2DIRS(10),S4S(10),LX(10),MX(10),WT(14),NXN(5),NYN(10)               ETM00
 1          COMMON /AA1/ X(14,14,5),Y(14,10),P(14,14),QX(14,14),QD(14),QO(14), GRA01
           1BB(10),FHX(10,10),YO(14,10),YD(14,10)
 1          COMMON /AA2/ AR(100),SIR(4),NQX(14,14),NQD(14),QXP(14,14),         GRA01
           1NQXP(14,14),NQOP(14),NQDP(14),NQO(14),NQXP1(14,14),BT1(10),BT2(10) GRA01
 1          COMMON /AA3/ BX(5),CY(10),XM(14,5),YM(14,10),XXM(14,5,5),XYM(14,5  GRA01
           1,10),YYM(14,10,10),DQP(14,14),DOP(14),DXM(14,14,5),DYM(14,14,10),  GRA01
           2DSX(14,5),DSY(14,10),NDW                                           GRA01
 1          PRINT 10
 4       10 FORMAT (*1*,4X,17HLOG OF ITERATIONS)
 4          IB=0
 5          NQ=0                                                               ETM01
 6          N=0                                                                ETM01
 7          DO 287 I=1,NX                                                      ETM01
 4          N=N+1                                                              ETM01
 6      287 B0(N)=BX(I)                                                        ETM01
 2          DO 288 I=1,NY                                                      ETM01
 4          N=N+1                                                              ETM01
 6      288 B0(N)=CY(I)                                                        ETM01
 2          INIT=0                                                             ETM02
 3      501 NQ=0                                                               ETM02
 4          ND=NX+NY                                                           ETM02
 5          N2=ND*ND                                                           ETM02
 7          NG=1                                                               ETM02
 0          NH=1                                                               ETM02
 1          NAP=0
 2          CALL GRVMOD (INIT,NG,NH,NX,NY,ND,NR,B0,FL0,FG0,FH0,WT,NU1,NU2)     ETM02
 4          FSUM=FL0
 6          IC=0
 7          DO 301 I=4,NR
 4          DO 301 J=1,NR
 5          IC=IC+1
 7          IF (IC.GT.ND) GO TO 300
 2      301 X(J,I,5)=B0(IC)
 3      300 S=0.0
      C     PUT NAP=1 HERE.
 4          IF (NAP.NE.1) GO TO 201
 6          DO 202 I=1,ND
 0          DO 202 J=1,ND
 1          IF (I.NE.J) FH0(I,J)=0.0
 6          IF (I.EQ.J) FH0(I,J)=1.0/FH0(I,J)
 4      202 CONTINUE
 1          GO TO 291
 1      201 S=0.0
 2          CALL MINV (FH0,ND,D,LX,MX,N2)                                      ETM02
 7          IF (D.NE.0) GO TO 291                                              ETM02
 4          PRINT 8                                                            ETM02
 7        8 FORMAT (//4X,44HHESSIAN SINGULAR AT INITIAL PARAMETER VALUES)      ETM02
 7      291 DO 401 J=1,ND                                                      ETM03
 4          DO 401 I=1,ND                                                      ETM03
 5      401 FHV(I,J)=FH0(I,J)                                                  ETM03
 3          MOD=1                                                              EST01
 4          MOD=2
```

```
C       PUT *MOD=2* HERE.
        INIT=1
        ICONV=1
        ITR=0
        NQ=0
        JK=0
        JS=0
        JT=0
        GM=0.0
        NU2=0
        FLMX=FLO
   500  ITR=ITR+1
        IF (FLMX.EQ.FLO) GO TO 121
        NU2=0
        FLMX=FLO
        GO TO 122
   121  NU2=NU2+1
        IF (NU2.LE.3) GO TO 122
        PRINT 4, ITR
        NOD=4
        GO TO 407
   122  S=0.0
     1  FORMAT (///4X,21HSUMMARY OF ITERATIONS)
     2  FORMAT (//4X,4X,26HCONVERGENCE ACHIEVED AFTER,I5,13H  ITERATIONS.,
       15X,30HROOT MEAN SQUARE OF GRADIENT =,F15.7)
     3  FORMAT (//8X,57HTERMINATION DUE TO UNDERFLOW IN CALCULATION OF LIK
       1ELIHOOD)
     4  FORMAT (//8X,27HCONVERGENCE NOT ACHIEVED IN,I5,12H  ITERATIONS)
  1003  JK=0
        MOD=2
        CALL NLMAXH (ND,ITR,ICONV,NR,NX,NY,MSTP,INIT,NH,FLO,B0,B1,B2,BZ,
       1FG0,FG1,FG2,FG3,FGZ,FH1,FH2,FHV,DIRS,S4S,LX,MX,JK,JS,JT,GM,NOD,WT,
       2NAP,NU1,NU2)
        IF (FLO.LE.FSUM) GO TO 304
        IC=0
        DO 305 I=4,NR
        DO 305 J=1,NR
        IC=IC+1
        IF (IC.GT.ND) GO TO 304
   305  X(J,I,5)=B0(IC)
   304  S=0.0
        IF (JS.EQ.1) GO TO 610
        IF (JK.EQ.1) GO TO 610
        IF (ITR.LE.19) GO TO 614
C       PUT *IF (ITR.LE.**) GO TO 614* HERE.
        NOD=4
        GO TO 610
   614  IF (JT.NE.1) GO TO 500
        PRINT 1
   610  IF (JS.EQ.1) GO TO 611
        IF (JK.EQ.1) GO TO 612
        IF (NOD.EQ.4) GO TO 613
        PRINT 2, ITR,GM
        GO TO 407
   611  PRINT 6
     6  FORMAT(//4X,43HTERMINATION DUE TO ABSENCE OF MAXIMUM POINT)
        MSTP=1
        GO TO 407
```

```
  612 PRINT 5, ITR                                                      EST04
    5 FORMAT (//4X,69HTERMINATION DUE TO SINGULARITY OF HESSIAN MATRIX A EST05
     1T ITERATION NUMBER, I5)                                           EST051
      MSTP=1                                                            EST05
      GO TO 407                                                         EST05
  613 PRINT 4, ITR                                                      EST05
  407 RETURN                                                            EST05
      END                                                               EST05
```

```
            SUBROUTINE GRVMOD (INIT,NG,NH,NX,NY,ND,NR,A,SUM,FG,FH,WT,NU1,NU2)
21          DIMENSION A(10),FG(10),FH(10,10),WT(14)
21          COMMON /AA1/ X(14,14,5),Y(14,10),P(14,14),OX(14,14),OD(14),OO(14),
           1RP(10),FHX(10,10),YO(14,10),YD(14,10)
21          COMMON /AA3/ BX(5),CY(10),XM(14,5),YM(14,10),XXM(14,5,5),XYM(14,5
           1,10),YYM(14,10,10),DQP(14,14),DOP(14),DXM(14,14,5),DYM(14,14,10),
           2DSX(14,5),DSY(14,10),NDW
21          IF (INIT.EQ.0) GO TO 251
22          N=0
23          DO 190 I=1,NX
24          N=N+1
26      190 BX(I)=A(N)
33          DO 191 I=1,NY
34          N=N+1
36      191 CY(I)=A(N)
43      251 DO 201 I=1,NR
45          DO 202 J=1,NR
46          S=0.0
47          DO 203 K=1,NX
51      203 S=S+X(I,J,K)*BX(K)
64          T=0.0
65          DO 204 K=1,NY
66      204 T=T+Y(I,K)*CY(K)
77          P(I,J)=S+T
04      202 CONTINUE
06      201 CONTINUE
10          PMAX=P(1,1)
11          PMIN=P(1,1)
12          DO 101 I=1,NR
14          DO 101 J=1,NR
15          IF (P(I,J).LE.PMAX) GO TO 101
22          PMAX=P(I,J)
24      101 CONTINUE
31          DO 102 I=1,NR
32          DO 102 J=1,NR
33          IF (P(I,J).GE.PMIN) GO TO 102
40          PMIN=P(I,J)
42      102 CONTINUE
47          PU=ABS(PMAX)
51          PL=ABS(PMIN)
53          T=-PMIN
54          IF (PU.GE.PL) T=-PMAX
57          DO 103 I=1,NR
61          DO 103 J=1,NR
62          S=P(I,J)+T
66      103 P(I,J)=EXP(S)
05          DO 205 J=1,NR
06          S=0.0
07          DO 206 I=1,NR
11      206 S=S+P(I,J)
21          DO 207 I=1,NR
22      207 P(I,J)=P(I,J)/S
32      205 CONTINUE
34          DO 208 K=1,NX
35          DO 209 J=1,NR
36          S=0.0
37          DO 210 L=1,NR
```

```
  210 S=S+P(L,J)*X(L,J,K)
  209 XM(J,K)=S
  208 CONTINUE
      DO 211 K=1,NY
      DO 211 J=1,NR
      S=0.0
      DO 212 L=1,NR
  212 S=S+P(L,J)*Y(L,K)
  211 YM(J,K)=S
      DO 213 K=1,NX
      DO 213 L=1,NX
      DO 213 J=1,NR
      S=0.0
      DO 214 I=1,NR
  214 S=S+P(I,J)*X(I,J,K)*X(I,J,L)
  213 XXM(J,K,L)=S
      DO 215 K=1,NX
      DO 215 L=1,NY
      DO 215 J=1,NR
      S=0.0
      DO 216 I=1,NR
  216 S=S+P(I,J)*X(I,J,K)*Y(I,L)
  215 XYM(J,K,L)=S
      DO 217 K=1,NY
      DO 217 L=1,NY
      DO 217 J=1,NR
      S=0.0
      DO 218 I=1,NR
  218 S=S+P(I,J)*Y(I,K)*Y(I,L)
  217 YYM(J,K,L)=S
      DO 219 I=1,NR
      DO 219 J=1,NR
  219 DQP(I,J)=QX(I,J)/QD(J)-P(I,J)
      DO 220 I=1,NR
      S=0.0
      DO 221 J=1,NR
  221 S=S+P(I,J)*QD(J)
  220 DOP(I)=QO(I)-S
      GO TO (1001,1002,1003), NU1
 1001 S=0.0
      DO 301 I=1,NR
      DO 301 J=1,NR
      T=100.0/QX(I,J)
      IF (QX(I,J).LE.10.0) T=100.0
      IF (NDW.EQ.0) T=1.0
  301 S=S+(DQP(I,J)**2)*T
      SUM=-S
      IF (NG.EQ.0) GO TO 245
      DO 302 K=1,NX
      DO 302 J=1,NR
      DO 302 I=1,NR
  302 DXM(I,J,K)=X(I,J,K)-XM(J,K)
      DO 303 K=1,NY
      DO 303 I=1,NR
      DO 303 J=1,NR
  303 DYM(I,J,K)=Y(I,K)-YM(J,K)
      DO 304 K=1,NX
      S=0.0
```

```
71          DO 305 I=1,NR
73          DO 305 J=1,NR
74          T=100.0/QX(I,J)
00          IF (QX(I,J).LE.10.0) T=100.0
07          IF (NDW.EQ.0) T=1.0
12      305 S=S+DQP(I,J)*P(I,J)*DXM(I,J,K)*T
34      304 FG(K)=2.0*S
41          DO 306 K=1,NY
43          S=0.0
44          DO 307 I=1,NR
46          DO 307 J=1,NR
47          T=100.0/QX(I,J)
53          IF (QX(I,J).LE.10.0) T=100.0
62          IF (NDW.EQ.0) T=1.0
65      307 S=S+DQP(I,J)*P(I,J)*DYM(I,J,K)*T
07      306 FG(NX+K)=2.0*S
15          IF (NH.EQ.0) GO TO 245
16          DO 308 K=1,NX
17          DO 308 L=1,NX
20          S1=0.0
21          S2=0.0
22          DO 309 I=1,NR
23          DO 309 J=1,NR
24          T=100.0/QX(I,J)
30          IF (QX(I,J).LE.10.0) T=100.0
37          IF (NDW.EQ.0) T=1.0
42          S1=S1+(P(I,J)**2)*DXM(I,J,K)*DXM(I,J,L)*T
60      309 S2=S2+DQP(I,J)*P(I,J)*(DXM(I,J,K)*DXM(I,J,L)+XM(J,K)*XM(J,L)-
           1XXM(J,K,L))*T
23      308 FH(K,L)=-2.0*(S1-S2)
35          DO 310 K=1,NX
36          DO 310 L=1,NY
37          S1=0.0
40          S2=0.0
41          DO 311 J=1,NR
42          DO 311 I=1,NR
43          T=100.0/QX(I,J)
47          IF (QX(I,J).LE.10.0) T=100.0
55          IF (NDW.EQ.0) T=1.0
60          S1=S1+(P(I,J)**2)*DXM(I,J,K)*DYM(I,J,L)*T
76      311 S2=S2+DQP(I,J)*P(I,J)*(DXM(I,J,K)*DYM(I,J,L)+XM(J,K)*YM(J,L)-
           1XYM(J,K,L))*T
41      310 FH(K,NX+L)=-2.0*(S1-S2)
55          DO 312 I=1,NX
56          DO 312 J=1,NY
57      312 FH(NX+J,I)=FH(I,NX+J)
76          DO 313 K=1,NY
77          DO 313 L=1,NY
00          S1=0.0
01          S2=0.0
02          DO 314 J=1,NR
03          DO 314 I=1,NR
04          T=100.0/QX(I,J)
10          IF (QX(I,J).LE.10.0) T=100.0
16          IF (NDW.EQ.0) T=1.0
21          S1=S1+(P(I,J)**2)*DYM(I,J,K)*DYM(I,J,L)*T
37      314 S2=S2+DQP(I,J)*P(I,J)*(DYM(I,J,K)*DYM(I,J,L)+YM(J,K)*YM(J,L)-
           1YYM(J,K,L))*T
```

```
  313 FH(NX+K,NX+L)=-2.0*(S1-S2)
      GO TO 245
 1002 S=0.0
      T=0.0
      DO 252 I=1,NR
      S=S+(DOP(I)**2)*WT(I)
      DO 252 J=1,NR
  252 T=T+DQP(I,J)**2
      SUM=-(T+S)
      IF (NG.EQ.0) GO TO 245
      DO 222 K=1,NX
      DO 222 J=1,NR
      DO 222 I=1,NR
  222 DXM(I,J,K)=X(I,J,K)-XM(J,K)
      DO 223 K=1,NY
      DO 223 I=1,NR
      DO 223 J=1,NR
  223 DYM(I,J,K)=Y(I,K)-YM(J,K)
      DO 224 K=1,NX
      S=0.0
      DO 225 I=1,NR
      DO 225 J=1,NR
  225 S=S+DQP(I,J)*P(I,J)*DXM(I,J,K)
      T=0.0
      DO 226 I=1,NR
      U=0.0
      DO 227 J=1,NR
  227 U=U+QD(J)*P(I,J)*DXM(I,J,K)
      DSX(I,K)=U
  226 T=T+WT(I)*DOP(I)*U
  224 FG(K)=2.0*(S+T)
      DO 228 K=1,NY
      S=0.0
      DO 229 I=1,NR
      DO 229 J=1,NR
  229 S=S+DQP(I,J)*P(I,J)*DYM(I,J,K)
      T=0.0
      DO 230 I=1,NR
      U=0.0
      DO 231 J=1,NR
  231 U=U+QD(J)*P(I,J)*DYM(I,J,K)
      DSY(I,K)=U
  230 T=T+WT(I)*DOP(I)*U
  228 FG(NX+K)=2.0*(S+T)
      NA=1
      IF (NH.EQ.0) GO TO 245
      DO 232 K=1,NX
      DO 232 L=1,NX
      S1=0.0
      S2=0.0
      DO 233 I=1,NR
      DO 233 J=1,NR
      S1=S1+(P(I,J)**2)*DXM(I,J,K)*DXM(I,J,L)
  233 S2=S2+DOP(I,J)*P(I,J)*(DXM(I,J,K)*DXM(I,J,L)+XM(J,K)*XM(J,L)-
     1XXM(J,K,L))
      S1=2.0*(S1-S2)
      S3=0.0
      S4=0.0
```

```
      DO 234 I=1,NR
      S3=S3+WT(I)*DSX(I,K)*DSX(I,L)
      S5=0.0
      DO 235 J=1,NR
  235 S5=S5+QD(J)*(P(I,J)*DXM(I,J,K)*DXM(I,J,L)+P(I,J)*(XM(J,K)*XM(J,L)-
     1XXM(J,K,L)))
  234 S4=S4+WT(I)*DOP(I)*S5
  232 FH(K,L)=-S1-2.0*(S3-S4)
      DO 236 K=1,NX
      DO 236 L=1,NY
      S1=0.0
      S2=0.0
      DO 237 J=1,NR
      DO 237 I=1,NR
      S1=S1+(P(I,J)**2)*DXM(I,J,K)*DYM(I,J,L)
  237 S2=S2+DQP(I,J)*P(I,J)*(DXM(I,J,K)*DYM(I,J,L)+XM(J,K)*YM(J,L)-
     1XYM(J,K,L))
      S3=0.0
      S4=0.0
      DO 238 I=1,NR
      S3=S3+WT(I)*DSX(I,K)*DSY(I,L)
      S5=0.0
      DO 239 J=1,NR
  239 S5=S5+QD(J)*P(I,J)*(DXM(I,J,K)*DYM(I,J,L)+XM(J,K)*YM(J,L)-XYM(J,K,
     1L))
  238 S4=S4+WT(I)*DOP(I)*S5
  236 FH(K,NX+L)=-2.0*(S1-S2+S3-S4)
      DO 240 I=1,NX
      DO 240 J=1,NY
  240 FH(NX+J,I)=FH(I,NX+J)
      DO 241 K=1,NY
      DO 241 L=1,NY
      S1=0.0
      S2=0.0
      DO 242 J=1,NR
      DO 242 I=1,NR
      S1=S1+(P(I,J)**2)*DYM(I,J,K)*DYM(I,J,L)
  242 S2=S2+DQP(I,J)*P(I,J)*(DYM(I,J,K)*DYM(I,J,L)+YM(J,K)*YM(J,L)-YYM(J
     1,K,L))
      S3=0.0
      S4=0.0
      DO 243 I=1,NR
      S3=S3+WT(I)*DSY(I,K)*DSY(I,L)
      S5=0.0
      DO 244 J=1,NR
  244 S5=S5+QD(J)*P(I,J)*(DYM(I,J,K)*DYM(I,J,L)+YM(J,K)*YM(J,L)-YYM(J,K,
     1L))
  243 S4=S4+WT(I)*DOP(I)*S5
  241 FH(NX+K,NX+L)=-2.0*(S1-S2+S3-S4)
      GO TO 245
 1003 S=0.0
  245 RETURN
      END
```

```
      SUBROUTINE MINV (AH,N,D,L,M,N2)
      DIMENSION AH(10,10),L(10),M(10)
      COMMON /AA2/ A(100)
      K=0
      DO 201 I=1,N
      DO 201 J=1,N
      K=K+1
  201 A(K)=AH(J,I)
      D=1.0
      NK=-N
      DO 80 K=1,N
      NK=NK+N
      L(K)=K
      M(K)=K
      KK=NK+K
      BIGA=A(KK)
      DO 20 J=K,N
      IZ=N*(J-1)
      DO 20 I=K,N
      IJ=IZ+I
      S=ABS(A(IJ))
      T=ABS(BIGA)
   10 IF (T-S) 15,20,20
   15 BIGA=A(IJ)
      L(K)=I
      M(K)=J
   20 CONTINUE
      J=L(K)
      IF (J-K) 35,35,25
   25 KI=K-N
      DO 30 I=1,N
      KI=KI+N
      HOLD=-A(KI)
      JI=KI-K+J
      A(KI)=A(JI)
   30 A(JI)=HOLD
   35 I=M(K)
      IF (I-K) 45,45,38
   38 JP=N*(I-1)
      DO 40 J=1,N
      JK=NK+J
      JI=JP+J
      HOLD=-A(JK)
      A(JK)=A(JI)
   40 A(JI)=HOLD
      T=ABS(BIGA)
   45 IF (T-1.0E-20) 46,46,48
   46 D=0.0
      RETURN
   48 DO 55 I=1,N
      IF (I-K) 50,55,50
   50 IK=NK+I
      A(IK)=A(IK)/(-BIGA)
   55 CONTINUE
      DO 65 I=1,N
      IK=NK+I
      HOLD = A(IK)
```

```
66         IJ=I-N                                          INV0
67         DO 65 J=1,N                                     INV0
71         IJ =IJ+N                                        INV0
73         IF (I-K) 60,65,60                               INV0
75     60  IF (J-K) 62,65,62                               INV0
77     62  KJ=IJ-I+K                                       INV0
02         A(IJ)=HOLD*A(KJ)+A(IJ)                          INV0
05     65  CONTINUE                                        INV0
12         KJ=K-N                                          INV0
13         DO 75 J=1,N                                     INV0
15         KJ=KJ+N                                         INV0
17         IF (J-K) 70,75,70                               INV0
20     70  A(KJ)=A(KJ)/BIGA                                INV0
23     75  CONTINUE                                        INV0
26         D=D*BIGA                                        INV
27         A(KK)=1.0/BIGA                                  INV0
31     80  CONTINUE                                        INV0
33         K=N                                             INV0
34    100  K=(K-1)                                         INV0
36         IF (K) 150,150,105                              INV0
37    105  I=L(K)                                          INV0
41         IF (I-K) 120,120,108                            INV0
43    108  JQ=N*(K-1)                                      INV0
46         JR=N*(I-1)                                      INV0
52         DO 110 J=1,N                                    INV0
53         JK=JQ+J                                         INV
55         HOLD=A(JK)                                      INV
57         JI=JR+J                                         INV
60         A(JK)=-A(JI)                                    INV
62    110  A(JI)=HOLD                                      INV
66    120  J=M(K)                                          IN
70         IF (J-K) 100,100,125                            IN
72    125  KI=K-N                                          IN
74         DO 130 I=1,N                                    IN
75         KI=KI+N                                         IN
77         HOLD = A(KI)                                    IN
00         JI=KI-K+J                                       IN
02         A(KI)=-A(JI)                                    IN
04    130  A(JI)=HOLD                                      IN
10         GO TO 100                                       IN
10    150  K=0                                             IN
11         DO 202 I=1,N                                    INV
13         DO 202 J=1,N                                    IN
14         K=K+1                                           INV
16    202  AH(J,I)=A(K)                                    INV
26         RETURN                                          INV
26         END                                             INV
```

```
      SUBROUTINE RESULT (NX,NY,ND,MSTP,NOD,INIT,N2,B0,B1,FG0,FG1,FH1,FH2 RLT00
     1,FHV,LX,MX,SE,TV,FL0,KTN,WT,NXN,NYN,NR,NU1,NU2,IX)                RLT002
      DIMENSION B0(10),B1(10),FG0(10),FG1(10),FH1(10,10),FH2(10,10),     RLT00
     1FHV(10,10),LX(10),MX(10),SE(10),TV(10),WT(14),NXN(5),NYN(10)      RLT004
      COMMON /AA1/ X(14,14,5),Y(14,10),P(14,14),QX(14,14),QD(14),QQ(14), GRA01
     1BR(10),FHX(10,10),YO(14,10),YD(14,10)
      COMMON /AA2/ AR(100),STR(4),NQX(14,14),NQD(14),QXP(14,14),         GRA01
     1NQXP(14,14),NQOP(14),NQDP(14),NQQ(14),NQXP1(14,14),BT1(10),BT2(10) GRA01
      COMMON /AA3/ BX(5),CY(10),XM(14,5),YM(14,10),XXM(14,5,5),XYM(14,5  GRA014
     1,10),YYM(14,10,10),DQP(14,14),DQP(14),DXM(14,14,5),DYM(14,14,10),  GRA01
     2DSX(14,5),DSY(14,10),NDW                                          GRA01
      IF (NX.EQ.0) GO TO 103
      N=0                                                               RLT01
      DO 606 I=1,NX                                                     RLT01
      N=N+1                                                             RLT02
  606 BX(I)=B0(N)                                                       RLT021
  103 IF (NY.EQ.0) GO TO 104
      DO 607 I=1,NY                                                     RLT02
      N=N+1                                                             RLT027
  607 CY(I)=B0(N)                                                       RLT02
  104 IF (NOD.EQ.3.OR.NOD.EQ.4) GO TO 601                               RLT011
      PRINT 1                                                           RLT01
    1 FORMAT (///4X,24HCONVERGENCE NOT ACHIEVED)                        RLT01
  602 PRINT 2                                                           RLT01
    2 FORMAT (//4X,59HTHE FOLLOWING ARE THE ESTIMATES OBTAINED BEFORE TE RLT01
     1RMINATION/4X)                                                     RLT01
      IF (NX.EQ.0) GO TO 603                                            RLT017
      PRINT 4, ((I,BX(I)),I=1,NX)                                       RLT02
    4 FORMAT (/9X,7(5X,2HB(,I2,2H)=,F6.3)//9X,7(5X,2HB(,I2,2H)=,F6.3)/  RLT02
     14X)                                                               RLT024
  603 IF (NY.EQ.0) GO TO 604                                            RLT02
      PRINT 5, ((I,CY(I)),I=1,NY)                                       RLT02
    5 FORMAT (/9X,7(5X,2HC(,I2,2H)=,F6.3)//9X,7(5X,2HC(,I2,2H)=,F6.3)/  RLT03
     14X)                                                               RLT031
  604 GO TO 1000                                                        RLT03
  601 NH=1                                                              RLT03
      NG=1                                                              RLT034
      D=1.0                                                             RLT034
      N2=ND*ND                                                          RLT034
      INIT=1                                                            RLT034
      IF (IX.EQ.1) GO TO 310                                            RLT034
      DO 119 I=1,ND                                                     RLT034
      IF (KTN.EQ.0) GO TO 120                                           RLT034
      BT2(I)=B0(I)                                                      RLT034
      GO TO 119                                                         RLT034
  120 BT1(I)=B0(I)                                                      RLT034
  119 CONTINUE                                                          RLT034
  310 IF (IX.NE.1) GO TO 121                                            RLT034
      DO 122 I=1,ND                                                     RLT034
      IF (KTN.EQ.0) GO TO 123                                           RLT034
      B0(I)=BT2(I)                                                      RLT034
      GO TO 122                                                         RLT034
  123 B0(I)=BT1(I)                                                      RLT034
  122 CONTINUE                                                          RLT034
  121 S=0.0                                                             RLT034
      IF (IX.EQ.1)  INIT=2
      CALL GRVMOD (INIT,NG,NH,   NX,NY,ND,NR,B0,FL0,FG0,FH2,WT,NU1,NU2) RLT03
```

```
53        DO 507 I=1,ND                                                    RLT0
60        DO 507 J=1,ND                                                    RLT0
61    507 FHV(I,J)=-FH2(I,J)                                               RLT0
77        CALL MINV (FHV,ND,D,LX,MX,N2)                                    RLT0
04        MM=0
05        KO=0                                                             RLT04
06        IF (D.NE.0.0) GO TO 608                                          RLT04
12        PRINT 6                                                          RLT04
16      6 FORMAT (8X,77HHESSIAN SINGULAR AT CONVERGENCE, INDICATING THAT THE RLT04
         1 CONSTRAINTS ARE JUST MET)                                       RLT04
16        IF (D.EQ.0.0) MM=1
24        IF (MM.EQ.1) GO TO 608
26        GO TO 602                                                        RLT04
27    608 N1=0                                                             RLT04
30        S1=0.0                                                           RLT0
31        T=0,0                                                            RLT0
32        V=0.0                                                            RLT0
33        DO 609 I=1,NR                                                    RLT0
34        N=0                                                              RLT0
35        DO 610 J=1,NR                                                    RLT04
37        NQX(J,I)=QX(J,I)+0.5                                             RLT0
46    610 N=N+NQX(J,I)                                                     RLT0
52        NQD(I)=N                                                         RLT0
54        DO 611 J=1,NR                                                    RLT0
55        S1=S1+(P(J,I)*NQD(I)-QX(J,I))                                    RES3
66        V=V+(QX(J,I)-P(J,I)*NQD(I))**2                                   RLT0
76    611 T=T+(QX(J,I)-P(J,I)*NQD(I))**2/(P(J,I)*NQD(I))                   RLT0
14        N1=N1+NQD(I)                                                     RLT0
16    609 CONTINUE                                                         RLT0
20        N=N-2*NR                                                         RLT0
22        N1=0                                                             RLT0
22        S=0.0                                                            RLT0
23        DO 612 I=1,NR                                                    RLT0
24        N1=N1+NQD(I)                                                     RLT0
26        DO 618 J=1,NR                                                    RLT0
30        N2=P(J,I)*NQD(I)+0.5                                             RLT0
37        T1=NQX(J,I)-N2                                                   RLT0
44        IF (T1.LT.0.0) GO TO 618                                         RLT0
45        S=S+T1                                                           RLT0
47    618 CONTINUE                                                         RLT0
52    612 CONTINUE                                                         RLT0
54        IF (KO.EQ.1) GO TO 621                                           RLT0
56        PCP=S/N1*100.0                                                   RLT0
61        FRM=S1/(NR*NR)                                                   RLT0
65        RM2R=V/(NR*NR)                                                   RLT0
71        RM2R=SQRT(RM2R)                                                  RLT0
74        SRX2=T                                                           RLT0
75        U=(NR-1)*(NR-1)                                                  RLT0
01        DF=U-ND                                                          RLT0
07        CF=U/DF                                                          RLT0
10        SR2=V                                                            RLT0
12        DO 701 J=1,NR                                                    RLT0
13        DO 701 I=1,NR                                                    RLT0
14        QXP(I,J)=P(I,J)*NQD(J)                                           RLT0
23    701 NQXP(I,J)=QXP(I,J)+0.5                                           RLT0
33        DO 702 I=1,NR                                                    RLT0
34        N1=0                                                             RLT0
35        N2=0                                                             RLT0
```

```
      N3=0
      DO 703 J=1,NR
      N1=N1+NQXP(I,J)
      N2=N2+NQXP(J,I)
  703 N3=N3+NQX(I,J)
      NQOP(I)=N1
      NQDP(I)=N2
  702 NQD(I)=N3
      S1=0.0
      S2=0.0
      S3=0.0
      DO 201 J=1,NR
      T=NQD(J)-NQX(J,J)
      DO 202 I=1,NR
      IF (I.EQ.J) GO TO 202
      S1=S1+(NQXP(I,J)-NQX(I,J))
      S2=S2+(NQXP(I,J)-NQX(I,J))**2
      S3=S3+(T/(NR-1)-NQX(I,J))**2
  202 CONTINUE
  201 CONTINUE
      TRM=S1/(NR*NR-NR)
      S=S2/(NR*NR-NR)
      TRM2=SQRT(S)
      S=S3/(NR*NR-NR)
      TRM20=SQRT(S)
      TR2=1.0-S2/S3
      GO TO 622
  621 SRX20=T
      FRM0=S1/(NR*NR)
      RM2R0=V/(NR*NR)
      RM2R0=SQRT(RM2R0)
      SR20=V
      IF (KTN.EQ.0) SR21=V
      PCP0=S/N1*100.0
  622 IF (KO.EQ.1) GO TO 623
      IF (MM.EQ.1) GO TO 301
      GO TO 624
  623 IF (MM.EQ.1) GO TO 626
      DO 625 I=1,ND
      B1(I)=0.0
      DO 625 J=1,ND
  625 B1(I)=B1(I)+FH1(I,J)*FG1(J)
      GO TO 626
  624 DO 619 I=1,ND
      S=FHV(I,I)*CF
      IF (S.LT.0.0) S=-S
      SE(I)=SQRT(S)
  619 TV(I)=B0(I)/SE(I)
  301 S=0.0
      DO 620 I=1,ND
  620 B1(I)=0.0
      NH=1
      NG=1
      CALL GRVMOD (INIT,NG,NH,NX,NY,ND,NR,B1,FL1,FG1,FH1,WT,NU1,NU2)
      CALL MINV (FH1,ND,D,LX,MX,N2)
      KO=1
      GO TO 608
  626 PRINT 7
```

```
53     7 FORMAT (*1*,3X,18HESTIMATION RESULTS//4X,19HPARAMETER ESTIMATES//
        111X,8HVARIABLE,7X,8HVARIABLE,10X,5HLOGIT,7X,8HSTANDARD,13X,2HT-/
        213X,6HNUMBER,11X,4HNAME,6X,9HESTIMATOR,10X,5HERROR,6X,9HSTATISTIC/
        34X)
53       N=0
54       IF (NX.EQ.0) GO TO 627
61       IF (MM.NE.1) GO TO 302
63       DO 303 I=1,NX
64       N=N+1
66   303 PRINT 8, I,NXN(I),B0(N)
07       GO TO 306
07   302 S=0.0
10       DO 628 I=1,NX
12       N=N+1
14   628 PRINT 8,I,NXN(I),B0(N),SE(N),TV(N)
43     8 FORMAT (14X,2HB(,I2,1H),5X,A10,4F15.5)
43   306 S=0.0
44       PRINT 9
50     9 FORMAT (* *)
50   627 IF (NY.EQ.0) GO TO 629
55       IF (MM.NE.1) GO TO 304
57       DO 305 I=1,NY
60       N=N+1
62   305 PRINT 10, I,NYN(I),B0(N)
03       GO TO 629
03   304 S=0.0
04       DO 630 I=1,NY
06       N=N+1
10   630 PRINT 10, I,NYN(I),B0(N),SE(N),TV(N)
37    10 FORMAT (11X,3X,2HC(,I2,1H),5X,A10,4F15.5)
37   629 PCP=100.0-PCP
41       PCP0=100.0-PCP0
43       PRINT 25
46    25 FORMAT (//4X,20HAUXILIARY STATISTICS/36X,14HAT CONVERGENCE,13X,
        17HAT ZERO/4X)
46       PRINT 11,FL0,FL1
56    11 FORMAT (9X,19HVALUE OF L FUNCTION,7X,F15.5,5X,F15.5/4X)
56       PRINT 12,SRX2,SRX20
66    12 FORMAT (9X,19HRESIDUAL CHI-SQUARE,7X,F15.5,5X,F15.5/4X)
66       PRINT 13, SR2,SR20
76    13 FORMAT (9X,24HSUM OF SQUARED RESIDUALS,2X,F15.5,5X,F15.5/4X)
76       PRINT 14, DF,DF
06    14 FORMAT (9X,18HDEGREES OF FREEDOM,8X,F15.5, 5X,F15.5/4X)
06       PRINT 15,PCP,PCP0
16    15 FORMAT (9X,27HPERCENT CORRECTLY PREDICTED,4X,F10.5,10X,F10.5/4X)
16       IF (SR20.EQ.0.0) GO TO 101
23       R2=1.0-SR2/SR20
25       PRINT 19,R2
33    19 FORMAT (9X,28HCOEFFICIENT OF DETERMINATION,23X,F15.5/4X)
33       PRINT 53, FRM,FRM0
43    53 FORMAT (/9X,22HMEAN FORECASTING ERROR,4X,F15.5,5X,F15.5)
43       PRINT 54, RM2R,RM2R0
53    54 FORMAT (/9X,34HROOT MEAN SQUARE ERROR OF FORECAST,F15.5,5X,F15,5)
53       PRINT 55, TRM,TRM2,        TR2
55    55 FORMAT (//9X,30HFOR OFF-DIAGONAL ELEMENTS ONLY//12X,22HMEAN FORECA
        1STING ERROR,12X,F15.5//12X,34HROOT MEAN SQUARE ERROR OF FORECAST,
        2F15.5//12X,28HCOEFFICIENT OF DETERMINATION,6X,F15.5)
55   101 PRINT 20, (I,I=1,ND)
```

```
4       20 FORMAT (*1*,3X,13HMOMENT MATRIX//9X,7(11X,1H(,I2,1H))/9X,7(11X,     RLT15
          11H(,I2,1H)))                                                         RLT15
4          DO 631 I=1,ND                                                        RLT15
1          DO 632 J=1,ND                                                        RLT15
2      632 FH1(I,J)=-FH2(I,J)                                                   RLT15
6      631 PRINT 45, I, (FH1(I,J),J=1,ND)                                       RLT15
0       45 FORMAT (/4X,1H(,I2,1H),7E15.6/8X,7E15.6)                             RLT15
0       21 FORMAT (/4X,1H(,I2,1H),8I15)                                         RLT15
0.         IF (MM.EQ.1) GO TO 308
2          PRINT 22, (I,I=1,ND)                                                 RLT15
5       22 FORMAT (///4X,17HCOVARIANCE MATRIX//9X,7(11X,1H(,I2,1H))/9X,7(11X,  RLT15
          11H(,I2,1H)))                                                         RLT15
5          DO 633 I=1,ND                                                        RLT16
2          DO 634 J=1,ND                                                        RLT16
3      634 FHV(I,J)=CF*FHV(I,J)                                                 RLT16
2      633 PRINT 45, I,(FHV(I,J),J=1,ND)                                        RLT16
4      308 S=0.0
5          IF (NR.GT.7) GO TO 131                                               RLT16
1          PRINT 31, (I,I=1,NR)                                                 RLT16
3       31 FORMAT (*1*/4X,15HPREDICTION TEST//9X,19HINTER-REGIONAL FLOW//9X,   RLT16
          18(11X,I2,1H)))                                                       RLT16
3          IF (KTN.EQ.0) GO TO 711                                              RLT16
0          DO 712 I=1,NR                                                        RLT16
2          PRINT 21, I,(NQXP(J,I),J=1,NR),NQDP(I)                               RLT16
3      712 PRINT 32, (NQX(J,I),J=1,NR),NQD(I)                                   RLT17
2          PRINT 33, (NQOP(J),J=1,NR)                                           RLT17
5          PRINT 34, (NQO(J),J=1,NR)                                            RLT17
1          GO TO 801                                                            RLT17
5      711 DO 707 I=1,NR                                                        RLT17
7          PRINT 21, I,(NQXP(I,J),J=1,NR),NQOP(I)                               RLT17
7      707 PRINT 32, (NQX(I,J),J=1,NR),NQO(I)                                   RLT17
5       32 FORMAT (8X,8I15)                                                     RLT17
5,         PRINT 33, (NQDP(J),J=1,NR)                                           RLT17
0          PRINT 34, (NQD(J),J=1,NR)                                            RLT17
4       33 FORMAT (/4X,4HSUMP,8I15)                                             RLT17
4       34 FORMAT (4X,4HSUMT,8I15)                                              RLT18
4      801 GO TO 132                                                            RLT18
0      131 PRINT 31, (I,I=1,8)                                                  RLT18
1          IF (KTN.EQ.0) GO TO 133                                              RLT65
6          DO 134 I=1,NR                                                        RLT65
0          PRINT 21, I,(NQXP(J,I),J=1,8)                                        RLT66
5      134 PRINT 32, (NQX(J,I),J=1,8)                                           RLT66
0          PRINT 33, (NQOP(J),J=1,8)                                            RLT66
1          PRINT 34, (NQO(J),J=1,8)                                             RLT66
3          PRINT 31, (I,I=9,NR)                                                 RLT66
6          DO 135 I=1,NR                                                        RLT66
3          PRINT 21, I,(NQXP(J,I),J=9,NR),NQDP(I)                               RLT66
4      135 PRINT 32, (NQX(J,I),J=9,NR),NQD(I)                                   RLT66
3          PRINT 33, (NQOP(J),J=9,NR)                                           RLT66
6          PRINT 34, (NQO(J),J=9,NR)                                            RLT66
2          GO TO 132                                                            RLT67
6      133 DO 136 I=1,NR                                                        RLT67
0.         PRINT 21, I,(NQXP(I,J),J=1,8)                                        RLT67
4      136 PRINT 32, (NQX(I,J),J=1,8)                                           RLT67
6          PRINT 33, (NQDP(J),J=1,8)                                            RLT67
7          PRINT 34, (NQD(J),J=1,8)                                             RLT67
1          PRINT 31, (I,I=9,NR)                                                 RLT67
4          DO 137 I=1,NR                                                        RLT67
```

```
      PRINT 21, I,(NQXP(I,J),J=9,NR),NQOP(I)
  137 PRINT 32, (NQX(I,J),J=9,NR),NQO(I)
      PRINT 33, (NQDP(J),J=9,NR)
      PRINT 34, (NQD(J),J=9,NR)
  132 S=0.0
      DO 708 I=1,NR
      DO 708 J=1,NR
      T=(NQXP(I,J)-NQX(I,J))**2
      IF (NQX(I,J).LE.5) GO TO 709
  708 S=S+T/QX(I,J)
      X2P=S
      DFX2P=(NR-1)*(NR-1)
      PRINT 35, X2P,DFX2P
   35 FORMAT (///4X,26HCHI-SQUARE FOR PREDICTION=,F15.5/4X,19HDEGREES OF
     1 FREEDOM=,F15.5)
      GO TO 710
  709 PRINT 36
   36 FORMAT (///4X,91HCHI-SQUARE VALUE WAS NOT CALCULATED SINCE ONE OF
     1CELL FREQUENCIES WAS FOUND TO BE 5 OR LESS)
  710 PRINT 37
   37 FORMAT (//4X,89HROW NUMBERS INDICATE THE REGIONS OF ORIGIN, AND CO
     1LUMN NUMBERS THE REGIONS OF DESTINATION//4X,96HFOR EACH ENTRY THE
     2UPPER FIGURE DENOTES THE PREDICTED VALUE, AND THE LOWER FIGURE THE
     3 TRUE VALUE//4X,52HLAST ROW AND COLUMN ELEMENTS ARE THE RESPECTIVE
     4 SUMS)
      IF (KTN.NE.0) GO TO 150
      DO 112 I=1,NR
      DO 112 J=1,NR
  112 NQXP1(J,I)=NQXP(I,J)
      GO TO 1000
  150 IC=0
      IC=0
      IC=1
      IC=2
C     PUT *IC=1* FOR HIGA METHOD OR *IC=2* FOR FURNESS METHOD
      NID=6
      NID=20
C     PUT *NID=XX* HERE FOR THE NUMBER OF ITERATIONS.
      DO 113 I=1,NR
      DO 113 J=1,NR
  113 QXP(I,J)=(NQXP(I,J)+NQXP1(I,J))/2.0
      IF (IC.EQ.1) GO TO 151
      ID=0
  158 DO 152 J=1,NR
      S=0.0
      DO 153 I=1,NR
  153 S=S+QXP(I,J)
      DO 154 I=1,NR
  154 QXP(I,J)=QXP(I,J)*NQD(J)/S
  152 CONTINUE
      DO 155 I=1,NR
      S=0.0
      DO 156 J=1,NR
  156 S=S+QXP(I,J)
      DO 157 J=1,NR
  157 QXP(I,J)=QXP(I,J)*NQD(I)/S
  155 CONTINUE
      ID=ID+1
```

```
      IF (ID.LE.NID) GO TO 158
  151 IF (IC.EQ.2) GO TO 171
      ID=0
  175 DO 160 J=1,NR
      IE=0
  164 S1=0.0
      S2=0.0
      DO 161 I=1,NR
      IF (QXP(I,J).LE.0.0) QXP(I,J)=0.0
      IF (QXP(I,J).EQ.0.0) GO TO 161
      S1=S1+QXP(I,J)
      S2=S2+NQO(I)
  161 CONTINUE
      S3=NQD(J)-S1
      IF (S3.LE.0.0) GO TO 110
      S2=0.0
      DO 111 I=1,NR
  111 S2=S2+NQO(I)
  110 IF (S3.LT.0.5.AND.S3.GT.-0.5) GO TO 162
      DO 163 I=1,NR
      IF (S3.GT.0.0) GO TO 165
      IF (QXP(I,J).EQ.0.0) GO TO 163
  165 QXP(I,J)=QXP(I,J)+S3*NOO(I)/S2
  163 CONTINUE
      IE=IE+1
      IF (IE.LE.NID) GO TO 164
  160 CONTINUE
  162 DO 166 I=1,NR
      IE=0
  174 S1=0.0
      S2=0.0
      DO 167 J=1,NR
      IF (QXP(I,J).LE.0.0) QXP(I,J)=0.0
      IF (QXP(I,J).EQ.0.0) GO TO 167
      S1=S1+QXP(I,J)
      S2=S2+NOD(J)
  167 CONTINUE
      S3=NOO(I)-S1
      IF (S3.LE.0.0) GO TO 169
      S2=0.0
      DO 170 J=1,NR
  170 S2=S2+NOD(J)
  169 IF (S3.LT.0.5.AND.S3.GT.-0.5) GO TO 171
      DO 172 J=1,NR
      IF (S3.GT.0.0) GO TO 173
      IF (QXP(I,J).EQ.0.0) GO TO 172
  173 QXP(I,J)=QXP(I,J)+S3*NOD(J)/S2
  172 CONTINUE
      IE=IE+1
      IF (IE.LE.5) GO TO 174
  166 CONTINUE
      ID=ID+1
      IF (ID.LE.3) GO TO 175
  171 DO 176 I=1,NR
      DO 176 J=1,NR
  176 NQXP(I,J)=QXP(I,J)+0.5
      DO 177 I=1,NR
      S1=0.0
```

```
17          S2=0.0                                                          RLT7
20          DO 178 J=1,NR                                                   RLT7
21          S1=S1+QXP(I,J)                                                  RLT7
25      178 S2=S2+QXP(J,I)                                                  RLT7
34          NQOP(I)=S1+0.5                                                  RLT7
40      177 NQDP(I)=S2+0.5                                                  RLT7
44          S=0.0                                                           RLT6
45          DO 118 I=1,NR                                                   RLT6
46      118 S=S+NQDP(I)                                                     RLT6
53          NQTP=S                                                          RLT6
55          S1=0.0                                                          RLT6
56          S2=0.0                                                          RLT6
57          S3=0.0                                                          RLT6
60          T=0.0                                                           RLT
61          DO 116 I=1,NR                                                   RLT
62          DO 116 J=1,NR                                                   RLT
63          T=T+NQX(I,J)                                                    RLT
70          S=NQXP(I,J)-NQX(I,J)                                            RES
76          S3=S3+S                                                         RLT
00          S1=S1+S**2                                                      RLT
02          IF (S.LT.0.0) GO TO 116                                         RLT
03          S2=S2+S                                                         RLT
04      116 CONTINUE                                                        RLT
11          PCP=100.0-S2*100.0/T                                            RL
14          R2=1.0-S1/((SR21+SR20)/2.0)                                     RL
21          FRM=S3/(NR*NR)                                                  RL
26          RM2R=S1/(NR*NR)                                                 RL
32          RM2R=SQRT(RM2R)                                                 RL
34          NQTT=T                                                          RL
36          S1=0.0                                                          RE
37          S2=0.0                                                          RE
40          S3=0.0                                                          RE
41          DO 203 J=1,NR                                                   RF
46          T=NQD(J)-NQX(J,J)                                               RES
53          DO 204 I=1,NR                                                   RES
54          IF (I.EQ.J) GO TO 204                                           RES
56          S1=S1+(NQXP(I,J)-NQX(I,J))                                      RES
65          S2=S2+(NQXP(I,J)-NQX(I,J))**2                                   RES
73          S3=S3+(T/(NR-1)-NQX(I,J))**2                                    RES
04      204 CONTINUE                                                        RES3
07      203 CONTINUE                                                        RES3
11          TRM=S1/(NR*NR-NR)                                               RES
15          S=S2/(NR*NR-NR)                                                 RES
22          TRM2=SQRT(S)                                                    RES
24          S=S3/(NR*NR-NR)                                                 RES
31          TRM20=SQRT(S)                                                   RF
33          TR2=1.0-S2/S3                                                   RES
36          IF (NR.GT.7) GO TO 138                                          RLT6
45          PRINT 51, (I,I=1,NR)                                            RLT6
57       51 FORMAT (*1*,69HPREDICTTON WITH TWO METHODS COMBINED, AND DOUBLE CO RLT6
           1NSTRAINTS IMPOSED.//9X,8(11X,1H(,I2,1H)))                       RLT6
57          DO 117 I=1,NR                                                   RLT6
64          PRINT 21, I,(NQXP(J,I),J=1,NR),NQDP(I)                          RLT6
75      117 PRINT 32, (NQX(J,I),J=1,NR),NQD(I)                              RLT6
24          PRINT 33, (NQOP(J),J=1,NR),NQTP                                 RLT6
30          PRINT 34, (NQO(J),J=1,NR),NQTT                                  RLT6
35          GO TO 139                                                       RLT6
71      138 PRINT 51, (I,I=1,8)                                             RLT6
```

```
      DO 140 I=1,NR                                                     RLT68
      PRINT 21, I,(NQXP(J,I),J=1,8)                                     RLT68
  140 PRINT 32, (NOX(J,I),J=1,8)                                        RLT68
      PRINT 33, (NQOP(J),J=1,8)                                         RLT69
      PRINT 34, (NQO(J),J=1,8)                                          RLT68
      PRINT 51, (I,I=9,NR)                                              RLT69
      DO 141 I=1,NR                                                     RLT69
      PRINT 21, I,(NQXP(J,I),J=9,NR),NQDP(I)                            RLT69
  141 PRINT 32, (NQX(J,I),J=9,NR),NQD(I)                                RLT69
      PRINT 33, (NQOP(J),J=9,NR),NQTP                                   RLT69
      PRINT 34, (NQO(J),J=9,NR),NQTT                                    RLT69
  139 PRINT 37                                                          RLT69
      PRINT 52                                                          RLT65
   52 FORMAT (///)                                                      RLT65
      PRINT 15, PCP                                                     RLT65
      PRINT 19, R2                                                      RLT65
      PRINT 53, FRM                                                     RLT20
      PRINT 54, RM2R                                                    RLT20
      PRINT 55, TRM,TRM2,       TR2
 1000 RETURN                                                            RLT20
      END                                                               RLT20
```

```
      SUBROUTINE NLMAXH(ND,IT,ICONV,NR,NX,NY,MSTP,INIT,NH,FL0,B0,B1,B2,
     1BZ,FG0,FG1,FG2,FG3,FGZ,FH1,FH2,FHV,DIR,S4,LX,MX,JK,JS,JT,GM,NOD,
     2WT,NAP,NU1,NU2)
      DIMENSION B0(10),B1(10),B2(10),BZ(10),FG0(10),FG1(10),FG2(10),
     1FG3(10),FGZ(10),FH1(10,10),FH2(10,10),FHV(10,10),DIR(10),S4(10),
     2LX(10),MX(10),WT(14)
      COMMON /AA1/ X(14,14,5),Y(14,10),P(14,14),QX(14,14),QD(14),Q0(14),
     1BR(10),FHX(10,10),Y0(14,10),YD(14,10)
      COMMON /AA2/ AR(100),SIR(4)
      COMMON /AA3/ BX(5),CY(10),XM(14,5),YM(14,10),XXM(14,5,5),XYM(14,5
     1,10),YYM(14,10,10),DQP(14,14),DOP(14),DXM(14,14,5),DYM(14,14,10),
     2DSX(14,5),DSY(14,10),NOW
      IA=1
      JV=0
      DO 501 I=1,ND
      FGZ(I)=FG0(I)
  501 BZ(I)=B0(I)
      FLZ=FL0
      DO 502 I=1,ND
      S=0.0
      DO 503 J=1,ND
  503 S=S+FHV(I,J)*FG0(J)
  502 DIR(I)=S
      NH=0
      DO 201 I=1,ND
  201 FG1(I)=B0(I)
      SIR(1)=FL0
      JJ=1
      ST1=1.0
  317 ST2=ST1+ST1
      DO 202 I=1,ND
  202 FG2(I)=FG1(I)+ST1*DIR(I)
      NG=0
      CALL GRVMOD (INIT,NG,NH,NX,NY,ND,NR,FG2,FL1,S4,FH1,WT,NU1,NU2)
      SIR(2)=FL1
      IF (FL1-FL0) 101,102,103
  101 ST1=-ST1
      DO 204 I=1,ND
  204 FG3(I)=FG1(I)+ST1*DIR(I)
      CALL GRVMOD(INIT,NG,NH,NX,NY,ND,NR,FG3,FL2,S4,FH1,WT,NU1,NU2)
      IF (FL2.GT.FL0) GO TO 311
      ST3=ST1
      ST2=ST1+ST1
      DO 312 I=1,ND
      B0(I)=FG2(I)
      B1(I)=FG1(I)
  312 B2(I)=FG3(I)
      FL3=FL2
      FL2=FL0
      GO TO 120
  311 DO 313 I=1,ND
  313 FG2(I)=FG3(I)
      FL1=FL2
      ST2=ST1+ST1
      GO TO 103
  102 ST3=ST1/2.0
      ST2=ST1
```

```
      DO 216 I=1,ND                                                     MAX042
  216 FG0(I)=FG1(I)+ST3*DIR(I)                                          MAX043
      CALL GRVMOD (INIT,NG,NH,NX,NY,ND,NR,FG0,FL2,S4,FH1,WT,NU1,NU2)    MAX044
      IF (FL2.GE.FL0) GO TO 217                                         MAX046
      PRINT 10,     IT,(FG0(I),I=1,ND)
   10 FORMAT (//4X,47HTHE FUNCTION IS NOT CONCAVE AT ITERATION NUMBER,  MAX048
     1I5,3HAND//4X,2HB=,10F10.5)                                        MAX049
      DO 218 I=1,ND                                                     MAX050
  218 B0(I)=FG2(I)                                                      MAX051
      GO TO 1002                                                        MAX052
  217 DO 219 I=1,ND                                                     MAX053
      B0(I)=FG1(I)                                                      MAX054
      B1(I)=FG0(I)                                                      MAX055
  219 B2(I)=FG2(I)                                                      MAX056
      FL1=S1R(1)                                                        MAX057
      FL3=S1R(2)                                                        MAX058
      GO TO 120                                                         MAX059
  103 DO 205 I=1,ND                                                     MAX060
  205 FG3(I)=FG2(I)+ST2*DIR(I)                                          MAX061
      CALL GRVMOD (INIT,NG,NH,NX,NY,ND,NR,FG3,FL2,S4,FH1,WT,NU1,NU2)    MAX062
      S1R(4)=FL2                                                        MAX064
  215 S1=S1R(2)                                                         MAX065
      S2=S1R(4)                                                         MAX066
      IF (S2-S1) 111,111,113                                            MAX067
  111 ST3=ST2/2.0                                                       MAX068
      DO 206 I=1,ND                                                     MAX069
  206 FG0(I)=FG2(I)+ST3*DIR(I)                                          MAX070
      CALL GRVMOD (INIT,NG,NH,NX,NY,ND,NR,FG0,FL3,S4,FH1,WT,NU1,NU2)    MAX071
      S1R(3)=FL3                                                        MAX073
      IF (S1R(2).LT.S1R(3)) GO TO 208                                   MAX074
      DO 209 I=1,ND                                                     MAX082
      B0(I)=FG1(I)                                                      MAX083
      B1(I)=FG2(I)                                                      MAX084
  209 B2(I)=FG0(I)                                                      MAX085
      FL1=S1R(1)                                                        MAX286
      FL2=S1R(2)                                                        MAX087
      FL3=S1R(3)                                                        MAX088
      GO TO 120                                                         MAX089
  208 DO 210 I=1,ND                                                     MAX090
      B0(I)=FG2(I)                                                      MAX091
      B1(I)=FG0(I)                                                      MAX092
  210 B2(I)=FG3(I)                                                      MAX093
      FL1=S1R(2)                                                        MAX094
      FL2=S1R(3)                                                        MAX095
      FL3=S1R(4)                                                        MAX096
      GO TO 120                                                         MAX097
  113 DO 211 I=1,ND                                                     MAX098
      FG1(I)=FG2(I)                                                     MAX099
  211 FG2(I)=FG3(I)                                                     MAX100
      S1R(1)=S1R(2)                                                     MAX101
      S1R(2)=S1R(4)                                                     MAX102
      ST2=ST2+ST2                                                       MAX103
      DO 212 I=1,ND                                                     MAX104
  212 FG3(I)=FG2(I)+ST2*DIR(I)                                          MAX105
      CALL GRVMOD (INIT,NG,NH,NX,NY,ND,NR,FG3,FL2,S4,FH1,WT,NU1,NU2)    MAX106
      S1R(4)=FL2                                                        MAX108
      JJ=JJ+1                                                           MAX109
      IF (JJ.LE.6) GO TO 215                                            MAX110
```

```
      JS=1
      GO TO 1000
  120 IA=IA+1
      IF (IA.GE.3) GO TO 315
      IF (IA.GE.4) GO TO 315
C     PUT *IF (IA.GE.4) GO TO 315* HERE.
      ST1=ST3/10.0
      DO 316 I=1,ND
  316 FG1(I)=B1(I)
      FL0=FL2
      S1R(1)=FL0
      GO TO 317
  315 R1=(FL1-FL2)/(-ST3)
      R2=(FL1-FL3)/(-ST2)
      ST5=ST3
   21 FORMAT (///4X,20HFL1,FL2,FL3,ST2,ST3=/9X,5E15.6)
      PRINT 21, FL1,FL2,FL3,ST2,ST3
C     PUT *GO TO 121* HERE FOR SKIPPING THE INTERPOLATION
      IF (R1.NE.R2) GO TO 301
      IF (R1.NE.0.0) GO TO 121
      ST4=ST3*1.0E-5
      ST3=ST3+ST4
      DO 302 I=1,ND
  302 B1(I)=B1(I)+ST4*DIR(I)
      CALL GRVMOD (INIT,NG,NH,NX,NY,ND,NR,B1,FL2,S4,FH1,WT,NU1,NU2)
      R1=(FL1-FL2)/(-ST3)
      R2=(FL1-FL3)/(-ST2)
      ST5=ST2/2.0-ST4
  301 S=0.0
      SC1=(R1*(-ST5))/(2.0*(R1-R2))
      DO 220 I=1,ND
  220 B0(I)=(B0(I)+B1(I))/2.0-SC1*DIR(I)
 1002 NG=1
      NH=1
      CALL GRVMOD (INIT,NG,NH,NX,NY,ND,NR,B0,FL0,FG0,FHV,WT,NU1,NU2)
      IF (FL0.GE.FL2) GO TO 404
  121 DO 405 I=1,ND
  405 B0(I)=B1(I)
      NG=1
      NH=1
      CALL GRVMOD(INIT,NG,NH,NX,NY,ND,NR,B0,FL0,FG0,FHV,WT,NU1,NU2)
  404 S=0.0
      DO 513 I=1,ND
  513 S=S+FG0(I)*FG0(I)
      S=S/ND
      IF (S.EQ.0.0) GO TO 514
      S=SQRT(S)
  514 GM=S
      S=0.0
      DO 230 I=1,ND
  230 S=S+(B0(I)-BZ(I))**2
      S=S/ND
      IF (S.EQ.0.0) GO TO 221
      S=SQRT(S)
  221 STZ=S
      BMAX=ABS(BZ(1)-B0(1))
      DO 231 I=1,ND
      S=ABS(BZ(I)-B0(I))
```

```
22         IF (S.LE.BMAX) GO TO 231                                           MAX08
25         BMAX=S                                                             MAX08
25     231 CONTINUE                                                           MAX08
30         FLCH=(FLZ-FLO)*100.0/FLO                                           MAX08
33         IF (NAP.NE.1) GO TO 306
36         DO 307 I=1,ND
37         DO 307 J=1,ND
40         IF (I.NE.J) FHV(I,J)=0.0
45         IF (I.EQ.J) FHV(I,J)=1.0/FHV(I,J)
53     307 CONTINUE
60         GO TO 528
60     306 S=0.0
61         CALL MINV (FHV,ND,D,LX,MX,N2)                                      MAX08
67         IF (GM.LE.1.0E-15) GO TO 1005
76         IF (D.EQ.0.0) JK=1                                                 MAX08
00     528 S=0.0
01         IF (ABS((FLZ-FLO)/FLO).GT.1.0E-6) GO TO 516                        MAX08
07         IF (BMAX.GE.1.0E-6) GO TO 516                                      MAX08
11    1005 NOD=3
13         JT=1                                                               MAX08
14         GO TO 516                                                          MAX09
15     516 PRINT 11,     IT,STZ,BMAX,GM,FLO,FLCH                              MAX09
35      11 FORMAT (//4X,17HITERATION NUMBER ,I3,2X,16HNEWTON-HIGA MODE/8X,    MAX09
          150HROOT MEAN SQUARE OF CHANGE IN PARAMETER ESTIMATES=,G15.6/8X,    EST16
          219HMAXIMUM ADJUSTMENT=,G15.6/8X,29HROOT MEAN SQUARE OF GRADIENT=,  MAX16
          3G15.6/8X,18HVALUE OF FUNCTION=,G15.8/8X,27HFUNCTION VALUE INCREASE
          1D BY,G10.3,2X,9H PERCENT.)
35    1000 RETURN                                                             MAX10
36         END                                                                MAX10
```

APPENDIX C

POLYREG:

Computer Program for Aggregation Function

```
      PROGRAM POLYREG (INPUT,OUTPUT)                                    PLY001
      DIMENSION XXV(10,10),LX(10),MX(10)                                PLY002
      COMMON /AA1/ YD(100,4),XD(100,4),Y(100),X(100,10),XX(10,10),XY(10) PLY003
     1,B(10,4),BSE(10,4),TV(10),RE(100),YP(100,4),YPP(100,4),YM(5)     PLY004
      COMMON /AA2/ A(100)                                               PLY005
      READ 1, NT,NV,NALT                                                PLY006
      NSET=1
  302 DO 101 I=1,NT                                                     PLY007
  101 READ 2, (YD(I,J),J=1,NALT),(XD(I,J),J=1,NALT)                     PLY008
    1 FORMAT (8I10)                                                     PLY009
    2 FORMAT (8F10.0)                                                   PLY010
      DO 301 II=1,NV                                                    PLY011
      NV1=II+1                                                          PLY011
      DO 200 M=1,NALT                                                   PLY011
      DO 102 J=1,NV1                                                    PLY012
      IF (J.EQ.1) GO TO 103                                             PLY013
      IF (J.EQ.2) GO TO 104                                             PLY014
      N=J-1                                                             PLY015
      DO 105 I=1,NT                                                     PLY016
  105 X(I,J)=XD(I,M)**N                                                 PLY017
      GO TO 102                                                         PLY018
  103 DO 106 I=1,NT                                                     PLY019
  106 X(I,J)=1.0                                                        PLY020
      GO TO 102                                                         PLY021
  104 DO 107 I=1,NT                                                     PLY022
  107 X(I,J)=XD(I,M)                                                    PLY023
  102 CONTINUE                                                          PLY024
      DO 201 I=1,NT                                                     PLY024
  201 Y(I)=YD(I,M)                                                      PLY024
      DO 108 I=1,NV1                                                    PLY025
      DO 108 J=1,NV1                                                    PLY026
      S=0.0                                                             PLY027
      DO 109 K=1,NT                                                     PLY028
  109 S=S+X(K,I)*X(K,J)                                                 PLY029
  108 XX(I,J)=S                                                         PLY030
      DO 110 I=1,NV1                                                    PLY031
      S=0.0                                                             PLY032
      DO 111 K=1,NT                                                     PLY033
  111 S=S+X(K,I)*Y(K)                                                   PLY034
  110 XY(I)=S                                                           PLY035
      DO 112 I=1,NV1                                                    PLY036
      DO 112 J=1,NV1                                                    PLY037
  112 XXV(I,J)=XX(I,J)                                                  PLY038
      N2=NV1*NV1                                                        PLY039
      CALL MINV (XXV,NV1,D,LX,MX,N2)                                    PLY040
      DO 113 I=1,NV1                                                    PLY041
      S=0.0                                                             PLY042
      DO 114 J=1,NV1                                                    PLY043
  114 S=S+XXV(I,J)*XY(J)                                                PLY044
  113 B(I,M)=S                                                          PLY045
      DO 117 I=1,NT                                                     PLY046
      S=0.0                                                             PLY047
      DO 118 J=1,NV1                                                    PLY048
  118 S=S+X(I,J)*B(J,M)                                                 PLY049
      YP(I,M)=S                                                         PLY050
  117 RE(I)=Y(I)-S                                                      PLY051
      S=0.0                                                             PLY052
```

```
      DO 119 I=1,NT
  119 S=S+RE(I)**2
      SR2=S
      VAR=S/(NT-NV1)
      SE=SQRT(VAR)
      DO 115 I=1,NV1
      IF (XXV(I,I).EQ.0.0) GO TO 116
      BSE(I,M)=SQRT(VAR*XXV(I,I))
      TV(I)=B(I,M)/BSE(I,M)
      GO TO 115
  116 BSE(I,M)=0.0
  115 CONTINUE
      S=0.0
      DO 120 I=1,NT
  120 S=S+Y(I)
      YM(M)=S/NT
      S=0.0
      DO 121 I=1,NT
  121 S=S+(Y(I)-YM(M))**2
      R2=1.0-SR2/S
      PRINT 10, NSET
   10 FORMAT (*1*,4X,10HDATA SET =,I3)
      PRINT 11, NT,II
   11 FORMAT (//4X,8X,2HNT,4X,6HDEGREE//4X,2I10)
      PRINT 12
   12 FORMAT (///4X,4HDATA//4X,13X,7HDEP VAR,23X,7HIND VAR)
      PRINT 13
      N=0
      DO 122 I=1,NT
      N=N+1
      IF (N.NE.6) GO TO 122
      PRINT 13
   13 FORMAT (* *)
      N=1
  122 PRINT 14, I,Y(I),XD(I,M)
   14 FORMAT (4X,1H(,I3,1H),F15.6,15X,F15.6)
      PRINT 15
   15 FORMAT (*1*,4X,19HPARAMETER ESTIMATES///19X,5HCOEFF,8X,7HST.ERR.,
     18X,7HT-VALUE//)
      DO 123 I=1,NV1
  123 PRINT 16, I,B(I,M),BSE(I,M),TV(I)
   16 FORMAT (4X,1H(,I3,1H),3F15.6)
      PRINT 17, SE
   17 FORMAT (//4X,18HESTIMATE OF SIGMA=,F15.6)
      PRINT 18, R2
   18 FORMAT (//4X,29HCOEFFICIENT OF DETERMINATION=,F15.6)
      PRINT 19
   19 FORMAT (*1*,4X,13HMOMENT MATRIX/)
      DO 124 I=1,NV1
  124 PRINT 20, I,(XX(I,J),J=1,NV1)
   20 FORMAT (4X,1H(,I3,1H),8E15.6)
      PRINT 21
   21 FORMAT (///4X,17HCOVARIANCE MATRIX/)
      DO 125 I=1,NV1
      DO 126 J=1,NV1
  126 XXV(I,J)=XXV(I,J)*VAR
  125 PRINT 20, I,(XXV(I,J),J=1,NV1)
      PRINT 22
```

```
   22 FORMAT (*1*,4X,45HESTIMATES OF DEPENDENT VARIABLE AND RESIDUALS//    PLY109
     14X,12X,8HDEP.VAR.,6X,9HEST OF DV,7X,8HRESIDUAL)                      PLY110
      PRINT 13                                                             PLY111
      N=0                                                                  PLY112
      DO 127 I=1,NT                                                        PLY113
      N=N+1                                                                PLY114
      IF (N.NE.6) GO TO 127                                                PLY115
      PRINT 13                                                             PLY116
      N=1                                                                  PLY117
  127 PRINT 20, I,Y(I),YP(I,M),RE(I)                                       PLY118
  200 CONTINUE                                                             PLY1181
      S1=0.0                                                               PLY119
      S2=0.0                                                               PLY120
      S3=0.0
      DO 203 I=1,NT                                                        PLY121
      S=0.0                                                                PLY122
      DO 204 J=1,NALT                                                      PLY123
      IF (YP(I,J).GT.1.0) YP(I,J)=1.0                                      PLY1231
      IF (YP(I,J).LT.0.0) YP(I,J)=0.0                                      PLY1232
  204 S=S+YP(I,J)                                                          PLY124
      DO 205 J=1,NALT                                                      PLY125
      YPP(I,J)=YP(I,J)/S                                                   PLY126
      S1=S1+(YD(I,J)-YPP(I,J))**2                                          PLY127
      S3=S3+(YD(I,J)-1.0/NALT)**2
  205 S2=S2+(YD(I,J)-YM(J))**2                                             PLY128
  203 CONTINUE                                                             PLY129
      K=NT*(NALT-1)-NALT*NV1                                               PLY130
      VAR=S1/K                                                             PLY131
      SE=SQRT(VAR)                                                         PLY132
      R2=1.0-S1/S2                                                         PLY133
      R2P=1.0-S1/S3
      PRINT 30, (I,I=1,NALT),(J,J=1,NALT)                                  PLY134
   30 FORMAT (*1*,4X,57HADJUSTED ESTIMATES OF AGGREGATE MODE-CHOICE PROB  PLY135
     1ABILITIES//9X,10HTRUE VALUE,35X,8HESTIMATE//9X,3(12X,1H(,I1,1H)),   PLY136
     23(12X,1H(,I1,1H))/)                                                  PLY137
      N=0                                                                  PLY138
      DO 206 I=1,NT                                                        PLY139
      N=N+1                                                                PLY140
      IF (N.NE.6) GO TO 206                                                PLY141
      N=1                                                                  PLY142
      PRINT 13                                                             PLY143
  206 PRINT 31, I,(YD(I,J),J=1,NALT),(YPP(I,J),J=1,NALT)                   PLY144
   31 FORMAT (4X,1H(,I3,1H),6F15.6)                                        PLY145
      PRINT 17,SE                                                          PLY146
      PRINT 18, R2                                                         PLY147
      PRINT 23, R2P
   23 FORMAT (//4X,45HCOEFFICIENT OF DETERMINATION IN PROBABILITY =,
     1F15.6)
  301 CONTINUE
      NSET=NSET+1
      IF (NSET.LE.2) GO TO 302
      STOP                                                                 PLY148
      END                                                                  PLY149
```

```
      SUBROUTINE MINV (AH,N,D,L,M,N2)
      DIMENSION AH(10,10),L(10),M(10)
      COMMON /AA2/ A(100)
      K=0
      DO 201 I=1,N
      DO 201 J=1,N
      K=K+1
  201 A(K)=AH(J,I)
      D=1.0
      NK=-N
      DO 80 K=1,N
      NK=NK+N
      L(K)=K
      M(K)=K
      KK=NK+K
      BIGA=A(KK)
      DO 20 J=K,N
      IZ=N*(J-1)
      DO 20 I=K,N
      IJ=IZ+I
      S=ABS(A(IJ))
      T=ABS(BIGA)
   10 IF (T-S) 15,20,20
   15 BIGA=A(IJ)
      L(K)=I
      M(K)=J
   20 CONTINUE
      J=L(K)
      IF (J-K) 35,35,25
   25 KI=K-N
      DO 30 I=1,N
      KI=KI+N
      HOLD=-A(KI)
      JI=KI-K+J
      A(KI)=A(JI)
   30 A(JI)=HOLD
   35 I=M(K)
      IF (I-K) 45,45,38
   38 JP=N*(I-1)
      DO 40 J=1,N
      JK=NK+J
      JI=JP+J
      HOLD=-A(JK)
      A(JK)=A(JI)
   40 A(JI)=HOLD
      T=ABS(BIGA)
   45 IF (T-1.0E-20) 46,46,48
   46 D=0.0
      RETURN
   48 DO 55 I=1,N
      IF (I-K) 50,55,50
   50 IK=NK+I
      A(IK)=A(IK)/(-BIGA)
   55 CONTINUE
      DO 65 I=1,N
      IK=NK+I
      HOLD = A(IK)
```

```
      IJ=I-N                                                            INV054
      DO 65 J=1,N                                                       INV055
      IJ =IJ+N                                                          INV056
      IF (I-K) 60,65,60                                                 INV057
   60 IF (J-K) 62,65,62                                                 INV058
   62 KJ=IJ-I+K                                                         INV059
      A(IJ)=HOLD*A(KJ)+A(IJ)                                            INV060
   65 CONTINUE                                                          INV061
      KJ=K-N                                                            INV062
      DO 75 J=1,N                                                       INV063
      KJ=KJ+N                                                           INV064
      IF (J-K) 70,75,70                                                 INV065
   70 A(KJ)=A(KJ)/BIGA                                                  INV066
   75 CONTINUE                                                          INV067
      D=D*BIGA                                                          INV068
      A(KK)=1.0/BIGA                                                    INV069
   80 CONTINUE                                                          INV070
      K=N                                                               INV071
  100 K=(K-1)                                                           INV072
      IF (K) 150,150,105                                                INV073
  105 I=L(K)                                                            INV074
      IF (I-K) 120,120,108                                              INV075
  108 JQ=N*(K-1)                                                        INV076
      JR=N*(I-1)                                                        INV077
      DO 110 J=1,N                                                      INV078
      JK=JQ+J                                                           INV079
      HOLD=A(JK)                                                        INV080
      JI=JR+J                                                           INV081
      A(JK)=-A(JI)                                                      INV082
  110 A(JI)=HOLD                                                        INV083
  120 J=M(K)                                                            INV084
      IF (J-K) 100,100,125                                              INV085
  125 KI=K-N                                                            INV086
      DO 130 I=1,N                                                      INV087
      KI=KI+N                                                           INV088
      HOLD = A(KI)                                                      INV089
      JI=KI-K+J                                                         INV090
      A(KI)=-A(JI)                                                      INV091
  130 A(JI)=HOLD                                                        INV092
      GO TO 100                                                         INV093
  150 K=0                                                               INV094
      DO 202 I=1,N                                                      INV095
      DO 202 J=1,N                                                      INV096
      K=K+1                                                             INV097
  202 AH(J,I)=A(K)                                                      INV098
      RETURN                                                            INV099
      END                                                               INV100
```